International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 3, Issue 9, September 2015, PP 1-7
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

An Application of Games under Uncertainity in Marketing

S. C. Sharma[1]
Department of Mathematics
University of Rajasthan
Jaipur, India
sharma-sc@uniraj.ernet.in

Ganesh Kumar[2]
Department of Mathematics
University of Rajasthan
Jaipur, India
ganeshmandha1988@gmail.com

Abstract: *In real-world games, the agents are often lack of the information about the other agent's payoffs. In these problems all entries of payoff matrices are uncertain variables; this paper introduces a concept of uncertain fuzzy game, the correspondingpayoffs may be within certainranges ratherthanexact values.In this paper the method of determining the value of the game in fuzzy environment, using trapezoidal fuzzy numbers is proposed. An approach called α -cut method which provides a way for converting the given fuzzy game into the interval data problem. Relevant definitions and numerical examples are also given.*

Keywords: *Trapezoidal fuzzy number, interval data, saddle point, uncertain, strategy.*

1. Introduction

Game theory had its beginnings in the 1920s, and significantly advanced at Princeton University through the work of John Nash [1] and [2]. The method of finding the fuzzy gamevalue using intervals as elements of matrix is explained in [3, 5]. Definitions of intervals and interval arithmetic are studied in [4].By comparing the intervalsand using dominance principle in [6] a fuzzy environment we find the fuzzy game value of the interval valuedmatrix.

2. A Two Person Zero Sum Fuzzy Game

A two person zero sum fuzzy game can be defined as,

$FG \approx (\tilde{S}_1, \tilde{S}_2, \tilde{K}, \tilde{A})$ Where,

$$\tilde{s}_1 \approx \{\tilde{a} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4) : \tilde{a}_i \succeq \tilde{0}, \forall i, \sum_{i=1}^{m} \tilde{a}_i = \tilde{I}\}$$

$\tilde{s}_2 \approx \{\tilde{a} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4) : \tilde{b}_j \succeq \tilde{0}, \forall j, \sum_{j=1}^{n} \tilde{b}_i = \tilde{I}\}$ Are the strategy spaces for players 1 and 2 respectively. Then the fuzzy payoff or the fuzzy gain for maximizing player 1 is given by,

$\tilde{K}(\tilde{a}, \tilde{b}) \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{a}_i \otimes \tilde{b}_j \approx \tilde{a}^T \otimes \tilde{A} \otimes \tilde{b}$ Where $\tilde{A}$ is said to be payoff matrix for the maximizing player.

3. Trapezoidal Fuzzy Number Membership Function

$$\mu_A(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)}; a_1 \le x \le a_2 \\ 1; a_2 \le x \le a_3 \\ \dfrac{(x - a_4)}{(a_3 - a_4)}; a_3 \le x \le a_4 \\ 0; otherwise \end{cases}$$

3.1Ordered Relation on Fuzzy Numbers,

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be any two symmetric fuzzy numbers. Let us define $\tilde{a} \preceq \tilde{b}$ if and only if $a_1 \le b_1, a_2 \le b_2, a_3 \le b_3, a_4 \le b_4,$ we can verify that $\preceq$ is a partial ordered relation.

3.2 Operations on Fuzzy Numbers:

For two fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ the following operations are defined,

Addition:

$$\tilde{a} \oplus \tilde{b} \approx (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

Subtraction:

$$\tilde{a} \Theta \tilde{b} \approx (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

Multiplication:

$$\tilde{a} \otimes \tilde{b} \approx \left(\frac{a_1(b_1+b_2+b_3+b_4)}{4}, \frac{a_2(b_1+b_2+b_3+b_4)}{4}, \frac{a_3(b_1+b_2+b_3+b_4)}{4}, \frac{a_4(b_1+b_2+b_3+b_4)}{4}\right) \quad \text{If } \tilde{b} \ge \tilde{0}$$

$$\tilde{a} \otimes \tilde{b} \approx \left(\frac{a_4(b_1+b_2+b_3+b_4)}{4}, \frac{a_3(b_1+b_2+b_3+b_4)}{4}, \frac{a_2(b_1+b_2+b_3+b_4)}{4}, \frac{a_1(b_1+b_2+b_3+b_4)}{4}\right) \quad \text{If } \tilde{b} \le \tilde{0}$$

