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Numerical Solutions of Imbibition in Double Phase Flow through Porous Medium with Capillary Pressure Using Differential Transform Method

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Abstract: *The present paper discusses an approximate solution of an oil water imbibition phenomenon has been obtained in a homogeneous porous medium of some finite length. The Solution of the non-linear partial differential equation describing counter current imbibition has been derived by applying differential transform method. The RDTM reduces significantly the numerical computation.*

Keywords: *Imbibition phenomenon, Homogeneous porous medium, RDTM*

1. Introduction

The imbibition phenomenon in double phase flow during displacement process through homogeneous porous medium with capillary pressure, due to difference in wetting abilities of the two immiscible fluids flowing in the medium. It is well known that when a porous medium filled with some fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to difference in wetting abilities is called counter-current imbibitions. This phenomenon occurs during secondary recovery process when water is injected to push oil towards oil reservoir. It is necessary to develop mathematical model by selecting small part as cylindrical porous matrix. This Phenomenon has been discussed for homogeneous and heterogeneous porous media and also for cracked porous medium by many researchers.

This phenomenon has been investigated by many authors such as Brownscombe and Dyes [1]; Graham and Richardson [2]; Scheidegger [3]; Verma [4]; Mehta and Verma [5]. Many researchers have discussed this phenomenon with different viewpoints. Bokserman, Zheltov and Kocheshkov [6] have described the physics of oil-water flow in a cracked and heterogeneous porous medium. Torsaeter and Silseth [7] presented the effect of sample shape and boundary condition on capillary imbibition. Mehta [8] has discussed analytically the phenomenon of imbibition in porous media under certain condition by using a singular perturbation approach. Yadav and Mehta [9] discussed the mathematical model and similarity solution of Counter-current imbibition phenomenon in banded Porous matrix. In the present paper we have discussed the counter current imbibition phenomenon in homogeneous porous medium with capillary pressure.

2. Statement of the Problem

We consider here a cylindrical mass of porous matrix of length L containing a viscous oil, is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibitions phase and this end is exposed to an adjacent formation of 'injected' water. It is assumed that the injected water and the viscous oil are two immiscible liquids of different salinities with small viscosity difference; the former represents the preferentially wetting and less viscous phase. This arrangement gives rise to phenomenon of linear counter-current imbibitions, that is, a spontaneous linear flow of water into the porous medium and a linear counter flow of oil from the medium.

Formation of the problem:

The seepage velocity of water (V_w) and (V_o) are given by Darcy's Law,

$$V_w = -\left(\frac{K_w}{\delta_w}\right) K \left[\frac{\partial P_w}{\partial x}\right] \tag{1}$$

$$V_o = -\left(\frac{K_o}{\delta_o}\right) K \left[\frac{\partial P_o}{\partial x}\right] \tag{2}$$

Where

K = The permeability of the homogeneous medium

$K_w =$ Relative permeability of water, which is function of S_w

$K_o =$ Relative permeability of water, which is function of S_o

$S_w =$ The saturation of water

$S_o =$ The saturation of oil

$P_w =$ Pressure of water

$P_o =$ Pressure of oil

$\delta_w, \delta_o =$ Constant kinematics viscosities

g = acceleration due to gravity

Regarding the phase densities as content, the equation of continuity for water can be written as:

$$P\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial x}\right) = 0 \tag{3}$$

Where P is porosity of the medium. The analytical condition (Scheidegger, 1960) governing imbibitions phenomenon is

$$V_o = -V_w \tag{4}$$

From the definition of capillary pressure P_c as the pressure discontinuity between two phases yields

$$P_o = -P_w \tag{5}$$

Combining equation (1),(2),(4) and (5) we get

$$\frac{\partial P_w}{\partial x} = -\left[\frac{K_o/\delta_o}{K_o/\delta_o + K_w/\delta_w}\right]\left(\frac{\partial P_c}{\partial x}\right) \tag{6}$$

Substituting the above in equation (1) we have,

$$V_w = K\left[\frac{K_o/\delta_o \cdot K_w/\delta_w}{K_o/\delta_o + K_w/\delta_w}\right]\left(\frac{\partial P_c}{\partial x}\right) \tag{7}$$

Equation (3) and (7) yields

$$P\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[K\left(\frac{K_o/_{\delta_o}\cdot{}^{K_w}/_{\delta_w}}{K_o/_{\delta_o}+{}^{K_w}/_{\delta_w}}\right)\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}\right]=0 \tag{8}$$

This is the desired differential equation describing the imbibitions phenomenon.

