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Family of Circulant Graphs without Cayley Isomorphism Property with m_i = 5

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Abstract: *A circulant graph $C_n(R)$ is said to have the Cayley Isomorphism (CI) property if whenever $C_n(S)$ is isomorphic to $C_n(R)$, there is some $a \in Z_n^*$ for which S = aR. It is known that (i) for 2 ≤ n, 3 ≤k, 1 ≤2s-1 ≤ 2n-1, n≠ 2s-1, R = {2s-1, 4n-(2s-1), $2p_1,2p_2,\ldots, 2p_{k-2}$} and S = {2n-(2s-1), 2n+2s-1, $2p_1,2p_2,\ldots,2p_{k-2}$}, circulant graphs $C_{8n}(R)$ and $C_{8n}(S)$ are without CI-property with m_i = 2 and (ii) for 1 ≤ n, 3 ≤ k, R = {1, 9n-1, 9n+1, $3p_1,3p_2,\ldots,3p_{k-2}$}, S = {3n+1, 6n-1, 12n+1, $3p_1,3p_2,\ldots,3p_{k-2}$} and T = {3n-1, 6n+1, 12n-1, $3p_1,3p_2,\ldots,3p_{k-2}$}, circulant graphs $C_{27n}(R)$, $C_{27n}(S)$ and $C_{27n}(T)$ are without CI-property m_i = 3 where gcd($p_1,p_2,\ldots,p_{k-2}$) = 1 and n,s,$p_1,p_2,\ldots,p_{k-2} \in N$. In this paper, we prove that for 1 ≤n, 3 ≤k, 1 ≤i ≤ 5,d_i= 5n(i-1)+1 and R_i= {5,d_i, 25n-d_i,25n+d_i,50n-d_i,50n+d_i, $5p_1,5p_2,\ldots,5p_{k-2}$}, circulant graphs $C_{125n}(R_i)$ are without CI-property m_j= 5 where m_j = gcd(n, r_j), $r_j \in R_i$, gcd($p_1,p_2,\ldots,p_{k-2}$) = 1 and n,$p_1,p_2,\ldots,p_{k-2} \in N$.*

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Keywords: *Type-1 isomorphism, Type-2 isomorphism, Cayley Isomorphism (CI) property, symmetric equidistance condition, abelian groups $Ad_{125n}(C_{125n}(R)$, o) and ($V_{125n,5}(C_{125n}(R))$, o), Type-1 group of $C_{125n}(R)$, Type-2 group on $C_{125n}(R)$ w.r.t. r = 5.*

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1 INTRODUCTION

Circulant graphs have been investigated by many authors **[1]-[9]**. An excellent account can be found in the book by Davis **[2]** and in **[4]**.

Through-out this paper, for a set R = $\{r_1, r_2, \ldots, r_k\}$, $C_n(R)$ denotes circulant graph $C_n(r_1, r_2, \ldots, r_k)$ where $1 \le r_1 < r_2 < \cdots < r_k \le [n/2]$. We consider only connected circulant graphs of finite order, $V(C_n(R))$ = $\{v_0,v_1,v_2,\ldots,v_{n-1}\}$ with v_i adjacent to v_{i+r} for each $r \in R$, subscript addition taken modulo n and all cycles have length at least 3, unless otherwise specified, 0 ≤i≤n-1. However when $\frac{n}{2} \in R$, edge $v_i v_{i+\frac{n}{2}}$ is taken as a single edge for considering the degree of the vertex v_i or $v_{i+\frac{n}{2}}$ and as a double edge while counting the number of edges or cycles in $C_n(R)$, 0 ≤i≤n-1. Circulant graph is also defined as a Cayley graph or digraph of a cyclic group. If a graph G is circulant, then its adjacency matrix A(G) is circulant. It follows that if the first row of the adjacency matrix of a circulant graph is $[a_1,a_2,\ldots,a_n]$, then a_1 = 0 and $a_i = a_{n-i+2}$, 2 ≤i≤n **[2], [8]**. We will often assume, with-out further comment, that the vertices are the corners of a regular n-gon, labeled clockwise. Circulant graphs C_{16}(1,2,7) and C_{16}(2,3,5) are shown in Figures 1 and 2, respectively.

THEOREM 1.1 [8] *If $C_n(R) \cong C_n(S)$, then there is a bijection f from R to S so that for all r∈R, gcd(n, r) = gcd(n, f(r)).*

DEFINITION 1.2 [5] A circulant graph $C_n(R)$ is said to have the *CI-property* if whenever $C_n(S)$ is isomorphic to $C_n(R)$, there is some $a \in Z_n^*$ for which S = aR.

LEMMA 1.3 [8] *Let S be a non-empty subset of Z_n and x∈Z_n. Define a mapping $\Phi_{n,x}$: S→Z_n such that $\Phi_{n,x}(s)$ = xs for every s∈S under multiplication modulo n. Then $\Phi_{n,x}$ is bijective if and only if S = Z_n and gcd(n, x) = 1.*

DEFINITION 1.4 [1] Circulant graphs, $C_n(R)$ and $C_n(S)$ for $R = \{r_1,r_2,\ldots,r_k\}$ and $S = \{s_1,s_2,\ldots,s_k\}$ are *Adam's isomorphic* if there exists a positive integer x relatively prime to n with $S = \{xr_1, xr_2, \ldots, xr_k\}^*_n$ where $< r_i >^*_n$, the *reflexive modular reduction* of a sequence $<r_i>$ is the sequence obtained by reducing each r_i modulo n to yield r_i' and then replacing all resulting terms r_i' which are larger than $\frac{n}{2}$ by n-r_i' **[1]**.

