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Fixed Points in Menger Space for Occasionally Weak Compatible Mappings

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Abstract: *The purpose of this paper is to establish a unique common fixed point theorem for six self mappings using the concept of occasionally weak compatibility in Menger space which is an alternate result of Jain and Singh [5].*

Keywords and Phrases: *Menger space, Common fixed points, Compatible maps, Occasionally Weak Compatible mapping.*

AMS Subject Classification (2000): *Primary 47H10, Secondary 54H25.*

1. Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [8]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [11] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [12] obtained a generalization of Banach Contraction Principle on a complete Menger space which is a milestone in developing fixed point theory in Menger space.

Jungck and Rhoades [7] termed a pair of self maps to be coincidentally commuting or equivalently weakly compatible if they commute at their coincidence points. Sessa [13] initiated the tradition of improving commutativity in fixed point theorems by introducing the notion of weak commuting maps in metric spaces. Jungck [5] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [9]. Interesting results in the field of Menger space have been discussed in Jain et. al. [2, 3, 4], Singh et. al. [14, 15], Cho et. al. [1], Patel et. al. [10] and so on.

In this paper a fixed point theorem for six self maps has been proved using the concept of occasionally weak compatibility which turns out to be an alternate result of Jain et. al. [5].

2. Preliminaries

Definition 2.1.[8] A mapping $\mathcal{F} : \mathrm{R} \to \mathrm{R}^+$ is called a *distribution* if it is non-decreasing left continuous with

$$\inf \{ \mathcal{F}(t) \mid t \in \mathrm{R} \} = 0 \qquad \text{and} \qquad \sup \{ \mathcal{F}(t) \mid t \in \mathrm{R} \} = 1.$$

We shall denote by L the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0 & , \quad t \le 0 \\ 1 & , \quad t > 0 \end{cases}.$$

Definition 2.2. [2] A mapping t :[0, 1] × [0, 1] → [0, 1] is called a *t-norm* if it satisfies the following conditions :

(t-1) t(a, 1) = a, t(0, 0) = 0 ;

(t-2) t(a, b) = t(b, a) ;

(t-3) t(c, d) ≥ t(a, b) ; for c ≥ a, d ≥ b,

(t-4) t(t(a, b), c) = t(a, t(b, c)) for all a, b, c, d ∈ [0, 1].

Definition 2.3. [2] A *probabilistic metric space (PM-space)* is an ordered pair (X, $\mathcal{F}$) consisting of a non empty set X and a function $\mathcal{F}$: X × X → L, where L is the collection of all distribution functions and the value of $\mathcal{F}$ at (u, v) ∈ X × X is represented by $F_{u,v}$. The function $F_{u,v}$ assumed to satisfy the following conditions:

(PM-1) $F_{u,v}(x) = 1$, for all x > 0, if and only if u = v;

(PM-2) $F_{u,v}(0) = 0$;

(PM-3) $F_{u,v} = F_{v,u}$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(x + y) = 1$,

for all u,v,w ∈ X and x, y > 0.

Definition 2.4. [2] A *Menger space* is a triplet (X, $\mathcal{F}$, t) where (X, $\mathcal{F}$) is a PM-space and t is a t-norm such that the inequality

(PM-5) $F_{u,w}(x + y) \ge t\{F_{u,v}(x), F_{v,w}(y)\}$, for all u, v, w ∈ X, x, y ≥ 0.

Definition 2.5. [11] A sequence $\{x_n\}$ in a Menger space (X, $\mathcal{F}$, t) is said to be *convergent* and *converges to a point* x in X if and only if for each ε > 0 and λ > 0, there is an integer M(ε, λ) such that

$$F_{x_n, x}(\varepsilon) > 1 - \lambda \qquad \text{for all } n \ge M(\varepsilon, \lambda).$$

Further the sequence $\{x_n\}$ is said to be *Cauchy sequence* if for ε > 0 and λ > 0, there is an integer M(ε, λ) such that

$$F_{x_n, x_m}(\varepsilon) > 1 - \lambda \qquad \text{for all } m, n \ge M(\varepsilon, \lambda).$$

A Menger PM-space (X, $\mathcal{F}$, t) is said to be *complete* if every Cauchy sequence in X converges to a point in X.

