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A Note on Semitotal blocks in Fuzzy Graphs

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Abstract: *In this research paper, it was studied about degree of vertices of a semitotal blocks in fuzzy graphs. In process we obtain some interesting results regarding the degree of the vertices in semitotal blocks in fuzzy graphs. We observed that when 'B' is a block of a given fuzzy graph G:(V, σ, μ), then degree of the vertex B in semi total block fuzzy graph $T_{STB}F(G)$ is equal to the sum of the membership grade of the vertices in that block and the number of edges in $T_{STB}F(G)$ related to block B is $|V(B)|$ with membership grade minimum of σ(u),σ(B). Also, we obtained that When G:(V ,σ,μ) fuzzy graph and v be a fuzzy vertex with degree $d_{FG}(v)$ in G:(V ,σ,μ), then the degree of 'v' in semitotal block fuzzy graph $T_{STBF}(G)$, $d_{STFG}(v)$ equal to the sum of the degree of the vertex in fuzzy graph and the product of |{B/Bis a block in fuzzy graph containing v}| with minimum of the set {σ(v), σ(B)}. Finally, it is proved that the ring sum of given fuzzy graph and vertex block fuzzy graph is equals to the semitotal block fuzzy graph of given fuzzy graph.*

Keywords: *fuzzy graph, ring sum of fuzzy graphs, Degree of vertex in fuzzy graphs, Semitotal-block fuzzy graph, vertex block graph.*

1 INTRODUCTION

Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Fuzzy models playing a vital role in real situations and becoming useful in engineering and sciences. In this paper, we introduced an algebraic operation ring sum of two fuzzy graphs which is different from existed literature and provided necessary examples. In this connection it is observed that the ring sum of two fuzzy graphs is also a fuzzy graph. Several interesting results on semitotal block fuzzy graph $T_{STB}F(G)$ of a fuzzy graph are observed. Finally, it is proved that, When G:(V ,σ, μ) fuzzy graph and v be a fuzzy vertex with degree $d_{FG}(v)$ in G:(V,σ,μ), then the degree of 'v' in semitotal block fuzzy graph $T_{STBF}(G)$ is $d_{STFG}(v)$ equal to the sum of the degree of the vertex in fuzzy graph and the product of |{B/Bis a block in fuzzy graph containing v}| with min { σ(v) and σ(B)}. Also we proved that $T_{STBF}(G) = G:(V ,\sigma,\mu) \oplus B_v(STBF(G))$.

Throughout this paper, we assume that fuzzy graph G is simple and connected.

1.1 Definition : A connected non–trivial fuzzy graph having no fuzzy cut vertex is a block in fuzzy graph.

1.2 Note: The set of all Blocks of fuzzy graph G is denoted by SBF(G)

1.3 Example: Consider the following fuzzy graph G:(V, σ, μ). Where V = {V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{11}} with $\sigma(v_1) = 0.8$, $\sigma(v_2) = 1$, $\sigma(v_3) = 0.7$, $\sigma(v_4) = 0.8$, $\sigma(v_5) = 0.8$, $\sigma(v_6) = 0.8$, $\sigma(v_7) = 0.8$, $\sigma(v_8) = 0.8$, $\sigma(v_9) = 0.7$, $\sigma(v_{10}) = 0.8$, $\sigma(v_{11}) = 0.8$ and $\mu(v_1, v_2) = 0.4$, $\mu(v_2, v_3) = 0.2$, μ(v_1,

$v_3) = 0.2$, $\mu(v_3, v_4) = 0.5$, $\mu(v_3, v_5) = 0.2$, $\mu(v_3, v_6) = 0.2$, $\mu(v_5, v_6) = 0.5$, $\mu(v_6, v_7) = 0.2$,$\mu(v_7, v_8) = 0.5$, $\mu(v_8, v_9) = 0.2$, $\mu(v_8, v_{11}) = 0.2$, $\mu(v_8, v_{10}) = 0.2$, $\mu(v_{10}, v_{11}) = 0.2\}$.

