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Radio Coloring Phenomena and Its Applications

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Abstract: *This paper studies the concepts of Radio Colorings. The main results are*

1) Relation between radio chromatic number for a connected graph of order n having diameter d, and for integers k, l.

2) Bounds for radio number of a connected graph of order n and diameter d.

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1. Introduction

The concept of L(h,k)-colorings has been generalized in a natural way. For nonnegative integers dl, d2,..., dk, where k ≥ 2, and L (dl, d2,..., dk)-coloring c of a graph G is an assignment c of colors (nonnegative integers in this case) to the vertices of G such that $|c(u)-c(w)| \geq$ di whenever d(u, w) = i for 1 ≤ i≤ k. The L(d1, d2,…, dk)-colorings in which di = k + 1- i for each i (1 ≤i ≤ k) have proved to be of special interest.

The term "radio coloring" emanates from its connection with the Channel Assignment Problem. In the United States, one of the responsibilities of the Federal Communications Commission (FCC) concerns the regulation of FM radio stations. Each station is characterized by its transmission frequency, effective radiated power, and antenna height. Each FM station is assigned a station class, which depends on a number of factors, including its effective radiated power and antenna height. The FCC requires that FM radio stations located within a certain proximity to one another must be assigned distinct channels and that the nearer two stations are connected to each other, the greater the difference in their assigned channels must be minimum distance between stations (see [6]).

We have also mentioned that the use of graph theory to study the Channel Assignment Problem and related problems dates back at least to 1970(Metzger [5]) .In 1980, William Hale [2] modeled the Channel Assignment Problem as both a frequency-distance constrained and frequency constrained optimization problem and discussed applications to important real world problems.

2. Preliminaries

2.1.Definition:

A **radio coloring** of *G* is an assignment of colors to the vertices of *G* such that two colors *i* and *j* can be assigned to two distinct vertices *u* and *v* only if $d(u, v) + |i-j| \geq 1+k$ for some fixed positive integer *k.*

2.2.Definition:

For a connected graph *G* of diameter *d* and an integer *k* with $1 \leq k \leq d$, a ***k*-radio coloring** *c* of *G* (sometimes called a **radio *k*-coloring**) is an assignment of colors (positive integers) to the vertices of *G* such that

$$d(u, v) + |c(u)-c(v)| \geq 1 + k.$$

for every two distinct vertices u and v of G. Thus a 1-radio coloring of G is simply a proper coloring of G, while a 2-radio coloring is an $L(2, 1)$-coloring **[1].** Note that a k-radio coloring c does not imply that c is a k-coloring of the vertices of G (a vertex coloring using k colors).

2.3.Definition

The **value** $rc_k(c)$ of a k-radio coloring c of G is defined as the maximum color assigned to a vertex of G by c (where, again, we may assume that some vertex of G is assigned the color 1). The coloring $\bar{c}$ of G defined by

$$\bar{c}\,(v) = rc_k\,(c) + 1 - c(v)$$

for every vertex v of G is also a k-radio coloring of G, referred to as the **complementary coloring** of c. Because it is assumed that some vertex of G has been colored 1 by c, it follows that $rc_k(\bar{c}\,) = rc_k(c)$.

2.4.Definition

For a connected graph G with diameter d and an integer k with $1 \leq k \leq d$, the ***k*-radio chromatic number** (or simply the ***k*-radio number**) $rc_k(G)$ is defined as

$$rc_k(G) = \min\{rc_k(c)\},$$

Where the minimum is taken over all k-radio colorings c of G. Since a 1-radio coloring of G is a proper coloring, it follows that $rc_1(G) = \chi(G)$. On the other hand, a 2-radio coloring of G is an $L(2, 1)$-coloring of G, all of whose colors are positive integers. Thus

$$rc_2(G) = 1 + \lambda(G).$$

3. Radio Chromatic Number:

3.1.Proposition:

For a connected graph G of order n having diameter d and for integers k and ℓ with $1 \leq k < \ell \leq d$.