A complete metric space can be treated as a complete Menger space in the following way:

Proposition 2.1. [3] If (X, d) is a metric space then the metric d induces mappings $\mathcal{F}$: X × X → L, defined by $F_{p,q}(x) = H(x - d(p, q))$, p, q ∈ X, where

H(k) = 0, for k ≤ 0 and H(k) = 1, for k >0.

Further if, t : [0,1] × [0,1] → [0,1] is defined by t(a,b) = min {a, b}. Then (X, $\mathcal{F}$, t) is a Menger space. It is complete if (X, d) is complete.

The space (X, $\mathcal{F}$, t) so obtained is called the *induced Menger space.*

Proposition 2.2. [8] In a Menger space (X, $\mathcal{F}$, t) if $t(x, x) \geq x$, for all $x \in [0, 1]$ then $t(a, b) = \min\{a, b\}$, for all $a, b \in [0, 1]$.

Definition 2.6. [7] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be weak compatible if they commute at their coincidence points i.e. $Ax = Sx$ for $x \in X$ implies $ASx = SAx$.

Definition 2.7. [9] Self mappings A and S of a Menger space (X, $\mathcal{F}$, t) are said to be *compatible* if $F_{ASx_n, SAx_n}(x) \to 1$ for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n \to u$ for some u in X, as $n \to \infty$.

Definition 2.8. [2] Self maps A and S of a Menger space (X, $\mathcal{F}$, t) are said to be occasionally weakly compatible (owc) if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Now, we give an example of pair of self maps (A, S) which are occasionally weak compatible but not compatible.

Example 2.1. Let (X, d) be a metric space where $X \in R^+$ and (X, $\mathcal{F}$, t) be the induced Menger space with $F_{x,y} = \dfrac{t}{t + d(x,y)}$, $\forall$ x, y $\in$ X and t > 0.

Define self maps A and S as follows:

$$S(x) = \begin{cases} \dfrac{2}{x^2}, & \text{if} \quad x \neq 0 \\ 1, & \text{if} \quad x = 0 \end{cases} \quad \text{and} \quad T(x) = \begin{cases} \dfrac{2}{x}, & \text{if} \quad x \neq 0 \\ 1, & \text{if} \quad x = 0. \end{cases}$$

Taking $x_n = n$, we get $\lim_{n\to\infty} F_{ASx_n, SAx_n}(t) \neq 1$, for all t > 0.

Hence, (S, T) is not compatible. Also, 0 and 1 are coincidence points of S and T but (S, T) commute only at point 0.

Thus, the pair (S, T) is occasionally weak compatible.

Remark 2.2. In view of above example, it follows that the concept of occasionally weak compatible maps is more general than that of compatible maps and weak compatible maps.

Proposition 2.3. Let $\{x_n\}$ be a Cauchy sequence in a Menger space (X, $\mathcal{F}$, t) with continuous t-norm t. If the subsequence $\{x_{2n}\}$ converges to x in X, then $\{x_n\}$ also converges to x.

Proof. As $\{x_{2n}\}$ converges to x, we have

$$F_{x_n,x}(\varepsilon) \geq t\left(F_{x_n,x_{2n}}\left(\frac{\varepsilon}{2}\right), F_{x_{2n},x}\left(\frac{\varepsilon}{2}\right)\right).$$

Then

$$\lim_{n\to\infty} F_{x_n,x}(\varepsilon) \geq t(1,1), \text{ which gives } \lim_{n\to\infty} F_{x_n,x}(\varepsilon) = 1, \ \forall\ \varepsilon > 0 \text{ and the result follows.}$$

Lemma 2.1. [15] Let $\{p_n\}$ be a sequence in a Menger space (X, $\mathcal{F}$, t) with continuous t-norm and $t(x, x) \geq x$. Suppose, for all $x \in [0, 1]$, there exists $k \in (0, 1)$ such that for all x > 0 and $n \in N$,

$$F_{p_n, p_{n+1}}(kx) \geq F_{p_{n-1}, p_n}(x)$$

or ,

$$F_{p_n, p_{n+1}}(x) \geq F_{p_{n-1}, p_n}(k^{-1}x).$$

Then $\{p_n\}$ is a Cauchy sequence in X.