LEMMA 1.5 [8] *Let $j,m,q,r,t,x\in Z_n$ such that $gcd(n, r) = m > 1$, $x = j+qm$, $0 \leq j\leq m-1$ and $0 \leq q,t\leq \frac{n}{m}-1$. Then the mapping $\theta_{n,r,t}$: $Z_n \rightarrow Z_n$ defined by $\theta_{n,r,t}(x) = x+jtm$ for every $x\in Z_n$ under arithmetic modulo n is bijective.*

THEOREM 1.6 [8] *Let $V(C_n(R)) = \{v_0,v_1,v_2,\ldots,v_{n-1}\}$, $V(K_n) = \{u_0,u_1,u_2,\ldots,u_{n-1}\}$, $r\in R$ and $j,m,q,t,x\in Z_n$ such that $gcd(n, r) = m > 1$, $x = j+qm$, $0 \leq j\leq m-1$ and $0 \leq q,t\leq \frac{n}{m} -1$. Then the mapping $\theta_{n,r,t}$: $V(C_n(R)) \rightarrow V(C_n(1,2,\ldots,n-1)) = V(K_n)$ defined by $\theta_{n,r,t}(v_x) = u_{x+jtm}$ and $\theta_{n,r,t}((v_x,v_{x+s})) = (\theta_{n,r,t}(v_x), \theta_{n,r,t}(v_{x+s}))$ for every $x\in Z_n$ and $s\in R$, under subscript arithmetic modulo n, for a set $R = \{r_1,r_2,\ldots,r_k$, n-$r_k,n-r_{k-1},\ldots,r_1\}$ is one-to-one, preserves adjacency and $\theta_{n,r,t}(C_n(R)) \cong C_n(R)$ for $t = 0,1,2,\ldots,\frac{n}{m} - 1$.*

DEFINITION 1.7 [8] Let $V(C_n(R)) = \{v_0,v_1,v_2,\ldots,v_{n-1}\}$, $V(K_n) = \{u_0, u_1, u_2, \ldots , u_{n-1}\}$, $r\in R$ and $j,m,q,t,x\in Z_n$ such that $gcd(n, r) = m > 1$, $x = j+qm$, $0 \leq j\leq m-1$ and $0 \leq q,t\leq \frac{n}{m} -1$. Define one-to-one mapping $\theta_{n,r,t}$: $V(C_n(R)) \rightarrow V(K_n)$ such that $\theta_{n,r,t}(v_x) = u_{x+jtm}$ and $\theta_{n,r,t}((v_x,v_{x+s})) = (\theta_{n,r,t}(v_x),\theta_{n,r,t}(v_{x+s}))$ for every $x\in Z_n$ and $s\in R$, under subscript arithmetic modulo n. And if for a particular value of t, $\theta_{n,r,t}(C_n(R)) = C_n(S)$ for some $S\subseteq [1, [n/2]]$ and $S\neq xR$ for all $x\in \Phi_n$ under reflexive modulo n, then $C_n(R)$ and $C_n(S)$ are called *Type-2 isomorphic circulant graphs w.r.t. r.*

DEFINITION 1.8 [8] The *symmetric equidistance condition* with respect to v_i in $C_n(R)$ for a set $R = \{r_1,r_2,\ldots,r_k\}$ is that v_{i+j} is adjacent to v_i if and only if v_{n-j+i} is adjacent to v_i, using subscript arithmetic modulo n, $0 \leq i,j\leq n-1$.

THEOREM 1.9 [8] *For a set $R = \{r_1,r_2,\ldots,r_k\} \subseteq [1, n/2]$, $1 \leq i\leq k$ and $0 \leq t\leq \frac{n}{m} -1$, $\theta_{n,r_i,t}(C_n(R)) = C_n(S)$ for some $S\subseteq [1, n/2]$ if and only if $\theta_{n,r_i,t}(C_n(R))$ satisfies the symmetric equidistance condition w.r.t. v_0.*

THEOREM 1.10 [8] *For $2 \leq n$, $3 \leq k$, $1 \leq 2s-1 \leq 2n-1$, $n\neq 2s-1$, $R = \{2s-1, 4n-2s+1, 2p_1,2p_2,\ldots,2p_{k-2}\}$* and *$S = \{2n-2s+1, 2n+2s-1, 2p_1,2p_2,\ldots,2p_{k-2}\}$, circulant graphs $C_{8n}(R)$ and $C_{8n}(S)$ are Type-2 isomorphic and without CI-property where $gcd(p_1,p_2,\ldots,p_{k-2}) = 1$ and $n,s,p_1,p_2,\ldots,p_{k-2}\in N$.*

THEOREM 1.11 [9] *For $3 \leq k$, $R = \{1, 9n-1, 9n+1, 3p_1,3p_2,\ldots,3p_{k-2}\}$, $S = \{3n+1, 6n-1, 12n+1, 3p_1,3p_2,\ldots,3p_{k-2}\}$ and $T = \{3n-1, 6n+1, 12n-1, 3p_1,3p_2,\ldots,3p_{k-2}\}$, $C_{8n}(R)$ and $C_{8n}(S)$ are Type-2 isomorphic and without CI-property where $gcd(p_1,p_2,\ldots,p_{k-2}) = 1$ and $n,p_1,p_2,\ldots,p_{k-2}\in N$.*

THEOREM 1.12 [8] *For $R = \{2, 2s-1, 2s'-1\}$, $1 \leq t\leq [\frac{n}{2}]$, $1 \leq 2s-1 < 2s'-1 \leq [\frac{n}{2}]$ and $n,s,s',t\in N$, if $C_n(R)$ and $\theta_{n,2,t}(C_n(R))$ are Type-2 isomorphic circulant graphs for some t, then $n\equiv 0 \pmod 8$, $2s-1+2s'-1 = \frac{n}{2}$, $t = \frac{n}{8}$ or $\frac{3n}{8}$, $2s'-1 \neq \frac{n}{8}$, $1 \leq 2s-1 \leq \frac{n}{4}$ and $16 \leq n$.*

THEOREM 1.13 [8] *Let $x\in Z_n$. Define mapping $\Phi_{n,x}$: $V(C_n(R)) \rightarrow V(K_n)$ for a set $R = \{r_1,r_2,\ldots,r_k, n-r_k,n-r_{k-1},\ldots,n-r_1\}$ such that $\Phi_{n,x}(v_i) = u_{xi}$ and $\Phi_{n,x}((v_i , v_{i+s})) = (\Phi_{n,x}(v_i), \Phi_{n,x}(v_{i+s}))$ for every $s\in R$ and $i\in Z_n$ under subscript arithmetic modulo n where $V(C_n(R)) = \{v_0,v_1,\ldots,v_{n-1}\}$* and *$V(K_n) = \{u_0,u_1,\ldots,$*
$u_{n-1}\}$. Then $\Phi_{n,x}(C_n(R)) = C_n(xR)$ and the mapping $\Phi_{n,x}$ is one-to-one if and only if $gcd(n, x) = 1$.