$$rc_l\,(G) \leq rc_k(G) + (n - 1)\,(\ell - k).$$

Proof. Let c be a k-radio coloring of G such that $rc_k(c) = rc_k(G)$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ such that $c(v_i) \leq c(v_{i+1})$ for $1 \leq i \leq n - 1$. We define a coloring c' of G by

$$c'(v_i) = c(v_i) + (i - 1)(l - k).$$

For integers i and j with $1 \leq i < j \leq n$, we therefore have

$$|\, c'(v_i) - c'(v_j)\,| = |\, c\,(v_i) - c(v_j)\,| + (j - i)(l - k)$$

Since c is a k-radio coloring of G, it follows that

$$|\, c\,(v_i) - c(v_j)\,| \geq 1 + k - d(v_i, v_j).$$

Consequently,

$$\begin{aligned} |\, c'(v_i) - c'(v_j)\,| &\geq 1 + k + (j - i)(l - k) - d(v_i, v_j) \\ &\geq 1 + l - d\,(v_i, v_j) \end{aligned}$$

Thus c' is an l-radio coloring of G with

$$rc_l\,(c') = rc_k(G) + (n - 1)(\ell - k)$$

and so $rc_l\,(G) \leq rc_k(G) + (n - 1)(\ell - k)$.

3.2.Definition:

Even though k-radio colorings of a connected graph with diameter d are defined for every integer k with $1 \leq k \leq d$, it is the two smallest and two largest values of k that have received the most attention. For a connected graph G with diameter d, a d-radio coloring c of a connected graph G with diameter d requires that

$$d(u, v) + |c(u) - c(v)| \geq 1 + d$$

for every two distinct vertices u and v of G. A d-radio coloring is called a radio labeling and the d-radio chromatic number (or d-radio number) is sometimes called simply the radio number $rn(G)$ of G.

3.3.Proposition

If G is a connected graph of order n and diameter d, then

$$n \leq rn(G) \leq 1 + (n-1)d.$$

We have noted that if $d = 1$ and so $G = K_n$, then $rn(K_n) = n$. The graph C_5 and the Petersen graph P both have diameter 2 and their radio numbers also attain the lower bound, namely $rn(C_5)=5$ and $rn(P)=10$. Furthermore, for each integer $k \geq 2$, the graph $K_k \times K_2$ has order $n = 2k$, diameter 2, and

$rn(K_k \times K_2)=n$. The graph $C_3 \times C_5$ has order $n = 15$, diameter 3, and radio number 15.(Figure 1)

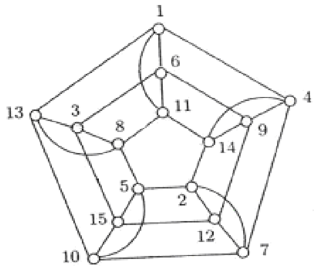

Figure1. *A radio labeling of* $C_3 \times C_5$

For a connected graph G of order n and diameter d, the upper bound for $rn(G)$ given in 2.3 can often be improved.

3.4.Proposition:

If G is connected graph of order n and diameter d containing an induced sub graph H of order p and diameter d such that $d_H(u, v) = d_G(u, v)$ for every two vertices u and v of H, then

$$rn(H) \leq rn(G) \leq rn(H) + (n-p)d.$$

A special case of Proposition 2.4 is when H is a path.

3.5.Corollary

If G is a connected graph of order n and diameter d,

Then $rn(P_{d+1}) \leq rn(G) \leq rn(P_{d+1}) + (n-d-1)d$

Corollary 2.5 illustrates the value of knowing the radio numbers of paths. The following result was obtained by Daphne Liu and Xuding Zhu **[4].**

3.6.Theorem:

For every integer $n \geq 3$,

$$rn(P_n) = \begin{cases} 2r^2+3 & if\ n = 2r+1 \\ 2r^2-2r+2 & if\ n = 2r \end{cases}$$

Combining Corollary 2.5 and Theorem2.6, we have the following.

3.7.Corollary:

Let G be a connected graph of order n and diameter d.

a) If $d=2$, then $4 \leq rn(G) \leq 2n-2$.

b) If $d=3$, then $6 \leq rn(G) \leq 3n-6$.

c) If $d=4$, then $11 \leq rn(G) \leq 4n-9$.