3. Main Result

Theorem 3.1. Let A, B, S, T, L and M be self mappings on a Menger space

$(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.1) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

(3.2) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

(3.3) One of ST(X), M(X), AB(X) or L(X) is complete;

(3.4) The pairs (L, AB) and (M, ST) are occasionally weak compatible;

(3.5) for all $p, q \in X$, $x > 0$ and $0 < a < 1$,

$$[F_{Lp, Mq}(x) + F_{ABp, Lp}(x)][F_{Lp, Mq}(x) + F_{STq, Mq}(x)] \geq 4[F_{ABp, Lp}(x/\alpha)][F_{Mq, STq}(x)].$$

Then A, B, S, T, L and M have a unique common fixed point in X.

Proof. Let $x_0 \in X$. From condition (3.1) there exist $x_1, x_2 \in X$ such that

$$Lx_0 = STx_1 = y_0 \quad \text{and} \quad Mx_1 = ABx_2 = y_1.$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Lx_{2n} = STx_{2n+1} = y_{2n} \quad \text{and} \quad Mx_{2n+1} = ABx_{2n+2} = y_{2n+1} \quad \text{for } n = 0, 1, 2, \ldots .$$

First of all, we show that $\{y_n\}$ is a Cauchy sequence in X.

Step 1. Putting $p = x_{2n}$, $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, Mx_{2n+1}}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)] \geq 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)]$$

or, $$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)][F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n}, y_{2n+1}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/\alpha)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$2 F_{y_{2n}, y_{2n+1}}(x) [F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 4[F_{y_{2n-1}, y_{2n}}(x/\alpha)][F_{y_{2n+1}, y_{2n}}(x)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x) [F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1},y_{2n}}(x/\alpha)][F_{y_{2n}, y_{2n+1}}(x)]$$

or, $$[F_{y_{2n}, y_{2n+1}}(x) + F_{y_{2n-1}, y_{2n}}(x)] \geq 2[F_{y_{2n-1}, y_{2n}}(x/\alpha)]$$

or, $$F_{y_{2n}, y_{2n+1}}(x) \geq F_{y_{2n-1}, y_{2n}}(x/\alpha). \tag{3.6}$$

Similarly,

$$F_{y_{2n-1}\, y_{2n}}(x/\alpha) \quad \geq \quad F_{y_{2n-2},\, y_{2n-1}}(x/\alpha^{2}). \tag{3.7}$$

From (3.6) and (3.7), it follows that

$$F_{y_{2n},\, y_{2n+1}}(x) \;\geq\; F_{y_{2n-1},\, y_{2n}}(x/\alpha) \;\geq\; F_{y_{2n-2},\, y_{2n-1}}(x/\alpha^{2}).$$

By repeated application of above inequality, we get

$$F_{y_{2n},\, y_{2n+1}}(x) \;\geq\; F_{y_{2n-1},\, y_{2n}}(x/\alpha) \;\geq\; F_{y_{2n-2},\, y_{2n-1}}(x/\alpha^{2})$$

$$\geq \; \ldots \geq \; F_{y_0,\, y_1}(x/\alpha^{n}).$$

Therefore, by lemma 2.1, $\{y_n\}$ is a Cauchy sequence in X.

Case I. ST(X) is complete. In this case $\{y_{2n}\} = \{STx_{2n+1}\}$ is a Cauchy sequence in ST(X), which is complete. Thus $\{y_{2n+1}\}$ converges to some $z \in ST(X)$. By proposition 2.3, we have

$$\{Mx_{2n+1}\} \to z \qquad \text{and} \qquad \{STx_{2n+1}\} \to z, \tag{3.8}$$

$$\{Lx_{2n}\} \to z \qquad \text{and} \qquad \{ABx_{2n}\} \to z. \tag{3.9}$$

As $z \in ST(X)$ there exists $v \in X$ such that $z = STv$.