1.4 Example: (i). Consider the fuzzy graph G:(V, σ, μ) in example-1.3. There are five blocks in fuzzy graph G:(V, σ, μ). They are $B_1F(G)$, $B_2F(G)$, $B_3F(G)$, $B_4F(G)$, $B_5F(G)$. Where **B_1F(G)** is a block having vertex set $\{V_1, V_2, V_3\}$ with $\sigma(v_1) = 0.8$, $\sigma(v_2) = 1$, $\sigma(v_3) = 0.7$ and $\mu(v_1, v_2) = 0.4$, $\mu(v_2, v_3) = 0.2$, $\mu(v_1, v_3) = 0.2$, **B_2F(G)** is a block having vertex set $\{V_3, V_4\}$ with $\sigma(v_3) = 0.7$, $\sigma(v_4) = 0.8$ and $\mu(v_3, v_4) = 0.5$, **B_3F(G)** is a block having the vertex set $\{V_3, V_5, V_6, V_7\}$ with $\sigma(v_3) = 0.7$, $\sigma(v_5) = 0.8$, $\sigma(v_6) = 0.8$, $\sigma(v_7) = 0.8$, and $\mu(v_3, v_5) = 0.2$, $\mu(v_3, v_6) = 0.2$, $\mu(v_5, v_6) = 0.5$, $\mu(v_6, v_7) = 0.2$, **B_4F(G)** is block in fuzzy graph having the vertex set $\{V_7, V_8\}$ with $\sigma(v_7) = 0.8$, $\sigma(v_8) = 0.8$, and $\mu(v_7, v_8) = 0.5$, **B_5F(G)** is block in fuzzy graph having the vertex set $\{V_8, V_9, V_{10}, V_{11}\}$ with $\sigma(v_8) = 0.8$, $\sigma(v_9) = 0.7$, $\sigma(v_{10}) = 0.8$, $\sigma(v_{11}) = 0.8$ and $\mu(v_8, v_9) = 0.2$, $\mu(v_8, v_{11}) = 0.2$, $\mu(v_8, v_{10}) = 0.2$, $\mu(v_{10}, v_{11}) = 0.2\}$.

In this example, SBF(G) = $\{B_1F(G), B_2F(G), B_3F(G), B_4F(G), B_5F(G)\}$

1.5 Note: All blocks in the fuzzy graph are fuzzy sub-graphs of the given fuzzy graph

1.6 Definition: Let $G_1:(V_1, \sigma_1, \mu_1)$, $G_2:(V_2, \sigma_2, \mu_2)$ be two fuzzy graphs. Then the ring sum of these two fuzzy graphs is a fuzzy graph G:(V, σ, μ) defined as $V = V_1 \cup V_2$,

$$\sigma(u) = \begin{cases} \sigma_1(u) \quad if \quad u \in V_1 \\ \sigma_2(u) \quad if \quad u \in V_2 \\ \sigma_1(u) \ \wedge \ \sigma_2(u) \quad if \quad u \in V_2 \cap V_2 \end{cases}$$

and

$$\mu(\overline{uv}) = \begin{cases} \mu_1(\overline{uv}) & if \quad \overline{uv} \ \in \ (E_1 - E_2) \\ \mu_2(\overline{uv}) & if \quad \overline{uv} \ \in \ (E_2 - E_1) \\ 0 & if \quad \overline{uv} \ \in \ (E_1 \cap E_2) \end{cases}.$$

Where $\overline{uv}$ denotes the edge between the vertex u and the vertex v. Where E_1 and E_2 are sets of edges of fuzzy graphs G_1 and G_2 respectively [2].

1.7 Note: The fuzzy graph obtained by the ring sum operation is also a fuzzy graph.

1.8 Definition: The semi-total block fuzzy graph denoted by $T_{STB}F(G)$ of a fuzzy graph $G:(V_{STBFG}, \sigma_{STBFG}, \mu_{STBFG})$ is defined as the fuzzy graph having the vertices set $V_{STBFG} = V(F(G)) \cup B(F(G))$ and two fuzzy vertices are adjacent if they corresponds to two adjacent vertices of F(G) or one corresponds to a block B of F(G) and other to fuzzy vertex v of F(G) and the membership grade of those new vertices and edges are defined as follows:

$$\sigma_{STBFG}(u) = \begin{cases} \sigma_{FG}(u) \quad if \quad u \in \ V(F(G)) \\ \max \ \{\sigma(u) \ / \ if \quad u \in B_i, 1 \le i \le n \end{cases}$$ and

$$\mu_{STBFG}(\overline{UV}) = \begin{cases} \mu_{FG}(\overline{uv}) \quad if \quad \overline{uv} \ \in \ V(F(G)) \\ \min\{ \ \sigma_{STBFG}(u), \ \ \sigma_{STBFG}(\mathrm{B}_i) \ \ , 1 \le \mathrm{i} \le \mathrm{n}\} \end{cases}$$

where B_i denotes the i^{th} block in Fuzzy Graph[2].

1.9 Note: Since every edge of fuzzy graph G:(V, σ, μ) is also an edge in the semi total block fuzzy graph with same μ -value (membership grade), every vertex of fuzzy graph G:(V, σ, μ) in the semi total block fuzzy graph with same μ -value (membership grade), the fuzzy graph is a fuzzy sub-graph of semi total block fuzzy graph. That is G:(V, σ, μ) contained in $G:(V_{STBFG}, \sigma_{STBFG}, \mu_{STBFG})$.