DEFINITION 1.14 [8] Let $Ad_n(C_n(R)) = T1_n(C_n(R)) = \{\Phi_{n,x}(C_n(R)) : x\in \Phi_n\} = \{C_n(xR) / x\in \Phi_n\}$ for a set $R = \{r_1,r_2,\ldots,r_k, n-r_k,n-r_{k-1},\ldots,n-r_1\}$. Define 'o' in $Ad_n(C_n(R))$ such that $\Phi_{n,x}(C_n(R))$ o $\Phi_{n,y}(C_n(R)) = \Phi_{n,xy}(C_n(R))$ and $C_n(xR)$ o $C_n(yR) = C_n((xy)R)$ for every $x,y\in \Phi_n$, under arithmetic modulo n. Clearly, $Ad_n(C_n(R)) = (T1_n(C_n(R))$, o) is the set of all circulant graphs which are Adam's isomorphic to $C_n(R)$ and $(Ad_n(C_n(R))$, o) is an abelian group called *the Adam's group* or *the Type-1 group on* $C_n(R)$ under 'o'.

DEFINITION 1.15 [8] Let *S* be a non-empty subset of Z_n, $r \in S$, $m,q,t,t',x \in Z_n$ such that $gcd(n, r) = m > 1$, $x = j+qm$, $0 \le j \le m-1$ and $0 \le q,t,t' \le \frac{n}{m} -1$. Define $\theta_{n,r,t}: Z_n \to Z_n$ such that $\theta_{n,r,t}(x) = x+jtm$ for every $x \in Z_n$ under arithmetic modulo *n*, $V_{n,r} = \{\theta_{n,r,t}: t = 0,1,\ldots,\frac{n}{m} -1\}$ and for $s \in Z_n$, $V_{n,r}(s) = \{\theta_{n,r,t}(s): t = 0,1,\ldots,\frac{n}{m} -1\}$ and $V_{n,r}(S) = \{V_{n,r}(s) : s \in S\}$. Define 'o' in $V_{n,r}$ such that $\theta_{n,r,t} o\, \theta_{n,r,t'} = \theta_{n,r,t+t'}$ and $(\theta_{n,r,t} o \theta_{n,r,t'})(x)$ ($= \theta_{n,r,t}(\theta_{n,r,t'}(x)) = \theta_{n,r,t}(x+jt'm) = (x+jt'm)+jtm = x+j(t+t')m$) $= \theta_{n,r,t+t'}(x)$ where $t+t'$ is calculated under addition modulo $\frac{n}{m}$. Clearly, for every $s \in Z_n$, $(V_{n,r}(s)$, o) is an *abelian group*.

DEFINITION 1.16 [8] Let $V(C_n(R)) = \{v_0,v_1,v_2,\ldots,v_{n-1}\}$, $V(K_n) = \{u_0,u_1,u_2,\ldots,u_{n-1}\}$, $r \in R$ and $j,m,q,t,x \in Z_n$ such that $gcd(n, r) = m > 1$, $x = j+qm$, $0 \le j \le m-1$ and $0 \le q,t \le \frac{n}{m} -1$. Define $\theta_{n,r,t}$: $V(C_n(R)) \to V(C_n(1,2,\ldots,n-1)) = V(K_n)$ such that $\theta_{n,r,t}(v_x) = u_{x+jtm}$ and $\theta_{n,r,t}((v_x,v_{x+s})) = (\theta_{n,r,t}(v_x), \theta_{n,r,t}(v_{x+s}))$ for every $x \in Z_n$ and $s \in R$, under subscript arithmetic reflexive modulo *n*. Let $V_{n,r} = \{\theta_{n,r,t}: t = 0,1,\ldots,\frac{n}{m} -1\}$ and $V_{n,r}(C_n(R)) = \{\theta_{n,r,t}(C_n(R)): t = 0,1,\ldots,\frac{n}{m} -1\}$. Define 'o' in $V_{n,r}$ such that $\theta_{n,r,t} o\, \theta_{n,r,t'} = \theta_{n,r,t+t'}$ and $\theta_{n,r,t}(C_n(R)) o \theta_{n,r,t'}(C_n(R)) = \theta_{n,r,t+t'}(C_n(R))$ for every $\theta_{n,r,t},\theta_{n,r,t'} \in V_{n,r}$ where $t+t'$ is calculated under addition modulo $\frac{n}{m}$. Then $(V_{n,r}(C_n(R))$, o) is an abelian group.

Clearly $V_{n,r}(C_n(R))$ contains all isomorphic circulant graphs of Type 2 of $C_n(R)$, if exist. Let $T2_{n,r}(C_n(R)) = \{C_n(R)\} \cup \{C_n(S): C_n(S)$ is Type-2 isomorphic to $C_n(R)$ w.r.t. $r\}$. Thus, $T2_{n,r}(C_n(R)) = \{C_n(R)\} \cup \{\theta_{n,r,t}(C_n(R)): \theta_{n,r,t}(C_n(R)) = C_n(S)$ and $C_n(S)$ is Type-2 isomorphic to $C_n(R)$ w.r.t. *r*, $0 \le t \le \frac{n}{m} -1\} \subseteq V_{n,r}(C_n(R))$ and $(T2_{n,r}(C_n(R))$, o) is a subgroup of $(V_{n,r}(C_n(R))$, o). Clearly, $T1_n(C_n(R)) \cap T2_{n,r}(C_n(R)) = \{C_n(R)\}$. $C_n(R)$ has Type-2 isomorphic circulant graph w.r.t. *r* iff $T2_{n,r}(C_n(R)) \neq \{C_n(R)\}$ iff $T2_{n,r}(C_n(R)) \cap \{C_n(R)\} \neq \Phi$ iff $|T2_{n,r}(C_n(R))| > 1$.

Definition 1.17 For any circulant graph $C_n(R)$, if $T2_{n,r}(C_n(R)) \neq \{C_n(R)\}$, then $(T2_{n,r}(C_n(R))$, o) is called *the Type-2 group of* $C_n(R)$ *w.r.t. r* under 'o'.

Cayley Isomorphism (CI) *problem* determines which graphs (or which groups) have the CI-property and its investigation started with the investigation of isomorphism of circulant graphs. An important achievement is the complete classification of cyclic CI-groups by Muzychuk in 1997 **[5],[6]**. But study on non-CI-graphs is not much done. Type-2 isomorphic circulant graphs are clearly graphs without CI-property. Theorems 1.10 and 1.11 gave classes of circulant graphs without CI-property. In this paper Theorem 2.3 gives new class of circulant graphs without CI-property.