Step I. Putting $p = x_{2n}$ and $q = v$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n},\, Mv}(x) + F_{ABx_{2n},\, Lx_{2n}}(x)][F_{Lx_{2n},\, Mv}(x) + F_{STv,\, Mv}(x)]$$

$$\geq \; 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{Mv,\, STv}(x)].$$

Letting $n \to \infty$, we get

$$[F_{z,\, Mv}(x) + F_{z,\, z}(x)][F_{z,\, Mv}(x) + F_{z,\, Mv}(x)] \;\geq\; 4[F_{z,\, z}(x/\alpha)][F_{Mv,\, z}(x)],$$

i.e. $F_{z,\, Mv}(x) \geq 1$, yields $Mv = z$.

Hence, $STv = z = Mv$.

As (M, ST) is occasionally weakly compatible, we have

$$STMv = MSTv$$

or, $$STz = Mz.$$

Step II. Putting $p = x_{2n}$, $q = z$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n},\, Mz}(x) + F_{ABx_{2n},\, Lx_{2n}}(x)][F_{Lx_{2n},\, Mz}(x) + F_{STz,\, Mz}(x)]$$

$$\geq \; 4[F_{ABx_{2n},\, Lx_{2n}}(x/\alpha)][F_{Mz,\, STz}(x)].$$

Letting $n \to \infty$ and using $STz = Mz$, we get

$$[F_{z, Mz}(x) + F_{z, z}(x)][F_{z, Mz}(x) + F_{Mz, Mz}(x)] \geq 4[F_{z, z}(x/\alpha)][F_{Mz, Mz}(x)],$$

i.e. $F_{z, Mz}(x) \geq 1$, yields $z = Mz$.

Step III. Putting $p = x_{2n}$ and $q = Tz$ for $x > 0$ in (3.5), we get

$$[F_{Lx_{2n}, MTz}(x) + F_{ABx_{2n}, Lx_{2n}}(x)][F_{Lx_{2n}, MTz}(x) + F_{STTz, MTz}(x)]$$

$$\geq 4[F_{ABx_{2n}, Lx_{2n}}(x/\alpha)][F_{MTz, STTz}(x)].$$

As $MT = TM$ and $ST = TS$ we have $MTz = TMz = Tz$ and $ST(Tz) = T(STz) = Tz$.

Letting $n \to \infty$, we get

$$[F_{z,Tz}(x) + F_{z,z}(x)][F_{z,Tz}(x) + F_{Tz,Tz}(x)] \geq 4[F_{z,z}(x/\alpha)][F_{Tz,Tz}(x)],$$

i.e. $F_{z, Tz}(x) \geq 1$, yields $Tz = z$.

Now $STz = Tz = z$ implies $Sz = z$.

Hence $Sz = Tz = Mz = z$.

Step IV. As $M(X) \subseteq AB(X)$, there exists $w \in X$ such that

$$z = Mz = ABw.$$

Putting $p = w$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lw, Mx_{2n+1}}(x) + F_{ABw,Lw}(x)][F_{Lw,Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)]$$

$$\geq 4[F_{ABw, Lw}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{Lw, z}(x) + F_{z, Lw}(x)][F_{Lw, z}(x) + F_{z, z}(x)] \geq 4[F_{z, Lw}(x/a)][F_{z, z}(x)],$$

i.e. $F_{Lw, z}(x) \geq 1$, yields $Lw = z$.

Therefore, $ABz = Lz$.

Step V. Putting $p = z$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{Lz, Mx_{2n+1}}(x) + F_{ABz, Lz}(x)][F_{Lz, Mx_{2n+1}}(x) + F_{STx_{2n+1}, Mx_{2n+1}}(x)]$$

$$\geq 4[F_{ABz,Lz}(x/\alpha)][F_{Mx_{2n+1}, STx_{2n+1}}(x)].$$

Letting $n \to \infty$, we get

$$[F_{Lz, z}(x) + F_{z, Lz}(x)][F_{Lz, z}(x) + F_{z, z}(x)] \geq 4[F_{z, Lz}(x/\alpha)][F_{z, z}(x)],$$

i.e. $F_{Lz, z}(x) \geq 1$, yields $Lz = z$.