Division:

$$\tilde{a} / \tilde{b} \approx \left(\frac{4a_1}{(b_1+b_2+b_3+b_4)}, \frac{4a_2}{(b_1+b_2+b_3+b_4)}, \frac{4a_3}{(b_1+b_2+b_3+b_4)}, \frac{4a_4}{(b_1+b_2+b_3+b_4)}\right) \quad \text{If } \tilde{b} \ge \tilde{0}$$

$$\tilde{a} / \tilde{b} \approx \left(\frac{4a_4}{(b_1+b_2+b_3+b_4)}, \frac{4a_3}{(b_1+b_2+b_3+b_4)}, \frac{4a_2}{(b_1+b_2+b_3+b_4)}, \frac{4a_1}{(b_1+b_2+b_3+b_4)}\right) \quad \text{If } \tilde{b} \le \tilde{0}$$

In case of division $\tilde{b}$ is not zero fuzzy number.

Scalar multiplication:

If $k \ne 0$ is a scalar, then scalar multiplication is defined as,

$$k\tilde{a} \approx \begin{cases} (ka_1, ka_2, ka_3, ka_4); k > 0 \\ (ka_4, ka_3, ka_2, ka_1); k < 0 \end{cases}$$

4. THE INTERVAL NUMBER SYSTEM

Closed interval denoted by $[X, Y]$ is the set of real numbers given by,

$$[X, Y] = \{x \in \mathbb{R} : X \le x \le Y\}$$

Although various other types of intervals (open, half-open) appear throughout mathematics, our work will center primarily on closed intervals. In this paper, the term intervalwill mean closedinterval. We will adopt the convention of denoting intervals and their endpoints by capital letters. The left and right endpoints of an interval X will be denoted by $\underline{X}$ and $\overline{X}$, respectively. Thus,

$X = [\underline{X}, \overline{X}]$

Two intervals X and Y are said to be equal if they are the same sets. Operationally, this happens if their corresponding endpoints are equal:

$X = Y \Leftrightarrow \underline{X} = \underline{Y}$ And $\overline{X} = \overline{Y}$

We say that X is degenerate if $\underline{X} = \overline{X}$. Such an interval contains a single real number x. By convention, we agree to identify a degenerate interval $[X, X]$ with the real number x.

Operations on intervals:

Intersection of the intervals : $X \cap Y = [\max\{\underline{X}, \underline{Y}\}, \min\{\overline{X}, \overline{Y}\}]$

Union of intervals : $X \cup Y = [\min\{\underline{X}, \underline{Y}\}, \max\{\overline{X}, \overline{Y}\}]$

Interval hull of intervals : $X \underline{\cup} Y = [\min\{\underline{X}, \underline{Y}\}, \max\{\overline{X}, \overline{Y}\}]$

Intersection plays a key role in interval analysis. If we have two intervals containing a result of interest, regardless of how they were obtained, then the intersection, which may be narrower, also contains the result.