Since the present investigation involves water and viscous oil, therefore according to schidegger (1960) approximation, we may write equation (8) in the form

$$P\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[K\frac{K_o}{\delta_o}\cdot\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}\right]=0 \tag{9}$$

$$As\ \frac{K_o/_{\delta_o}\cdot{}^{K_w}/_{\delta_w}}{K_o/_{\delta_o}+{}^{K_w}/_{\delta_w}}\approx\frac{K_o}{\delta_o}$$

At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Muskat [10] , between phase saturation and relative permeability as,

$$K_w=(S_w)^3,$$

$$K_o=1-\alpha S_w,\alpha=0.1$$

and

$$P_c=-\beta S_w \tag{10}$$

substituting the values from equation (10) into (9) we get

$$P\frac{\partial S_w}{\partial t}-\frac{K\beta_o}{\delta_o}\frac{\partial}{\partial x}\left[(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right]=0 \tag{11}$$

Equation (11) is reduced to dimensionless from by setting

$X=\frac{x}{L}$, $\qquad T=\frac{K\beta t}{\delta_o L^2 P}$, $\qquad S_w(x,t)=\ S_w{}^*(x,t)$

And then equation (11) takes the form

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial x}\left[(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right] \tag{12}$$

with auxiliary conditions

$$S_w(x,0)=0 \qquad ;0<x\le L$$

$$S_w(x,0)=\emptyset \qquad for\ all\ \ t$$

$$\frac{\partial S_w}{\partial x}(L,t)=0 \qquad for\ all\ \ t$$

Where $\emptyset$ is the mean saturation at the imbibition face and regarded as a simplicity. Equation (12) is desired non-linear differential equation of motion for the flow of two immiscible liquids in homogeneous medium.

The numerical values are shown by table. Curves indicate the behavior of saturation of water corresponding to various time periods.

3. Solution using RDTM Method

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial x}\left[(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right] \tag{12}$$

Taking the initial condition $S_w(x,0)=S_{w0}=f(x)$

$$f(x)=\frac{e^x-1}{e-1} \tag{13}$$

The problem is solved by reduced differential transform method because our equation is partial differential equation.

Reduced differential Transform Method

The Basic definition of RDTM is given below

If the function u(x,t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function . u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right] t^k$$

Apply RDTM on (12)

$$(k+1)S_{w(k+1)}(x) = \left[\sum_{r=0}^{k} (1-\alpha S_{wr})\left(S_{w(k-r)}\right)_x\right] - [\alpha((S_{wk})_x)^2] \tag{14}$$

Now let k=0 then put initial condition (13) into eq. (12), So we have the values of $S_{wk}(x)$ as following

$$(1)S_{w1}(x) = \left[\sum_{r=0}^{0} (1-\alpha S_{w0})(S_{w0})_{xx}\right] - [\alpha((S_{wk})_x)^2]$$

$$S_{w1}(x) = \left(1-\alpha\left(\frac{e^x-1}{e-1}\right)\right)\left(\frac{e^x}{e-1}\right) - \alpha\left(\frac{e^x}{e-1}\right)^2$$

$$= \frac{e^{x+1} - e^x - \alpha e^{2x} + \alpha e^x - \alpha e^{2x}}{(e-1)^2}$$

$$= \frac{e^{x+1} + (\alpha-1)e^x - 2\alpha e^{2x}}{(e-1)^2}$$

Now for $\alpha = 0.1$

$$S_{w1}(x) = \frac{e^{x+1} - 0.9e^x - 0.2e^{2x}}{(e-1)^2}$$

$$(2)S_{w2}(x) = \left[\sum_{r=0}^{1} (1-\alpha S_{wr})\left(S_{w(k-r)}\right)_{xx}\right] - [\alpha((S_{wk})_x)^2]$$

$$= (1-\alpha S_{w0})(S_{w1})_{xx} + (1-\alpha S_{w1})(S_{w0})_{xx} - [\alpha((S_{w1})_x)^2]$$

$$= \left(1-\alpha\left(\frac{e^x-1}{e-1}\right)\right)\left(\frac{e^{x+1}-0.9e^x-0.8e^{2x}}{(e-1)^2}\right) + \left(1-\alpha\left(\frac{e^{x+1}-0.9e^x-0.2e^{2x}}{(e-1)^2}\right)\right)\left(\frac{e^x}{e-1}\right) - \left[\alpha\left(\left(\frac{e^{x+1}-0.9e^x-0.4e^{2x}}{(e-1)^2}\right)\right)^2\right]$$

$$= \frac{2e^{x+3} - 5.8e^{x+2} - 1.1e^{2(x+1)} + 2.08e^{2x+1} + 0.18e^{3x+1} + 5.61e^{x+1} - 0.98e^{2x} - 1.81e^x - 0.17e^{3x} - 0.016e^{4x}}{(e-1)^4}$$

In this way we can generated other polynomials by putting different values in equation (14)