Effort to obtain more circulant graphs without CI-property is the motivation for this work. For all basic ideas in graph theory, we follow **[3]**.

2 MAIN RESULT

THEOREM 2.1 *For i* = 1 to 5, $d_i = 5n(i-1)+1$ *and* $R_i = \{5, d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i\}$, *circulant graphs* $C_{125n}(R_i)$ *are isomorphic circulant graphs, n∈N.*

Proof: We prove that for *i* = 1 to 5, $d_i = 5n(i-1)+1$ and $R_i = \{5, d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i\}$, $\theta_{125n,5,in}(C_{125n}(R_1)) = C_{125n}(R_{i+1})$ where *i*+1 is calculated under addition modulo 5.

To simplify our calculation let us consider $R_i = \{5, d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i, 75n-d_i, 75n+d_i, 100n-d_i, 100n+d_i, 125n-d_i, 125n-5\}$, $d_i = 5n(i-1)+1$ and *i* = 1 to 5. In particular,

R_1 = {1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1},

R_2 = {5, 5*n*+1, 20*n*-1, 30*n*+1, 45*n*-1, 55*n*+1, 70*n*-1, 80*n*+1, 95*n*-1, 105*n*+1, 120*n*-1, 125*n*-5},

R_3 = {5, 10*n*+1, 15*n*-1, 35*n*+1, 40*n*-1, 60*n*+1, 65*n*-1, 85*n*+1, 90*n*-1, 110*n*+1, 115*n*-1, 125*n*-5},

R_4 = {5, 10*n*-1, 15*n*+1, 35*n*-1, 40*n*+1, 60*n*-1, 65*n*+1, 85*n*-1, 90*n*+1, 110*n*-1, 115*n*+1, 125*n*-5},

R_5 = {5, 5*n*-1, 20*n*+1, 30*n*-1, 45*n*+1, 55*n*-1, 70*n*+1, 80*n*-1, 95*n*+1, 105*n*-1, 120*n*+1, 125*n*-5}.

Using the definition of $\theta_{n,r,t}$ we get the following

$\theta_{125n,5,n}(R_1)$ = $\theta_{125n,5,n}$({1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1}) = {5*n*+1, 5, 20*n*-1, 30*n*+1, 45*n*-1, 55*n*+1, 70*n*-1, 80*n*+1, 95*n*-1, 105*n*+1, 125*n*-5, 120*n*-1} = R_2;

$\theta_{125n,5,2n}(R_1)$ = $\theta_{125n,5,2n}$({1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1}) = {10*n*+1, 5, 15*n*-1, 35*n*+1, 40*n*-1, 60*n*+1, 65*n*-1, 85*n*+1, 90*n*-1, 110*n*+1, 125*n*-5, 115*n*-1} = R_3;

$\theta_{125n,5,3n}(R_1)$ = $\theta_{125n,5,3n}$({1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1}) = {15*n*+1, 5, 10*n*-1, 40*n*+1, 35*n*-1, 65*n*+1, 60*n*-1, 90*n*+1, 85*n*-1, 115*n*+1, 125*n*-5, 110*n*-1} = R_4;

$\theta_{125n,5,4n}(R_1)$ = $\theta_{125n,5,4n}$({1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1}) = {20*n*+1, 5, 5*n*-1, 45*n*+1, 30*n*-1, 70*n*+1, 55*n*-1, 95*n*+1, 80*n*-1, 120*n*+1, 125*n*-5, 105*n*-1} = R_5.

Now the result follows from the definition of $\theta_{n,r,t}$.

THEOREM 2.2 *When*R_i = {5, d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i}, d_i = 5*n*(*i*-1)+1, *i*,*j* = 1 to 5 *and n*∈*N*, $\theta_{125n,5,jn}(C_{125n}(R_i))$ = $C_{125n}(R_{i+j})$*where i*+*jis calculated under addition modulo* 5 *and* $C_{125n}(R_i)$ *are Type*-2 *isomorphic circulant graphs*.

Proof: To prove that a set of circulant graphs {$C_n(R)$} are of Type-2 isomorphic, it is enough to prove that every pair of the circulant graphs are different (not the same), isomorphic and not of Adam's isomorphic (not of Type-1 isomorphic).

When R_i = {5, d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i}, d_i = 5*n*(*i*-1)+1, 1 ≤*i*,*j*≤ 5 *and n*∈*N*,R_i = R_j iff *i* = *j*. Thus for different *i*, the set of jump sizes of the five circulant graphs $C_{125n}(R_i)$ are different and thereby the five circulant graphs are also different.

In the proof of Theorem 2.1, we have $\theta_{125n,5,in}(C_{125n}(R_1))$ = $C_{125n}(R_{i+1})$ where *i*+1 is calculated under addition modulo 5, i = 1 to 5. Similarly it is easy to prove that $\theta_{125n,5,in}(C_{125n}(R_2))$ = $C_{125n}(R_{i+2})$, $\theta_{125n,5,in}(C_{125n}(R_3))$= $C_{125n}(R_{i+3})$, $\theta_{125n,5,in}(C_{125n}(R_4))$ = $C_{125n}(R_{i+4})$ and $\theta_{125n,5,in}(C_{125n}(R_5))$ = $C_{125n}(R_{i+5})$ = $C_{125n}(R_i)$ where R_i = {5, d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i}, d_i = 5*n*(*i*-1)+1, *i* = 1 to 5 and *n*∈*N*. This implies when R_i = {5, d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i}, d_i = 5*n*(*i*-1)+1, *i*,*j* = 1 to 5 and *n*∈*N*, $\theta_{125n,5,in}(C_{125n}(R_j))$ = $C_{125n}(R_{i+j})$ where*i*+*j* is calculated under addition modulo 5. This implies that for *i* = 1 to 5 all the five circulant graphs $C_{125n}(R_i)$ are isomorphic.

To complete the proof we are left with establishing their isomorphism is of Type-2. Now it is enough to prove that each pair of isomorphic circulant graphs $C_{125n}(R_i)$ and $C_{125n}(R_j)$ for *i*≠*j* are not of Type-1, 1 ≤*i*,*j*≤ 5. At first we prove that isomorphic circulant graphs $C_{125n}(R_1)$ and $C_{125n}(R_2)$ are Type-2.