In general, the union of two intervals is not an interval. However, the interval hull of two intervals is always an interval and can be used in interval computations. We have: $X \cup Y \subseteq X \underline{\cup} Y$

The width of an interval X is defined and denoted by: $\omega(X) = \overline{X} - \underline{X}$

The absolute value of X , denoted by $|X|$, and is the maximum of the absolute values of its endpoints: $|X| = \max\{|\underline{X}|, |\overline{X}|\}$ note that $|x| \le |X|, \forall x \in X$

The midpoint of X is given by: $m(X) = \frac{1}{2}(\underline{X} + \overline{X})$

Order Relations for Intervals: $X < Y$ means that $\overline{X} < \underline{Y}$

Transitive order relation for intervals is set inclusion: $X \subseteq Y \Leftrightarrow \underline{Y} \le \underline{X}$ and $\overline{X} \le \overline{Y}$

The sum of two intervals : $X + Y = \{x + y : x \in X, y \in Y\}$

Subtraction of two intervals : $X - Y = \{x - y : x \in X, y \in Y\}$

Product of two intervals : $X \odot Y = \{xy : x \in X, y \in Y\}$

Quotient of two intervals : $X / Y = \{x / y : x \in X, y \in Y\}$

5. $\alpha -$ Cut Method

By using $\alpha -$ cut method we can convert a fuzzy integer problem into an interval data problem.

Suppose that we have a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$, and then we can convert this number into interval data as following,

$(a_1, a_2, a_3, a_4) = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]; \alpha = 0,1$

6. The Algorithm

Step 1: formulate the given problem in the form of trapezoidal fuzzy number.

Step 2: convert the trapezoidal fuzzy numbers into interval numbers using α -cut method to get the interval matrix.

Step 3: For each row $\{1,2,.....,m\}$ find the entry g_{ij} that is less than or equal to all other entries in the i^{th} row.

Step 4: For each column $\{1,2,......,n\}$ find the entry g_{ij} that is greater than or equal to all other entries in the j^{th} column.

Step 5: Determine if there is an entry g_{ij} that is simultaneously the minimum of the i^{th} row and the maximum of the j^{th} column which is known as saddle point and value of saddle interval is known as value of game, If any of the above values cannot be found then saddle point does not exist. Then we go for dominance principle.

Step 6: if all the elements of the i^{th} row are less than or equal to the corresponding elements of j^{th} row then j^{th} row will be dominated row.

Step 7: if all the elements of k^{th} column are greater than or equal to the corresponding elements of r^{th} column then k^{th} column will be dominated column.

Step 7: dominated row or column can be deleted to reduce the size of payoff matrix as optimal strategies will remain unaffected. A given strategy can also said to be dominated if it is inferior to an average of two or more other pure strategies. More generally if some convex linear combination of some rows dominates the i^{th} row then i^{th} row will be deleted. Similar arguments follow for columns.

Step 8: using the above arguments matrix can be reduced to a simple matrix for game value can be evaluated easily.

Step 9: if there is no saddle point and dominance principle also failed then we define fuzzy membership of an interval being a minimum and a maximum of an interval vector and then we define the notion of a least and greatest interval in $\mathbb{R}$ as defined in next step.

Step 10: The binary fuzzy operator $\preceq$ of two intervals X and Y returns a real number between 0 and 1 as follows:

$$X \preceq Y = \begin{cases} 1, X = Y; \overline{X} \le \underline{Y}, X \ne Y; \underline{X} < \underline{Y} < \overline{X} < \overline{Y} \\ 0, \overline{Y} \le \underline{X}, X \ne Y; \underline{Y} < \underline{X} < \overline{Y} < \overline{X} \\ \dfrac{\overline{Y} - \overline{X}}{\omega(Y) - \omega(X)}, \underline{Y} \le \underline{X} \le \overline{X} \le \overline{Y}, \omega(Y) > 0, X \ne Y \\ \dfrac{\underline{Y} - \underline{X}}{\omega(X) - \omega(Y)}, \underline{X} \le \underline{Y} \le \overline{Y} \le \overline{X}, \omega(X) > 0, X \ne Y \end{cases}$$

The binary fuzzy operator $\succeq$ of two intervals X and Y is defined as,

$X \succeq Y = 1$ If $X = Y$ and $X \succeq Y = 1 - (X \preceq Y)$ otherwise i.e.

