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Connected Total Dominating Sets and Connected Total Domination Polynomials of Gem Graphs

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Abstract: *Let G = (V, E) be a simple graph. A set S of vertices in a graph G is said to be a total dominating set if every vertex $v \in V$ is adjacent to an element of S. A total dominating set S of G is called a connected total dominating set if the induced subgraph <S> is connected. In this paper, we study the concept of connected total domination polynomials of the Gem graph G_n. The connected total domination polynomial of a graph G of order n is the polynomial* $D_{ct}(G, x) = \sum_{i=\gamma_{ct}(G)}^{n} d_{ct}(G,i)x^i$ *, where $d_{ct}(G, i)$ is the number of connected total dominating sets of G of size i and $\gamma_{ct}(G)$ is the connected total domination number of G. We obtain some properties of $D_{ct}(G_n, x)$ and their coefficients. Also, we obtain the recursive formula to derive the connected total dominating sets of the Gem graph G_n.*

Keywords: *Gem graph, connected total dominating set, connected total domination number, connected total domination polynomial.*

1. INTRODUCTION

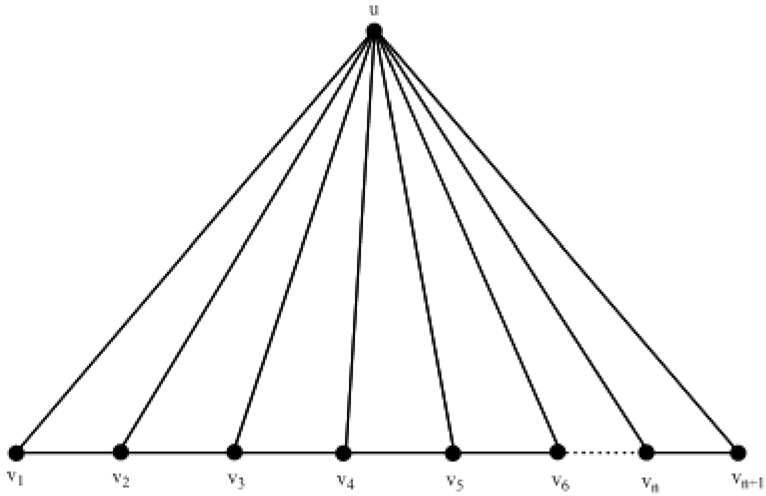
Let G = (V, E) be a simple graph of order |V | = n. A set S of vertices in a graph G is said to be a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S.

A set S of vertices in a graph G is said to be a total dominating set if every vertex $v \in V$ is adjacent to an element of S. A total dominating set S of G is called a connected total dominating set if the induced subgraph <S> is connected. The minimum cardinality of a connected total dominating set S of G is called the connected total domination number and is denoted by $\gamma_{ct}(G)$.

Let G_n be a Gem graph with n + 2 vertices. In the next section, we construct the families of the connected total dominating sets of G_n by recursive method. In section 3, we use the results obtained in section 2 to study the connected total domination polynomials of the Gem graph G_n. As usual, we use $\binom{n}{i}$ for the combination n to i.

2. CONNECTED TOTAL DOMINATING SETS OF A GEM GRAPH G_n

Gem graph [5] is a graph obtained by joining an additional vertex u to each vertex of a path P_{n+1} and is denoted by G_n.



Gem Graph G_n

Figure 1

Let G_n be a Gem graph with n + 2 vertices. Label the vertices of G_n as $v_1, v_2, v_3, \ldots, v_{n+1}, v_{n+2}$. Then , $V(G_n) = \{ v_1, v_2, \ldots, v_{n+1}, v_{n+2}\}$ and $E(G_n) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), \ldots, (v_1, v_{n+1}), (v_1, v_{n+2}), (v_2, v_3), (v_3, v_4), \ldots, (v_{n+1}, v_{n+2})\}$. Let $d_{ct}(G_n, i)$ be the number of connected total dominating sets of G_n with cardinality i.

Lemma 2.1

The following properties hold for all graph G with $| V(G) | = n + 2$ vertices.

(i) $d_{ct}(G, n + 2) = 1$.