Claim: For R_1 = {1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1}, R_2 = {5, 5*n*+1, 20*n*-1, 30*n*+1,45*n*-1, 55*n*+1} and *n*∈*N*, $C_{125n}(R_1)$ and $C_{125n}(R_2)$ are Type-2 isomorphic.

If not, they are of Adam's isomorphic. This implies, there exists *s*∈*N* such that $C_{125n}(sR_1)$ = $C_{125n}(R_2)$ where *s* = 5*x*-4 or *s* = 5*x*-3 or *s* = 5*x*-2 or *s* = 5*x*-1 and *gcd*(125*n*, *s*) = 1, *x*∈*N*. Now let us choose *s* such that *s* = 5*x*-4 such that gcd(125*n*, 5*x*-4) = 1, $C_{125n}((5x-4)R_1)$ = $C_{125n}(R_2)$ and *x*∈*N*. This implies, (5*x*-4){1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1, 75*n*-1, 75*n*+1, 100*n*-1, 100*n*+1, 125*n*-5, 125*n*-1} = {5, 5*n*+1, 20*n*-1, 30*n*+1, 45*n*-1, 55*n*+1, 70*n*-1, 80*n*+1, 95*n*-1, 105*n*+1, 120*n*-1, 125*n*-5} under arithmetic modulo 125*n*. This implies, 5(5*x*-4), (5*x*-4)(125*n*-5), 5+125*n*p_1 and 125*n*-5+125*n*p_2 are the only numbers, each is a multiple of 5, in the two sets for some p_1,p_2∈N_0. Thus when *s* = 5*x*-4 the following two cases arise.

Case i 5(5*x*-4) = 5+125*n*p_1, p_1∈N_0, *x*∈*N*, 1 ≤ 5*x*-4 ≤ 125*n*-1.

In this case, p_1 = 0 or 1 or 2 or 3 or 4 since 1 ≤ 5*x*-4 ≤ 125*n*-1 and *n*,*x*∈*N*. When p_1 = 0, 5*x*-4 = 1; p_1 = 1, 5*x*-4 = 25*n*+1; p_1 = 2, 5*x*-4 = 50*n*+1; p_1 = 3, 5*x*-4 = 75*n*+1; p_1 = 4, 5*x*-4 = 100*n*+1 and in each case, graph $C_{125n}((5x-4)R_1)$ is same as $C_{125n}(R_1)$. The jump sizes of the circulant graph $C_{125n}(sR_1)$

corresponding to Adam's isomorphism when s = 5x-4 = 25n+1, s = 5x-4 = 50n+1, s = 5x-4 = 75n+1 and s = 5x-4 = 100n+1 are given in Table 1.

Case ii 5(5x-4) = 125n-5+125np_2, $p_2 \in N_0$, $x \in N$, 1 ≤ 5x-4 ≤ 125n-1.

In this case, p_2 = 0 or 1 or 2 or 3 or 4 since 1 ≤ 5x-4 ≤ 125n-1 and $n,x \in N$. When p_2 = 0, 5x-4 = 25n-1; p_2 = 1, 5x-4 = 50n-1; p_2 = 2, 5x-4 = 75n-1; p_2 = 3, 5x-4 = 100n-1; p_2 = 2, 5x-4 = 125n-1 and in each case, graph $C_{125n}((5x-4)R_1)$ is same as $C_{125n}(R_1)$. The jump sizes of the circulant graph $C_{125n}(sR_1)$ corresponding to Adam's isomorphism when s = 5x-4 = 25n-1, s = 5x-4 = 50n-1, s = 5x-4 = 75n-1, s = 5x-4 = 100n-1 and s = 5x-4 = 125n-1 are given in Table 1.

Consider the case when s = 5x-3 such that $C_{125n}(sR_1) = C_{125n}(R_2)$ where gcd(125n,5x-3) = 1, 1 ≤ 5x-3 ≤ 125n-1 and $x \in N$. This implies, (5x-3){1, 5, 25n-1, 25n+1, 50n-1, 50n+1, 75n-1, 75n+1, 100n-1, 100n+1, 125n-5, 125n-1} = {5, 5n+1, 20n-1, 30n+1, 45n-1, 55n+1, 70n-1, 80n+1, 95n-1, 105n+1, 120n-1, 125n-5} under arithmetic modulo 125n. This implies, 5(5x-3), (5x-3)(125n-5), 5+125np_1 and 125n-5+125np_2 are the only numbers, each is a multiple of 5, in the two sets for some $p_1, p_2 \in N_0$. Thus when s = 5x-3 the following two cases arise.

Table 1. *Calculation of rs under arithmetic modulo* 125*n w.r.t.* R_1 *where s = 5x-i, i*= 1,2,3,4.

s \ *r*	**1**	**25*n*-1**	**25*n*+1**	**50*n*-1**	**50*n*+1**	**75*n*-1**	**75*n*+1**	**100*n*-1**	**100*n*+1**	**125*n*-1**
25*n*+1	25*n*+1	125*n*-1	50*n*+1	25*n*-1	75*n*+1	50*n*-1	100*n*+1	75*n*-1	1	100*n*-1
50*n*+1	50*n*+1	100*n*-1	75*n*+1	125*n*-1	100*n*+1	25*n*-1	1	50*n*-1	25*n*+1	75*n*-1
75*n*+1	75*n*+1	75*n*-1	100*n*+1	100*n*-1	1	125*n*-1	25*n*+1	25*n*-1	50*n*+1	50*n*-1
100*n*+1	100*n*+1	50*n*-1	1	75*n*-1	25*n*+1	100*n*-1	50*n*+1	125*n*-1	75*n*+1	25*n*-1
25*n*-1	25*n*-1	75*n*+1	125*n*-1	50*n*+1	100*n*-1	25*n*+1	75*n*-1	1	50*n*-1	100*n*+1
50*n*-1	50*n*-1	50*n*+1	25*n*-1	25*n*+1	125*n*-1	1	100*n*-1	100*n*+1	75*n*-1	75*n*+1
75*n*-1	75*n*-1	25*n*+1	50*n*-1	1	25*n*-1	100*n*+1	125*n*-1	75*n*+1	100*n*-1	50*n*+1
100*n*-1	100*n*-1	1	75*n*-1	100*n*+1	50*n*-1	25*n*+1	25*n*-1	50*n*+1	125*n*-1	25*n*+1
125*n*-1	125*n*-1	100*n*+1	100*n*-1	75*n*+1	75*n*-1	50*n*+1	50*n*-1	25*n*+1	25*n*-1	1