$$X \succeq Y = \begin{cases} 1, X = Y; \overline{Y} \le \underline{X}, X \ne Y \\ 0, \overline{X} \le \underline{Y}, X \ne Y; \underline{X} < \underline{Y} < \overline{X} < \overline{Y}; \underline{Y} < \underline{X} < \overline{Y} < \overline{X} \\ \dfrac{\underline{X} - \underline{Y}}{\omega(Y) - \omega(X)}, \underline{Y} \le \underline{X} \le \overline{X} \le \overline{Y}, \omega(Y) > 0, X \ne Y \\ \dfrac{\overline{X} - \overline{Y}}{\omega(X) - \omega(Y)}, \underline{X} \le \underline{Y} \le \overline{Y} \le \overline{X}, \omega(X) > 0, X \ne Y \end{cases}$$

6.1 Numerical example 1.

In a small town there are only two stores and that handle sundry goods A and B. The total number of customers is divided between the two, because price and quality are equal. Both stores have reputation in community for the equally good services they render. Both the stores plan to run pre-Diwali sales during first week of October. Sales are advertised through local press, radio and TV with the aid of an advertising firm. Here the estimates are not precise and hence taken as fuzzy.

Now using the algorithm:

The payoff matrix is given below:

Firm B→ Firm A↓	Press	Radio	TV
Press	(3,5,7,9)	(13,14,15,16)	(23,24,25,26)
Radio	(0,1,2,3)	(3,6,9,12)	(14,16,18,20)
TV	(0.25,0.50,0.75,1)	(2,4,6,8)	(1,2,3,4)

Using α -cut method i.e.

$(a_1, a_2, a_3, a_4) = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]; \alpha = 0,1$; The above matrix can be converted into interval data matrix. Hence we get,

$$\begin{pmatrix} [3,9] & [13,16] & [23,26] \\ [0,3] & [3,12] & [14,20] \\ [0.25,1] & [2,8] & [1,4] \end{pmatrix}$$

Here

$$\min\{[3,9],[13,16],[17,26]\} = [3,9]$$
$$\min\{[0,3],[3,12],[14,20]\} = [0,3]$$
$$\min\{[0.25,1],[2,8],[1,4]\} = [0.25,1]$$
$$\max\{[3,9],[0,3],[0.25,1]\} = [3,9]$$
$$\max\{[13,16],[3,12],[2,8]\} = [13,16]$$
$$\max\{[23,26],[14,20],[1,4]\} = [23,26]$$

So in above problem it is clear that at (1, 1) place [3, 9] is saddle interval. So first store select any row containing a saddle interval and second store select any column containing a saddle interval.

6.2 Numerical example: 2

Two companies Fumco and Tabacs are competing for their business. The Strategies for each firm are don't advertise or advertise. Also, Tabacs is a newly opened one and so not known to all. Assume that the following payoff matrix describes the increase in business for Fumco and the decrease for Tabacs.

Tabacs → Fumco↓	Don’t Advertise	Advertise
Don’t Advertise	(2,3,4,5)	(1,2,3,4)
Advertise	(3,5,7,9)	(2,5,8,11)

Using α -cut method i.e.

$(a_1, a_2, a_3, a_4) = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]; \alpha = 0,1$; The above matrix

can be converted into interval data matrix. Hence we get,

$$\begin{pmatrix} [2,5] & [1,4] \\ [3,9] & [2,11] \end{pmatrix}$$

There is no saddle point and dominance principle also failed so we will find fuzzy membership of an interval being a minimum and a maximum of an interval vector and then we define the notion of a least and greatest interval in $\mathbb{R}$ as following:

First of all we will find least interval for this,

$$\min\{[2,5]\prec[1,4],[2,5]\prec[3,9],[2,5]\prec[2,11]\}$$
$$=\min\{0,1,1\}=0$$
$$\min\{[1,4]\prec[2,5],[1,4]\prec[3,9],[1,4]\prec[2,11]\}$$
$$=\min\{1,1,1\}=1$$
$$\min\{[3,9]\prec[2,5],[3,9]\prec[1,4],[3,9]\prec[2,11]\}$$
$$=\min\{0,0,\frac{2}{3}\}=0$$
$$\min\{[2,11]\prec[2,5],[2,11]\prec[1,4],[2,11]\prec[3,9]\}$$
$$=\min\{0,0,\frac{1}{3}\}=0$$

Therefore, max {0, 1, 0, 0} =1

Which is corresponding to interval [1, 4] Hence least interval is [1, 4].