(ii) $d_{ct}(G, n + 1) = n + 2$.

(iii) $d_{ct}(G, i) = 0$ if $i > n + 2$.

(iv) $d_{ct}(G, 1) = 0$.

Proof:

Let G = (V, E) be a simple graph of order n + 2.

(i)We have $\mathcal{D}_{ct}(G, n + 2) = \{ v_1, v_2, \ldots, v_{n+1}, v_{n+2}\}$.

Therefore, $d_{ct}(G, n + 2) = 1$.

(ii) Also, $\mathcal{D}_{ct}(G, n + 1) = \{\{v_1, v_2, \ldots, v_{n+1}, v_{n+2}\} - x \,/\, x \in \{ v_1, v_2, \ldots, v_{n+1}, v_{n+2} \}\}$.

Therefore, $d_{ct}(G, n + 1) = n + 2$.

(iii) There does not exist a subgraph H of G such that $| V(H) | > | V(G) |$. Therefore,

$d_{ct}(G, i) = 0$ if $i > n + 2$.

(iv) By the definition of total domination, a single vertex cannot dominate totally.

Therefore, $d_{ct}(G, 1) = 0$.

Lemma 2.2

For all $n \in Z+$, $\binom{n}{i} = 0$ if $i > n$ or $i < 0$.

Lemma 2.3

For any path graph P_n with n vertices,

(i) $d_{ct}(P_n, n) = 1$.

(ii) $d_{ct}(P_n, n - 1) = 2$.

(iii) $d_{ct}(P_n, n-2) = 1$.

(iv) $d_{ct}(P_n, i) = 0$ if $i < n-2$ or $i > n$.

Theorem 2.4

For any path graph P_n with n vertices, $D_{ct}(P_n, x) = x^{n-2} + 2x^{n-1} + x^n$.

Proof:

The proof is given in [6].

Theorem 2.5

Let S_n be a star graph with n vertices, then $d_{ct}(S_n, i) = \binom{n}{i} - \binom{n-1}{i}$ for all $n \geq 3$.

Proof:

The proof is given in [7].

Theorem 2.6

Let G_n be a Gem graph with n + 2 vertices, then $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all i.

Proof:

Let G_n be a Gem graph with n + 2 vertices. Let $V(G_n) = \{v_1, v_2, \ldots, v_{n+1}, v_{n+2}\}$ and $E(G_n) = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), \ldots, (v_1, v_{n+1}), (v_1, v_{n+2}), (v_2, v_3), (v_3, v_4), \ldots, (v_{n+1}, v_{n+2})\}$. Let S_{n+2} be the star graph with n + 2 vertices and P_{n+1} be the path graph with n + 1 vertices. $v_1 \in V(S_{n+2})$ is the vertex adjacent to all the vertices of P_{n+1}. We have S_{n+2} is a spanning subgraph of G_n and since $G_n - v_1 = P_{n+1}$, $S_{n+2} \cup P_{n+1} = G_n$. Therefore, the number of connected total dominating sets of the Gem graph G_n with cardinality i is the sum of the connected total dominating sets of the star graph S_{n+2} with cardinality i and the number of connected total dominating sets of the Path graph P_{n+1} with cardinality i.

Hence, $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all i.

Theorem 2.7

Let G_n be a Gem graph with n + 2 vertices, then

(i) $d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i}$ for all $i < n-1$, $n \geq 4$.

(ii) $d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i} + 1$ if $i = n-1, n+1$.

(iii) $d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i} + 2$ if $i = n$.

Proof:

(i) By Theorem 2.6, we have, $d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)$ for all i.

Since, $d_{ct}(P_{n+1}, i) = 0$ for all $i < n-1$, we have,

$d_{ct}(G_n, i) = d_{ct}(S_{n+2}, i)$ for all $i < n-1$.

$= \binom{n+2}{i} - \binom{n+1}{i}$ for all $i < n-1$, by Theorem 2.5.

(ii) Since, $d_{ct}(P_{n+1}, i) = 1$ for $i = n-1, n+1$, we have,

$d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i} + 1$, if $i = n-1, n+1$.