Table 2. *Calculation of rs under arithmetic modulo* 125*n w.r.t.* R_2 *where s = 5x-i, i* = 1,2,3,4.

s \ *r*	**5*n*+1**	**20*n*-1**	**30*n*+1**	**45*n*-1**	**55*n*+1**	**70*n*-1**	**80*n*+1**	**95*n*-1**	**105*n*+1**	**120*n*-1**
25*n*+1	30*n*+1	120*n*-1	55*n*+1	20*n*-1	80*n*+1	45*n*-1	105*n*+1	70*n*-1	5*n*+1	95*n*-1
50*n*+1	55*n*+1	95*n*-1	80*n*+1	120*n*-1	105*n*+1	20*n*-1	5*n*+1	45*n*-1	30*n*+1	70*n*-1
75*n*+1	80*n*+1	70*n*-1	105*n*+1	95*n*-1	5*n*+1	120*n*-1	30*n*+1	20*n*-1	55*n*+1	45*n*-1
100*n*+1	105*n*+1	45*n*-1	5*n*+1	70*n*-1	25*n*+1	95*n*-1	55*n*+1	120*n*-1	80*n*+1	20*n*-1
25*n*-1	20*n*-1	80*n*+1	120*n*-1	55*n*+1	95*n*-1	30*n*+1	70*n*-1	5*n*+1	45*n*-1	105*n*+1
50*n*-1	45*n*-1	55*n*+1	20*n*-1	30*n*+1	120*n*-1	5*n*+1	95*n*-1	105*n*+1	70*n*-1	80*n*+1
75*n*-1	70*n*-1	30*n*+1	45*n*-1	5*n*+1	20*n*-1	105*n*+1	120*n*-1	80*n*+1	95*n*-1	55*n*+1
100*n*-1	95*n*-1	5*n*+1	70*n*-1	105*n*+1	45*n*-1	80*n*+1	20*n*-1	55*n*+1	120*n*-1	30*n*+1
125*n*-1	120*n*-1	105*n*+1	95*n*-1	80*n*+1	70*n*-1	55*n*+1	45*n*-1	30*n*+1	20*n*-1	5*n*+1

Table 3.*Calculation of rs under arithmetic modulo* 125*n* *w.r.t.* R_3 *where s = 5x-i, i* =1,2,3,4.

s \ r	**10*n*+1**	**15*n*-1**	**35*n*+1**	**40*n*-1**	**60*n*+1**	**65*n*-1**	**85*n*+1**	**90*n*-1**	**110*n*+1**	**115*n*-1**
25*n*+1	35*n*+1	115*n*-1	60*n*+1	15*n*-1	85*n*+1	40*n*-1	110*n*+1	65*n*-1	10*n*+1	90*n*-1
50*n*+1	60*n*+1	90*n*-1	85*n*+1	115*n*-1	110*n*+1	15*n*-1	10*n*+1	40*n*-1	35*n*+1	65*n*-1
75*n*+1	85*n*+1	65*n*-1	110*n*+1	90*n*-1	10*n*+1	115*n*-1	35*n*+1	15*n*-1	60*n*+1	40*n*-1
100*n*+1	110*n*+1	40*n*-1	10*n*+1	65*n*-1	35*n*+1	90*n*-1	60*n*+1	115*n*-1	85*n*+1	15*n*-1
25*n*-1	15*n*-1	85*n*+1	115*n*-1	60*n*+1	90*n*-1	35*n*+1	65*n*-1	10*n*+1	40*n*-1	110*n*+1
50*n*-1	40*n*-1	60*n*+1	15*n*-1	35*n*+1	115*n*-1	10*n*+1	90*n*-1	110*n*+1	65*n*-1	85*n*+1
75*n*-1	65*n*-1	35*n*+1	40*n*-1	10*n*+1	15*n*-1	110*n*+1	115*n*-1	85*n*+1	90*n*-1	60*n*+1
100*n*-1	90*n*-1	10*n*+1	65*n*-1	110*n*+1	40*n*-1	85*n*+1	15*n*-1	60*n*+1	115*n*-1	35*n*+1
125*n*-1	115*n*-1	110*n*+1	90*n*-1	85*n*+1	65*n*-1	60*n*+1	40*n*-1	35*n*+1	15*n*-1	10*n*+1

Case i $5(5x-3) = 5+125np_1$, $p_1 \in N_0$, $x \in N$, $1 \le 5x-3 \le 125n-1$.

In this case, $p_1 = 0$ or 1 or 2 or 3 or 4 since $1 \le 5x-3 \le 125n-1$ and $n,x \in N$. When $p_1 = 0$, $5x-3 = 1$; $p_1 = 1$, $5x-3 = 25n+1$; $p_1 = 2$, $5x-3 = 50n+1$; $p_1 = 3$, $5x-3 = 75n+1$; $p_1 = 4$, $5x-3 = 100n+1$ and in each case, graph $C_{125n}((5x-3)R_1)$ is same as graph $C_{125n}(R_1)$. The jump sizes of the circulant graph $C_{125n}(sR_1)$ corresponding to Adam's isomorphism when $s = 5x-3 = 25n+1$, $s = 5x-3 = 50n+1$, $s = 5x-3 = 75n+1$ and $s = 5x-3 = 100n+1$ are given in Table 1.

Case ii $5(5x-3) = 125n-5+125np_2$, $p_2 \in N_0$, $x \in N$, $1 \le 5x-3 \le 125n-1$.

In this case, $p_2 = 0$ or 1 or 2 or 3 or 4 since $1 \le 5x-3 \le 125n-1$ and $n,x \in N$. When $p_2 = 0$, $5x-3 = 25n-1$; $p_2 = 1$, $5x-3 = 50n-1$; $p_2 = 2$, $5x-3 = 75n-1$; $p_2 = 3$, $5x-3 = 100n-1$; $p_2 = 4$, $5x-3 = 125n-1$ and in each case, graph $C_{125n}((5x-3)R_1)$ is same as $C_{125n}(R_1)$. The jump sizes of the circulant graph $C_{125n}(sR_1)$ corresponding to Adam's isomorphism when $s = 5x-3 = 25n-1$, $s = 5x-3 = 50n-1$, $s = 5x-3 = 75n-1$, $s = 5x-3 = 100n-1$ and $s = 5x-3 = 125n-1$ are given in Table 1.