Therefore, $ABz = Lz = z$.

Step VI. Putting $p = Bz$ and $q = x_{2n+1}$ for $x > 0$ in (3.5), we get

$$[F_{LBz,\, Mx_{2n+1}}(x) + F_{ABBz,\, LBz}(x)][F_{LBz,\, Mx_{2n+1}}(x) + F_{STx_{2n+1},\, Mx_{2n+1}}(x)]$$

$$\geq 4[F_{ABBz,LBz}(x/\alpha)][F_{Mx_{2n+1},\, STx_{2n+1}}(x)].$$

As BL = LB, AB = BA, so we have

L(Bz) = B(Lz) = Bz and AB(Bz) = B(ABz) = Bz.

Letting $n \to \infty$, we get

$$[F_{Bz,\, z}(x) + F_{Bz,\, Bz}(x)][F_{Bz,\, z}(x) + F_{z,\, z}(x)] \geq 4[F_{Bz,\, Bz}(x/\alpha)][F_{z,\, z}(x)],$$

i.e. $F_{Bz,\, z}(x) \geq 1$, yields Bz = z and ABz = z implies Az = z.

Therefore, Az = Bz = Lz = z.

Combining the results from different steps, we get

Az = Bz = Lz = Mz = Tz = Sz = z.

Hence, the six self maps have a common fixed point in this case.

Case when L(X) is complete follows from above case as $L(X) \subseteq ST(X)$.

Case II. AB(X) is Complete. This case follows by symmetry. As $M(X) \subseteq AB(X)$, therefore the result also holds when M(X) is complete.

Uniqueness. Let u be another common fixed point of A, B, S, T, L and M; then Au = Bu = Su = Tu = Lu = Mu = u.

Putting p = z and q = u for $x > 0$ in (3.5), we get

$$[F_{Lz,\, Mu}(x) + F_{ABz,\, Lz}(x)][F_{Lz,\, Mu}(x) + F_{STu,\, Mu}(x)] \geq 4[F_{ABz,\, Lz}(x/\alpha)][F_{Mu,\, STu}(x)].$$

Letting $n \to \infty$, we get

$$[F_{z,\, u}(x) + F_{z,\, z}(x)][F_{z,\, u}(x) + F_{u,\, u}(x)] \geq 4[F_{z,\, z}(x/\alpha)][F_{u,\, u}(x)],$$

i.e. $F_{z,u}(x) \geq 1$, yields z = u.

Therefore, z is a unique common fixed point of A, B, S, T, L and M.

This completes the proof.

Remark 3.1. If we take B = T = I, the identity map on X in theorem 3.1, then the condition (3.2) is satisfied trivially and we get

Corollary 3.1. Let A, S, L and M be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.10) $L(X) \subseteq S(X)$, $M(X) \subseteq A(X)$;

(3.11) One of S(X), M(X), A(X) or L(X) is complete;

(3.12) The pairs (L, A) and (M, S) are occasionally weak compatible;

(3.13) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{Lp,\, Mq}(x) + F_{Ap,\, Lp}(x)][F_{Lp,\, Mq}(x) + F_{Sq,\, Mq}(x)]$$

$$\geq \quad 4[F_{Ap,\, Lp}(x/\alpha)][F_{Mq,\, Sq}(x)].$$

Then A, S, L and M have a unique common fixed point in X.

If we take A = I, the identity map in Corollary 3.1, we get

Corollary 3.2. Let S, L and M be self mappings on a complete Menger space

(X, F, t) satisfying :

(3.14) $L(X) \subseteq S(X)$;

(3.15) The pair (M, S) is occasionally weak compatible;

(3.16) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{Lp,\, Mq}(x) + F_{p,\, Lp}(x)][F_{Lp,\, Mq}(x) + F_{Sq,\, Mq}(x)]$$

$$\geq \quad 4[F_{p,\, Lp}(x/\alpha)][F_{Mq,\, Sq}(x)].$$

Then S, L and M have a unique common fixed point in X.