(iii) Since, $d_{ct}(P_{n+1}, i) = 2$ if $i = n$, we have,

$$d_{ct}(G_n, i) = \binom{n+2}{i} - \binom{n+1}{i} + 2 \text{ if } i = n.$$

Corollary 2.8

Let G_n be a Gem graph with n + 2 vertices, then

(i) $d_{ct}(G_n, i) = \binom{n+1}{i-1}$ for all $i < n - 1$, $n \geq 4$.

(ii) $d_{ct}(G_n, i) = \binom{n+1}{i-1} + 1$ for $i = n - 1$, $n + 1$.

(iii) $d_{ct}(G_n, i) = \binom{n+1}{i-1} + 2$ if $i = n$.

Proof:

Since, $\binom{n+2}{i} - \binom{n+1}{i} = \binom{n+1}{i-1}$, (i), (ii) and (iii) follows from Theorem 2.7 (i), (ii) and (iii).

Theorem 2.9

Let G_n be a Gem graph with n + 2 vertices, then

(i) $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + 2$ if $i = 2$.

(ii) $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1)$ for all $3 \leq i \leq n + 2$ and $i \neq n - 2$, $n - 1$, n.

(iii) $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) - 1$ for $i = n$, $n - 2$.

(iv) $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1)$ for $i = n - 1$.

Proof:

(i) When $i = 2$, $d_{ct}(G_n, 2) = \binom{n+1}{1}$, by Corollary 2.8 (i).

$= n + 1$.

Consider, $d_{ct}(G_{n-1}, 2) + 1 = \binom{n}{1} + 1$.

$= n + 1$.

$d_{ct}(G_{n-1}, 2) + 1 = d_{ct}(G_n, 2)$.

Therefore, $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + 1$ if $i = 2$.

(ii) By Corollary 2.8 (i), we have, $d_{ct}(G_n, i) = \binom{n+1}{i-1}$ for all $i < n - 1$.

Also, $d_{ct}(G_{n-1}, i) = \binom{n}{i-1}$ and $d_{ct}(G_{n-1}, i - 1) = \binom{n}{i-2}$.

Consider, $d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1) = \binom{n}{i-1} + \binom{n}{i-2}$.

$$= \binom{n+1}{i-1}.$$

$$= d_{ct}(G_n, i).$$

Therefore, $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i - 1)$ for all $3 \le i \le n + 2$ and $i \ne n-2, n-1, n$.

(iii) When $i = n$, we have, $d_{ct}(G_n, n) = \binom{n+1}{n-1} + 2$, by Corollary 2.8 (iii).

$$= \binom{n+1}{2} + 2.$$

$d_{ct}(G_{n-1}, n) = \binom{n}{n-1} + 2$, by Corollary 2.8 (iii).

$$= \binom{n}{1} + 2.$$

$d_{ct}(G_{n-1}, n-1) = \binom{n}{n-2} + 1$, by Corollary 2.8 (ii).

$$= \binom{n}{2} + 1.$$

Consider, $d_{ct}(G_{n-1}, n) + d_{ct}(G_{n-1}, n-1) = \binom{n}{1} + 2 + \binom{n}{2} + 1.$

$$= \binom{n+1}{2} + 2 + 1.$$

$$= d_{ct}(G_n, n) + 1.$$

Therefore, $d_{ct}(G_n, n) = d_{ct}(G_{n-1}, n) + d_{ct}(G_{n-1}, n-1) - 1$.

Hence, $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1) - 1$ if $i = n$.

When $i = n - 2$, we have,

$d_{ct}(G_n, n-2) = \binom{n+1}{n-3}$, by Corollary 2.8 (i).

$$= \binom{n+1}{4}.$$

$d_{ct}(G_{n-1}, n-2) = \binom{n}{n-3} + 1$, by Corollary 2.8 (ii).

$$= \binom{n}{3} + 1.$$

$d_{ct}(G_{n-1}, n-3) = \binom{n}{n-4}$, by Corollary 2.8 (i).

$$= \binom{n}{4}.$$

Consider, $d_{ct}(G_{n-1}, n-2) + d_{ct}(G_{n-1}, n-3) = \binom{n}{3} + 1 + \binom{n}{4}$.

$$= \binom{n+1}{4} + 1.$$

$$= d_{ct}(G_n, n-2) + 1.$$

Therefore, $d_{ct}(G_n, n-2) = d_{ct}(G_{n-1}, n-2) + d_{ct}(G_{n-1}, n-3) - 1$.