Now for greatest interval:

$$\max\{[2,5]\succ[1,4],[2,5]\succ[3,9],[2,5]\succ[2,11]\}$$
$$=\max\{0,0,0\}=0$$
$$\max\{[1,4]\succ[2,5],[1,4]\succ[3,9],[1,4]\succ[2,11]\}$$
$$=\max\{0,0,0\}=0$$
$$\max\{[3,9]\succ[2,5],[3,9]\succ[1,4],[3,9]\succ[2,11]\}$$
$$=\max\{0,0,\frac{1}{3}\}=\frac{1}{3}$$
$$\max\{[2,11]\succ[2,5],[2,11]\succ[1,4],[2,11]\succ[3,9]\}$$
$$=\max\{1,0,\frac{2}{3}\}=1$$

Therefore $\min\{0,0,\frac{1}{3},1\}=0$

Which is corresponding to intervals [2, 5] and [1, 4] which are the greatest intervals. It is clear from the above discussion that the least and the greatest interval which is saddle interval is [1, 4].

7. CONCLUSION

Marketing is one of the toughest occupations in the present scenarios. Agents make cooperation and communication between the several departments and companies. When companies are allowed to operate multi class basis, the agents of the company are then compelled to divide the costs between different classes. When the game theory is applied to marketing problems on that occasion it is difficult to assess payoffs exactly because of inaccuracy of information and fuzzy understanding of situations by agents.Fuzzy theory provides us an alternative for modeling human uncertainty without the scope of probability theory. In such cases, games with fuzzy payoffs, in which payoffs are represented as fuzzy numbers, are often considered. We introduce fuzzy goals for payoffs in order to incorporate ambiguity of judgements and the Agent tries to maximize of attainment of the fuzzy goals.

REFERENCES

[1]. Dutta, P. Strategies and games. Theory and practice, MIT PRESS, 1999.

[2]. Winston, W. OperationsResearch, Applications and Algorithms, Brooks/Cole, 2004.

[3]. Loganathan, C. and Annie Christi, M.S. Fuzzy Game Value of the Interval Matrix, International Journal of Engineering Research andApplications, Vol: 2, Issue 5, 2012.

[4]. R. E. Moore, R. E. Methods and Applications of Interval Analysis, SIAM Studies in Applied Mathematics, 1995.

[5]. Collins W.D. and Chenyi H. Fuzzily determined Interval matrix games.

[6]. Porchelvi.T and Stephen Dinagar D. Fuzzy Dominance Principle, International journal of Algorithms, Computing and Mathematics, Volume 3, 2000.

[7]. Pal, M. and Nayak, P. K. Solutions of m $\times$ n Rectangular Fuzzy Games by dominance, 2004.

AUTHOR'S BIOGRAPHY

Dr. S. C. Sharma, is working as associate professor in department of mathematics, University of Rajasthan, Jaipur, India. He completed M. Sc. and Ph. D. from department of mathematics, University of Rajasthan, Jaipur. He made a great contribution in research work in the field of special function and operations research. A number of national and international conferences attended by him and presented many research papers.

Shri Ganesh Kumar, is working as assistant professor in department of mathematics, University of Rajasthan, Jaipur, India. He completed M. Sc. from department of mathematics, University of Rajasthan, Jaipur. Many conferences attended by him and presented papers to resolve many problems in present scenarios.