Similarly when $s = 5x-2$ and $s = 5x-1$ it is easy to see that $C_{125n}((5x-2)R_1) = C_{125n}(R_1)$ and $C_{125n}((5x-1)R_1) = C_{125n}(R_1)$. Thus $C_{125n}(sR_1) = C_{125n}(R_1)$ when $s = 5x-4$ or $s = 5x-3$ or $s = 5x-2$ or $s = 5x-1$ where $gcd(125n, s) = 1$ and $n,x \in N$. This implies $C_{125n}(sR_1) \neq C_{125n}(R_2)$ for every $s \in N$ such that $gcd(125n, s) = 1$ and $n \in N$.

This shows that the isomorphic circulant graphs $C_{125n}(R_1)$ and $C_{125n}(R_2)$ for R_1 = {1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1}, R_2 = {5, 5*n*+1, 20*n*-1, 30*n*+1,45*n*-1, 55*n*+1} are not of Type-1, $n \in N$. This implies, for R_1 = {1, 5, 25*n*-1, 25*n*+1, 50*n*-1, 50*n*+1}, R_2 = {5, 5*n*+1, 20*n*-1, 30*n*+1, 45*n*-1, 55*n*+1} and $n \in N$, $C_{125n}(R_1)$ and $C_{125n}(R_2)$ are Type-2 isomorphic.

By similar discussion and calculation it is easy to prove that circulant graphs $C_{125n}(R_1)$ and $C_{125n}(R_j)$ are Type-2 isomorphic for *j* = 3,4,5. Thus we could prove that $C_{125n}(R_1)$ and $C_{125n}(R_j)$ are Type-2 isomorphic for *j* = 2,3,4,5. Table-*i* corresponds to calculation of *rs* under arithmetic modulo 125*n* w.r.t R_i and R_{j+1} for *j* = *i*,*i*+1,…,4 and *i* = 1,2,3,4.

The above discussion and calculations prove that circulant graphs $C_{125n}(R_i)$ and $C_{125n}(R_j)$ for $i \neq j$ are Type-2 isomorphic,*i*,*j*= 1,2,3,4,5. Hence the result follows. □

THEOREM 2.3 *For* *i* = 1 to 5, d_i = 5*n*(*i*-1)+1, 3 ≤*k* *and* R_i = {d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i, 5p_1,5p_2,…,5p_{k-2}}, *circulant graphs* $C_{125n}(R_i)$ *are Type-*2 *isomorphic and without CI-property where* *gcd*(p_1,p_2,…,p_{k-2}) = 1 *and* *n*,p_1,p_2,…,$p_{k-2} \in N$.

Proof: For *i* = 1 to 5, d_i = 5*n*(*i*-1)+1, 3 ≤*k* and R_i = {5, d_i, 25*n*-d_i, 25*n*+d_i, 50*n*-d_i, 50*n*+d_i}, circulant graphs $C_{125n}(R_i)$ are Type-2 isomorphic, using Theorem 2.2, $n \in N$. Lemma 1.5 helps us while searching for possible value(s) of *t* such that the transformed graph $\theta_{n,r,t}(C_n(R))$ is circulant of the form $C_n(S)$ for some $S \subseteq [1, n/2]$, the calculation on r_j which are integer multiples of $m = \gcd(n,r)$ need not be done as there is no change in these r_j under the transformation $\theta_{n,r,t}$. This implies, for i = 1 to 5, $d_i = 5n(i-1)+1$ and $R_i = \{d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i, 5p_1, 5p_2, \ldots, 5p_{k-2}\}$, circulant graphs $C_{125n}(R_i)$ are Type-2 isomorphic circulant graphs where $3 \le k$, $gcd(p_1,p_2,\ldots,p_{k-2}) = 1$ and $n,p_1,p_2,\ldots,p_{k-2} \in N$. Type-2 isomorphic circulant graphs are graphs *without CI-property*. Hence the result follows.

Table 4. *Calculation of rs under arithmetic modulo* 125*n w.r.t.* R_4 *where s* = 5*x*-*i*, *i* = 1,2,3,4.

s \ r	$10n-1$	$15n+1$	$35n-1$	$40n+1$	$60n-1$	$65n+1$	$85n-1$	$90n+1$	$110n-1$	$115n+1$
$25n+1$	$110n-1$	$40n+1$	$10n-1$	$65n+1$	$35n-1$	$90n+1$	$60n-1$	$115n+1$	$85n-1$	$15n+1$
$50n+1$	$85n-1$	$65n+1$	$110n-1$	$90n+1$	$10n-1$	$115n+1$	$35n-1$	$15n+1$	$60n-1$	$40n+1$
$75n+1$	$60n-1$	$90n+1$	$85n-1$	$115n+1$	$110n-1$	$15n+1$	$10n-1$	$40n+1$	$35n-1$	$65n+1$
$100n+1$	$35n-1$	$115n+1$	$60n-1$	$15n+1$	$85n-1$	$40n+1$	$110n-1$	$65n+1$	$10n-1$	$90n+1$
$25n-1$	$90n+1$	$10n-1$	$65n+1$	$110n-1$	$40n+1$	$85n-1$	$15n+1$	$60n-1$	$115n+1$	$35n-1$
$50n-1$	$65n+1$	$35n-1$	$40n+1$	$10n-1$	$15n+1$	$110n-1$	$115n+1$	$85n-1$	$90n+1$	$60n-1$
$75n-1$	$40n+1$	$60n-1$	$15n+1$	$35n-1$	$115n+1$	$10n-1$	$90n+1$	$110n-1$	$65n+1$	$85n-1$
$100n-1$	$15n+1$	$85n-1$	$115n+1$	$60n-1$	$90n+1$	$35n-1$	$65n+1$	$10n-1$	$40n+1$	$110n-1$
$125n-1$	$115n+1$	$110n-1$	$90n+1$	$85n-1$	$65n+1$	$60n-1$	$40n+1$	$35n-1$	$15n+1$	$10n-1$

Table 5 Calculation of *rs* under arithmetic modulo 125*n* w.r.t. R_5 where *s* = 5*x*-*i*, *i* = 1,2,3,4.