Hence, $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1) - 1$, if $i = n - 2$.

(iv) When $i = n - 1$, we have,

$d_{ct}(G_n, n-1) = \binom{n+1}{n-2} + 1$, by Corollary 2.8 (ii) .

$$= \binom{n+1}{3} + 1.$$

$d_{ct}(G_{n-1}, n-1) = \binom{n}{n-2} + 2$, by Corollary 2.8 (iii).

$$= \binom{n}{2} + 2.$$

$d_{ct}(G_{n-1}, n-2) = \binom{n}{n-3} + 1$, by Corollary 2.8 (ii) .

$$= \binom{n}{3} + 1 .$$

Consider, $d_{ct}(G_{n-1}, n-1) + d_{ct}(G_{n-1}, n-2) = \binom{n}{2} + 2 + \binom{n}{3} + 1$.

$$= \binom{n+1}{3} + 1 + 2.$$

$$= d_{ct}(G_n, n-1) + 2.$$

Therefore, $d_{ct}(G_n, n-1) = d_{ct}(G_{n-1}, n-1) + d_{ct}(G_{n-1}, n-2) - 2$.

Hence, $d_{ct}(G_n, i) = d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1) - 2$ if $i = n - 1$.

3. CONNECTED TOTAL DOMINATION POLYNOMIALS OF A GEM GRAPH G_n

Definition 3.1

Let $d_{ct}(G_n, i)$ be the number of connected total dominating sets of a Gem graph G_n with cardinality i. Then, the connected total domination polynomial of G_n is defined as,

$$D_{ct}(G_n, x) = \sum_{i=\gamma_{ct}(G_n)}^{n+2} d_{ct}(G_n, i)\, x^i.$$

Remark 3.2

$\gamma_{ct}(G_n) = 2$

Proof:

Let G_n be a Gem graph with n + 2 vertices. Let $v_1 \in V(G_n)$ and v_1 is the vertex adjacent to all the vertices $v_2, v_3, \ldots, v_{n+2}$. The vertex v_1 and one more vertex from $\{v_2, v_3, \ldots, v_{n+2}\}$ is enough to cover all the other vertices. Therefore, the minimum cardinality is 2. Hence, $\gamma_{ct}(G_n) = 2$.

Theorem 3.3

Let S_n be a star graph with n vertices, then $D_{ct}(S_n, x) = x\,[\,(1 + x)^{n-1} - 1]$.

Proof:

The proof is given is [7].

Theorem 3.4

Let S_n, $n \geq 3$ be a star graph with n vertices, then

(i) $D_{ct}(S_n, x) = \sum_{i=2}^{n} \binom{n}{i} x^i - \sum_{i=2}^{n} \binom{n-1}{i} x^i$.

(ii) $D_{ct}(S_n, x) = \sum_{i=2}^{n} \binom{n-1}{i-1} x^i$.

Proof:

The proof is given in [7].

Theorem 3.5

Let G_n be a Gem graph with n + 2 vertices, then $D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x)$.

Proof:

By the definition of connected total domination polynomial, we have,

$$D_{ct}(G_n, x) = \sum_{i=2}^{n+2} d_{ct}(G_n, i)\, x^i.$$

$$= \sum_{i=2}^{n+2} [d_{ct}(S_{n+2}, i) + d_{ct}(P_{n+1}, i)]\, x^i \text{ , by Theorem 2.6.}$$

$$= \sum_{i=2}^{n+2} d_{ct}(S_{n+2}, i)\, x^i + \sum_{i=2}^{n+1} d_{ct}(P_{n+1}, i)\, x^i.$$

Therefore, $D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x)$.