Proof:

While the paths P_{d+1} show the sharpness of the lower bounds in Corollary 2.7 the sharpness of the upper bounds are less obvious. It is not difficult to show that for every integer $n \geq 3$, there exists a connected graph G of diameter 2 with $rn(G) = 2n-2$. The graph H of Figure 2(a) has order $n=6$, diam $(H) = 3$ and rn $(H) = 12 = 3n-6$. The graph F of Figure 2(b) has order n=6, diam (F) = 4, and rn(F) = 14 = $4n$-10. The number $4n$-10 does not attain the upper bound for the radio number of a graph of diameter 4 given in Corollary 2.7(c). In fact, it may be that the appropriate upper bound for this case is $4n$-10 rather than $4n$-9.

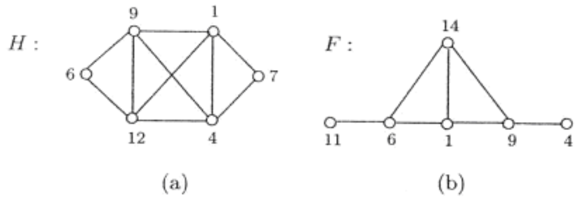

Figure2. *Radio numbers of graphs having diameters 3 & 4*

For a connected graph G of diameter d, a $(d$ - 1)-radio coloring c requires that

$$d(u, v) + |c(u)-c(v)| \geq d$$

for every two distinct vertices u and v of G. A $(d$-1)-radio coloring c is also referred to as a **radio antipodal coloring** (or simply an **antipodal coloring**) of G since $c(u)=c(v)$ only if u and v are antipodal vertices of G. The **radio antipodal number** or, more simply, the **antipodal number** $an(c)$ of c is the largest color assigned to a vertex of G by c. The antipodal chromatic number or the antipodal number $an(G)$ of G is

$$an(G) = min\{an(c)\},$$

where the minimum is taken over all radio antipodal colorings c of G. If c is a radio antipodal coloring of a graph G such that $an(c) = \ell$, then the complementary coloring $\bar{c}$ of G defined by

$$\bar{c}\,(v) = \ell + 1 - c(v)$$

for every vertex v of G is also a radio antipodal coloring of G.

A radio antipodal coloring of the graph H in Figure 3 is given with antipodal number 5. Thus $an(H) \leq 5$. Let c be a radio antipodal coloring of H with $an(c) = an(H) \leq 5$. Since diam$(H) = 3$, the colors of every two adjacent vertices of H must differ by at least 2 and the colors of two vertices at distance 2 must differ. We may assume that $c(v) \in \{1, 2\}$, for otherwise, $\bar{c}\,(v) \in \{1,2\}$ for the complementary radio antipodal coloring $\bar{c}$ of H.

Suppose that $c(v) = a \leq 2$. Then at least one of the vertices u, w, and y must have color at least $a + 2$, one must have color at least $a + 3$, and the other must have color at least $a + 4$. Since $a + 4 \geq 5$, it follows that $an(c) \geq 5$ and so $an(H) \geq 5$. Hence $an(H) = 5$

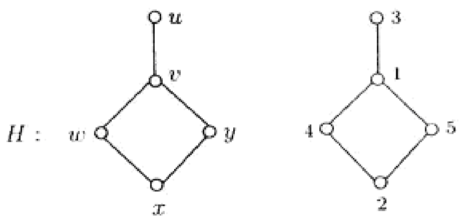

Figure 3. *A graph with antipodal number 5*

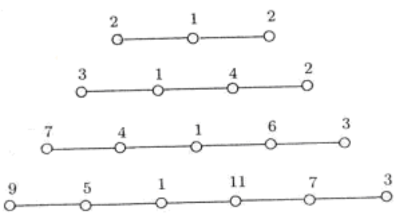

Figure 4. *Radio antipodal colorings of P_n $(3 \leq n \leq 6)$*

Figure 4 gives radio antipodal colorings of the paths P_n with $3 \leq n \leq 6$ that give $an(P_n)$ for these graphs. The antipodal numbers of all paths were determined by Khennoufa and Togni **[3].**

3.8.Remarks:

1) If we consider less diameter, Radio coloring concept vanishes.
2) If we consider maximum diameter, required radio chromatic number will occur.
3) The conditions of radio coloring is not applicable for chromatic polynomials.

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