Now by inverse Transform

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

$S_w(x,T) = S_{w0}(x)T^0 + S_{w1}(x)T^1 + S_{w2}(x)T^2 + \cdots$

$S_w(x,t) =$

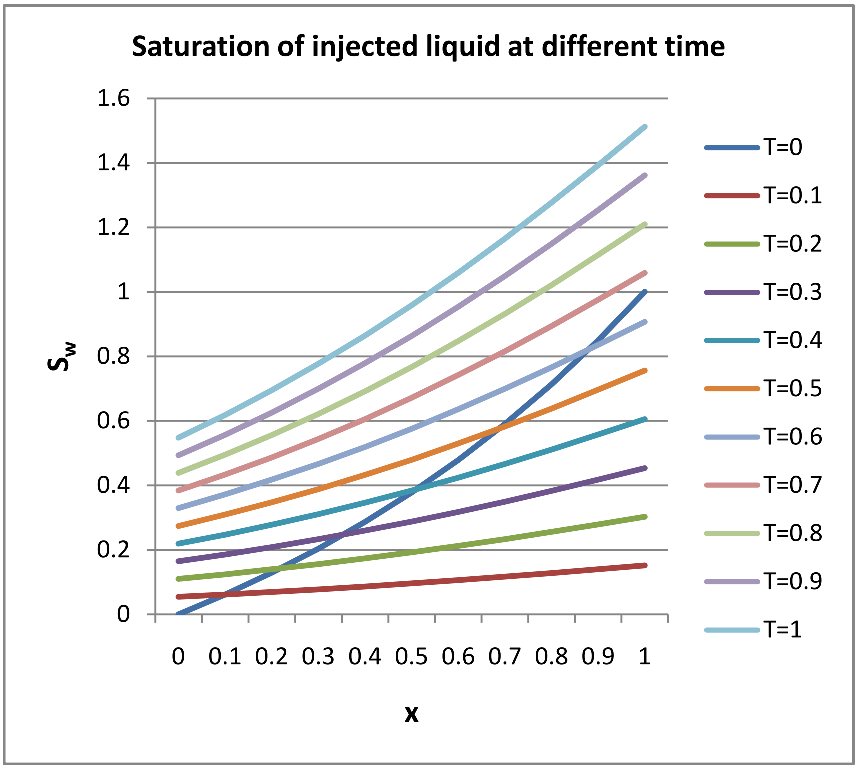
$\frac{e^x-1}{e-1}T^0 + \frac{e^{x+1}-0.9e^x-0.2e^{2x}}{(e-1)^2}T^1 +$

$\frac{2e^{x+3}-5.8e^{x+2}-1.1e^{2(x+1)}+2.08e^{2x+1}+0.18e^{3x+1}+5.61e^{x+1}-0.98e^{2x}-1.81e^{x}-0.17e^{3x}-0.016e^{4x}}{(e-1)^4}\frac{T^2}{2} + \cdots$

4. Table and Figure

The following table shows the approximate solution for saturation of injected liquid for different values of x at different time using RDTM

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
T=0	0	0.061207025	0.128851248	0.203609677	0.286230518	0.377540669	0.47845399	0.58998046	0.71323627	0.849455012	1
T=0.1	0.054810702	0.061860909	0.069478273	0.077683845	0.086492349	0.095909618	0.10592935	0.11652902	0.12766469	0.139264613	0.151221089
T=0.2	0.109621404	0.123721819	0.138956545	0.15536769	0.172984698	0.191819236	0.2118587	0.23305803	0.25532939	0.278529225	0.302442179
T=0.3	0.164432105	0.185582728	0.208434818	0.233051536	0.259477047	0.287728855	0.31778806	0.34958705	0.38299408	0.417793838	0.453663268
T=0.4	0.219242807	0.247443637	0.277913091	0.310735381	0.345969397	0.383638473	0.42371741	0.46611607	0.51065877	0.55705845	0.604884357
T=0.5	0.274053509	0.309304546	0.347391363	0.388419226	0.432461746	0.479548091	0.52964676	0.58264509	0.63832347	0.696323063	0.756105447
T=0.6	0.328864211	0.371165456	0.416869636	0.466103071	0.518954095	0.575457709	0.63557611	0.6991741	0.76598816	0.835587676	0.907326536
T=0.7	0.383674913	0.433026365	0.486347909	0.543786916	0.605446444	0.671367328	0.74150546	0.81570312	0.89365285	0.974852288	1.058547625
T=0.8	0.438485615	0.494887274	0.555826181	0.621470761	0.691938793	0.767276946	0.84743482	0.93223214	1.02131755	1.114116901	1.209768715
T=0.9	0.493296316	0.556748183	0.625304454	0.699154607	0.778431142	0.863186564	0.95336417	1.04876115	1.14898224	1.253381513	1.360989804
T=1	0.548107018	0.618609093	0.694782727	0.776838452	0.864923491	0.959096182	1.05929352	1.16529017	1.27664693	1.392646126	1.512210893



5. Conclusion

In graph, X-axis represents the different values of x and Y-axis represents saturation of injected liquid in saturated porous media.

It is interpreted from graph that at particular time level, saturation of injected liquid is increase with increase in value of x and as time increases, rate of increase of the saturation of injected liquid lessen at each layer.

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