Circulant graphs C_{125}(1,5,24,26,49,51), C_{125}(5,6,19,31,44,56), C_{125}(5,11,14,36,39,61),

s \ r	$5n-1$	$20n+1$	$30n-1$	$45n+1$	$55n-1$	$70n+1$	$80n-1$	$95n+1$	$105n-1$	$120n+1$
$25n+1$	$105n-1$	$45n+1$	$5n-1$	$70n+1$	$30n-1$	$95n+1$	$55n-1$	$120n+1$	$80n-1$	$20n+1$
$50n+1$	$80n-1$	$70n+1$	$105n-1$	$95n+1$	$5n-1$	$120n+1$	$30n-1$	$20n+1$	$55n-1$	$45n+1$
$75n+1$	$55n-1$	$95n+1$	$80n-1$	$120n+1$	$105n-1$	$20n+1$	$5n-1$	$45n+1$	$30n-1$	$70n+1$
$100n+1$	$30n-1$	$120n+1$	$55n-1$	$20n+1$	$80n-1$	$45n+1$	$105n-1$	$70n+1$	$5n-1$	$95n+1$
$25n-1$	$95n+1$	$5n-1$	$70n+1$	$105n-1$	$45n+1$	$80n-1$	$20n+1$	$55n-1$	$120n+1$	$30n-1$
$50n-1$	$70n+1$	$30n-1$	$45n+1$	$5n-1$	$20n+1$	$105n-1$	$120n+1$	$80n-1$	$95n+1$	$55n-1$
$75n-1$	$45n+1$	$55n-1$	$20n+1$	$30n-1$	$120n+1$	$5n-1$	$95n+1$	$105n-1$	$70n+1$	$80n-1$
$100n-1$	$20n+1$	$80n-1$	$120n+1$	$55n-1$	$95n+1$	$30n-1$	$70n+1$	$5n-1$	$45n+1$	$105n-1$
$125n-1$	$120n+1$	$105n-1$	$95n+1$	$80n-1$	$70n+1$	$55n-1$	$45n+1$	$30n-1$	$20n+1$	$5n-1$

C_{125}(5,9,16,34,41,66) = C_{125}(5,9,16,34,41,59) and C_{125}(4,5,21,29,71,76) = C_{125}(4,5,21,29,49,54) are isomorphic and are of Type 2.

THEOREM 2.4 *For* i = 1 to 5, $d_i = 5n(i-1)+1$, $3 \le k$ *and* $R_i = \{d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i, 5p_1, 5p_2, \ldots, 5p_{k-2}\}$, $(V_{125n,5}(C_{125n}(R_i)), \circ)$ *is an abelian group where* $gcd(p_1,p_2,\ldots,p_{k-2}) = 1$, $n,p_1,p_2,\ldots,p_{k-2} \in N$.

Proof: The result follows from Theorem 2.3 and definition of $V_{n,r}$.

Let $C_{125}(1,5,24,26,49,51) = R_1$, $C_{125}(5,6,19,31,44,56) = R_2$, $C_{125}(5,11,14,36,39,61) = R_3$, $C_{125}(5,9,16,34,41,66) = C_{125}(5,9,16,34,41,59) = R_4$ and $C_{125}(4,5,21,29,71,76) =$

$C_{125}(4,5,21,29,49,54) = R_5$. Then the corresponding Type 2 group is ($T2_{125,5}(C_{125}(R_i))$, o) where $T2_{125,5}(C_{125}(R_i))$ = {R_1, R_2, R_3, R_4, R_5} for i = 1,2,3,4,5.

Open Problem Find $T2_{125n,5}(C_{125n}(R_i))$ when R_i = {d_i, 25n-d_i, 25n+d_i, 50n-d_i, 50n+d_i, 5p_1,5p_2,…,5p_{k-2}},1 ≤i≤ 5,d_i = 5n(i-1)+1, 3 ≤k,gcd(p_1,p_2,…,p_{k-2}) = 1, n,p_1,p_2,…,p_{k-2}∈N.

3 CONCLUSION

In this paper and in **[12]**, **[14],** we obtained families of isomorphic circulant graphs of Type-2 (and without CI-property), each with 2, 3 or 5 copies of isomorphic circulant subgraphs. One can go for general result on circulant graphs with m_i= gcd(n, r_i) is odd and > 5.

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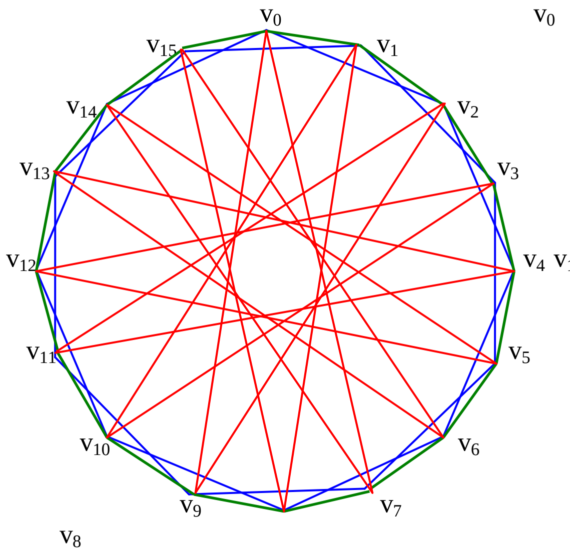

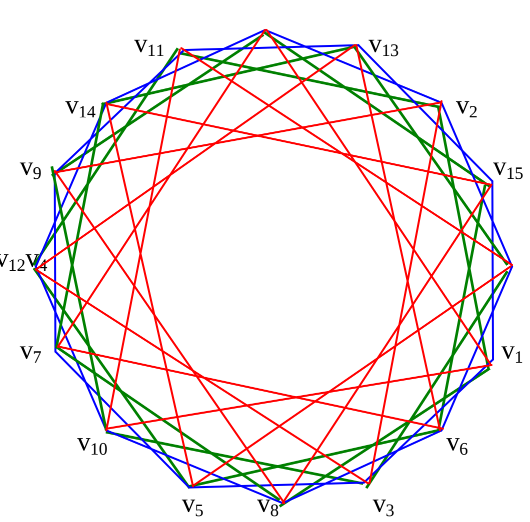

Fig. 1.$C_{16}(1,2,7)$**Fig. 2.**$C_{16}(2,3,5)$