Theorem 3.6

Let $D_{ct}(G_n, x)$ be the connected total domination polynomial of a Gem graph with n + 2 vertices, then $D_{ct}(G_n, x) = x\,[\,(1 + x)^{n+1} - 1] + x^{n-1} + 2x^n + x^{n+1}$.

Proof:

By Theorem 3.5, we have, $D_{ct}(G_n, x) = D_{ct}(S_{n+2}, x) + D_{ct}(P_{n+1}, x)$.

Therefore, $D_{ct}(G_n, x) = x\,[\,(1 + x)^{n+1} - 1] + x^{n-1} + 2x^n + x^{n+1}$, by Theorem 2.4 and Theorem 3.3.

Theorem 3.7

Let $D_{ct}(G_n, x)$ be the connected total domination polynomial of a Gem graph with n + 2 vertices, then

(i) $D_{ct}(G_n, x) = \sum_{i=2}^{n+2}\binom{n+2}{i}x^i - \sum_{i=2}^{n+2}\binom{n+1}{i}x^i + x^{n-1} + 2x^n + x^{n+1}$.

(ii) $D_{ct}(G_n, x) = \sum_{i=2}^{n+2}\binom{n+i}{i-1}x^i + x^{n-1} + 2x^n + x^{n+1}$.

Proof:

(i) follows from Theorem 3.5, Theorem 3.4 (i) and Theorem 2.4.

(ii) follows from Theorem 3.5, Theorem 3.4 (ii) and Theorem 2.4.

Theorem 3.8

Let $D_{ct}(G_n, x)$ be the connected total domination polynomial of a Gem graph with n + 2 vertices, then $D_{ct}(G_n, x) = (1 + x) D_{ct}(G_{n-1}, x) + x^2 - x^{n-2} - 2x^{n-1} - x^n$.

Proof:

By the definition of connected total domination polynomial, we have,

$$D_{ct}(G_n, x) = \sum_{i=2}^{n+2} d_{ct}(G_n, i)\, x^i.$$

$$= \sum_{i=2}^{n+2} [d_{ct}(G_{n-1}, i) + d_{ct}(G_{n-1}, i-1)]x^i, \text{ by Theorem 2.9.}$$

$$= \sum_{i=2}^{n+2} d_{ct}(G_{n-1}, i)\, x^i + \sum_{i=2}^{n+2} d_{ct}(G_{n-1}, i-1)\, x^i.$$

$$= \sum_{i=2}^{n+2} d_{ct}(G_{n-1}, i)\, x^i + x\sum_{i=3}^{n+2} d_{ct}(G_{n-1}, i-1)\, x^{i-1}.$$

$$= D_{ct}(G_{n-1}, x) + x\, D_{ct}(G_{n-1}, x).$$

Hence, $D_{ct}(G_n, x) = (1 + x) D_{ct}(G_{n-1}, x)$. (1)

When i = 2, $d_{ct}(G_n, 2)\, x^2 = [d_{ct}(G_{n-1}, 2) + 1]\, x^2$, by Theorem 2.9 (i).

Hence, $d_{ct}(G_n, 2)\, x^2 = d_{ct}(G_{n-1}, 2)\, x^2 + x^2$ (2)

When i = n – 2,

$d_{ct}(G_n, n-2)\, x^{n-2} = [d_{ct}(G_{n-1}, n-2) + d_{ct}(G_{n-1}, n-3) - 1]\, x^{n-2}$, by Theorem 2.9 (iii).

Hence,

$d_{ct}(G_n, n-2)\, x^{n-2} = d_{ct}(G_{n-1}, n-2)\, x^{n-2} + d_{ct}(G_{n-1}, n-3)\, x^{n-2} - x^{n-2}$ (3)

When i = n – 1,

$d_{ct}(G_n, n-1)\, x^{n-1} = [d_{ct}(G_{n-1}, n-1) + d_{ct}(G_{n-1}, n-2) - 2]\, x^{n-1}$, by Theorem 2.9 (iv).