If we take S = A = I, the identity map on X and writing $L = T_i$ and $M = T_j$ in Corollary 3.1, we get

Corollary 3.3. Let T_i and T_j be self mappings on a Menger space $(X, \mathcal{F}, t)$ with continuous t-norm t satisfying :

(3.17) for all $p, q \in X$, $x > 0$ and $0 < \alpha < 1$,

$$[F_{T_ip,\, T_jq}(x) + F_{p,\, T_ip}(x)][F_{T_ip,\, T_jq}(x) + F_{q,\, T_jq}(x)]$$

$$\geq \quad 4[F_{p,\, T_ip}(x/\alpha)][F_{T_jq,\, q}(x)].$$

Then T_i and T_j have a unique common fixed point in X.

4. CONCLUSION

In view of proposition 2.2, t(a, b) = min{a, b}. Thus, theorem 3.1 is an alternate result of Jain et. al. [5] reducing the compatibility of the pair (L, AB) and weak compatibility of the pair (M, ST) to occasionally weak compatibility and dropping the condition of continuity in a Menger space with continuous t-norm.

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REFERENCES

[1]. Cho, Y.J., Murthy, P.P. and Stojakovik, M., *Compatible mappings of type (A) and common fixed point in Menger space,* Comm. Korean Math. Soc. 7 (2), (1992), 325-339.

[2]. Jain, A. and Chaudhary, B., *On common fixed point theorems for semi-compatible and occasionally weakly compatible mappings in Menger space,* International Journal of Research and Reviews in Applied Sciences, Vol. 14 (3), (2013), 662-670.

[3]. Jain, A. and Singh, B., *Common fixed point theorem in Menger space through compatible maps of type (A),* Chh. J. Sci. Tech. 2 (2005), 1-12.

[4]. Jain, A. and Singh, B., *A fixed point theorem in Menger space through compatible maps of type (A),* V.J.M.S. 5(2), (2005), 555-568.

[5]. Jain, A. and Singh, B., *Common fixed point theorems in Menger spaces,* The Aligarh Bull. of Maths. Vol. 25 (1), (2006), 23-31.

[6]. Jungck, G., *Compatible mappings and common fixed points,* Internat. J. Math. and Math. Sci. 9(4), (1986), 771-779.

[7]. Jungck, G. and Rhoades, B.E., *Fixed points for set valued functions without continuity,* Indian J. Pure Appl. Math. 29(1998), 227-238.

[8]. Menger, K., *Statistical metrics,* Proc. Nat. Acad. Sci. USA. 28(1942), 535 -537.

[9]. Mishra, S.N., *Common fixed points of compatible mappings in PM-spaces,* Math. Japon. 36(2), (1991), 283-289.

[10]. Patel, R.N. and Patel, D., *Fixed points of compatible mappings of type (A) on Menger space,* V.J.M.S. 4(1), (2004), 185-189.

[11]. Schweizer, B. and Sklar, A., *Statistical metric spaces,* Pacific J. Math. 10 (1960), 313-334.

[12]. Sehgal, V.M. and Bharucha-Reid, A.T., *Fixed points of contraction maps on probabilistic metric spaces,* Math. System Theory 6(1972), 97- 102.

[13]. Sessa, S., *On a weak commutativity condition of mappings in fixed point consideration,* Publ. Inst. Math. Beograd 32(46), (1982), 146-153.

[14]. Singh, M., Sharma, R.K. and Jain, A., *Compatible mappings of type (A) and common fixed points in Menger space,* Vikram Math. J. 20 (2000), 68-78.

[15]. Singh, S.L. and Pant, B.D., *Common fixed point theorems in probabilistic metric spaces and extension to uniform spaces,* Honam. Math. J. 6 (1984), 1-12.

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