Hence, $d_{ct}(G_n, n-1)\, x^{n-1} = d_{ct}(G_{n-1}, n-1)\, x^{n-1} + d_{ct}(G_{n-1}, n-2)\, x^{n-1} - 2x^{n-1}$ (4)

When i = n,

$d_{ct}(G_n, n)\, x^n = [d_{ct}(G_{n-1}, n) + d_{ct}(G_{n-1}, n-1) - 1]\, x^n$, by Theorem 2.9 (iii).

Hence, $d_{ct}(G_n, n)\, x^n = d_{ct}(G_{n-1}, n)\, x^n + d_{ct}(G_{n-1}, n-1)x^n - x^n$ (5)

Combining (1), (2), (3), (4) and (5) we get,

$$D_{ct}(G_n, x) = (1 + x)\, D_{ct}(G_{n-1}, x) + x^2 - x^{n-2} - x^{n-1} - x^n$$

Example 3.9

$$D_{ct}(G_5, x) = 6x^2 + 15x^3 + 21x^4 + 17x^5 + 7x^6 + x^7.$$

By Theorem 3.8, we have,

$$D_{ct}(G_6, x) = (1 + x)(6x^2 + 15x^3 + 21x^4 + 17x^5 + 7x^6 + x^7) + x^2 - x^4 - 2x^5 - x^6.$$

$$= 7x^2 + 21x^3 + 35x^4 + 36x^5 + 23x^6 + 8x^7 + x^8.$$

We obtain $d_{ct}(G_n, i)$ for $1 \leq n \leq 15$ as shown in Table 1.

Table 1

n \ i	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	3	1														
2	5	4	1													
3	5	8	5	1												
4	5	11	12	6	1											
5	6	15	21	17	7	1										
6	7	21	35	36	23	8	1									
7	8	28	56	70	57	30	9	1								
8	9	36	84	126	126	85	38	10	1							
9	10	45	120	210	252	210	121	47	11	1						
10	11	55	165	330	462	462	330	166	57	12	1					
11	12	66	220	495	792	924	792	495	221	68	13	1				
12	13	78	286	715	1287	1716	1716	1287	715	287	80	14	1			
13	14	91	364	1001	2002	3003	3432	3003	2002	1001	365	93	15	1		
14	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	456	107	16	1	
15	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	561	122	17	1

In the following Theorem, we obtain some properties of $d_{ct}(G_n, i)$.

Theorem 3.10

The following properties hold for the coefficients of $D(G_n, x)$ for all n.

(i) $d_{ct}(G_n, 2) = n + 1$ for all $n \geq 4$.

(ii) $d_{ct}(G_n, n + 2) = 1$.

(iii) $d_{ct}(G_n, n + 1) = n + 2$.

(iv) $d_{ct}(G_n, i) = 0$,if $i < 2$ or $i > n + 2$.

(v) $d_{ct}(G_n, n) = \binom{n+1}{2} + 2$, for all $n \geq 2$.

(vi) $d_{ct}(G_n, n - 1) = \binom{n+1}{3} + 1$, for all $n \geq 3$.

(vii) $d_{ct}(G_n, n - 2) = \binom{n+1}{4}$, for all $n \geq 4$.

(viii) $d_{ct}(G_n, n - 3) = \binom{n+1}{5}$, for all $n \geq 5$.

(ix) $d_{ct}(G_n, n - 4) = \binom{n+1}{6}$, for all $n \geq 6$.

(x) $d_{ct}(G_n, n - i) = \binom{n+1}{i+2}$, for all $n \geq i + 2$.

Proof:

Proof of (i), (ii) and (iii) follows from Corollary 2.8.

(iv) From Table 1, We have, $d_{ct}(G_n, i) = 0$ if $i < 2$ or $i > n + 2$.

Proof of (v), (vi), (vii), (viii), (ix) and (x) follows from Corollary 2.8.

4. CONCLUSION

In this paper, the connected total domination polynomials of a Gem graph has been derived by identifying its connected total dominating sets. It also helps us to characterize the connected total dominating sets and to find the number of connected total dominating sets of cardinality i. We can generalize this study to any power of the Gem graph and some interesting properties can be obtained via the roots of the connected total domination polynomial of G_n^k.

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