1.10 Definition1.10: Let G:(V, σ, μ) be a fuzzy graph and B be a block. Then a fuzzy edge in semitotal block fuzzy graph is said to be fuzzy edge related to block B if one the end vertex of that fuzzy bridge is B.

2 DEGREE OF THE VERTEX IN SEMITOTAL BLOCK FUZZY GRAPH

In this section, we presented some necessary definitions lemmas and theorems which will help us for better understanding the main results. Also we provided the proofs for theorems at need.

2.1 Definition: Let G:(V, σ, μ) be a fuzzy graph. Then the degree of the vertex in G is defined as the sum of the degree of the membership grade of the fuzzy edges incident on that vertex.

2.2 Lemma: Let B be a block of given fuzzy graph G:(V, σ, μ) [2]. Then,

(i) degree of the vertex B in semi total block fuzzy graph $T_{STB}F(G)$ is equals to the summation of the membership grade of the vertices in that block.

(ii) The number of edges in $T_{STB}F(G)$ related to block B is $|V(B)|$ with membership grade minimum of σ(u), σ(B).

2.3 Theorem: $|E_{STB}F(G)| = |EF(G)| + |V(B_1)| + |V(B_2)| + \ldots + |V(B_k)|$.

2.4 Theorem: Let G:(V ,σ, μ) fuzzy graph and v be a fuzzy vertex with degree $d_{FG}(v)$ in G : (V ,σ, μ). Then the degree of v in semitotal block fuzzy graph $T_{STBF}(G)$ is $d_{STFG}(v)$ equal to the sum of the degree of the vertex in fuzzy graph and |{B/Bis a block in fuzzy graph containing v}| x min { σ(v) and σ(B)}.

Proof: Let v be a fuzzy vertex with degree $d_{FG}(v)$ in fuzzy graph G:(V ,σ,μ) and $T_{STBF}(G)$ semitotal block fuzzy graph of G. Suppose 'v' is a vertex in block B of fuzzy graph G:(V ,σ,μ). Then there is an edge between vertex v and vertex B in semitotal block fuzzy graph $T_{STBF}(G)$ with membership grade minimum of σ(v) and σ(B). This is true for all vertices in block B. Now the degree of the vertex 'v' in semitotal block fuzzy graph $T_{STBF}(G)$ is the summation of $d_{FG}(v)$ and minimum of σ(v) and σ(B). If 'v' appears in some block B_i (other than B) then there is an edge between the vertex 'v' and vertex 'B_i' in semitotal block fuzzy graph with membership grade minimum of σ(v) and σ(B_i). Suppose vertex v appears in n blocks the there are n edges from vertex to distinct blocks with membership grade of minimum of σ(v) and σ(B_i) for i = 1,2,. . . n. Therefore, $d_{STFG}(v)$ is equal to the sum of the degree of the vertex 'v' in fuzzy graph and the product of |{B/Bis a block in fuzzy graph containing v}| with minimum of {σ(v) and σ(B)}.

2.5 Note: (i) If 'v' is a fuzzy vertex lies in only one block then the degree of vertex v is equals to its total degree.

(ii) If 'v' appears more than one block in fuzzy graph then the degree not equals to its total degree.

3 VERTEX BLOCK FUZZY GRAPHS

This section contains main result that the ring sum of given fuzzy graph and the vertex block fuzzy graph gives the semitotal block fuzzy graphs. That is, $T_{STBF}(G) = G:(V ,\sigma,\mu) \oplus B_v(STBF(G))$.

3.1 Definition: The vertex block fuzzy graph $B_v(F(G))$ of a fuzzy graph G:(V ,σ,μ) is defined as follows: (i) the vertex set of vertex block fuzzy graph $B_v(F(G))$ is the union of vertex set of fuzzy graph and set of blocks of fuzzy graph SBF(G) and (ii) the edge set $E(B_v(F(G))) = \{uB_i$ / u is a vertex in B_i, $1< i < n\}$ where uB_i is an edge from vertex 'u' to vertex block B_i in SBF(G) with the membership grade minimum of σ(u) and σ(B_i)}. Moreover, the membership grade of the vertices in vertex block fuzzy graph is σ(u) = σ(v) if u is a fuzzy vertex in fuzzy graph G and σ(B) = maximum { σ(u) / u is vertex in block B}.

3.2 Example: Consider the fuzzy graph G:(V ,σ, μ) with vertex set V = { $v_1, v_2, v_3, v_4, v_5, v_6, v_7$} with σ ($v_1$) = 1, σ(v_2) = 0.8, σ(v_3) = 0.7, σ(v_4) = 0.8, σ(v_5) = 0.8, σ(v_6) = 0.8, σ(v_7) = 0.8 and the edge set EF(G) = { v_1v_2, v_1v_3, v_3v_4, v_3v_5, v_3v_6, v_5v_7, v_6v_7,} with membership grade of edges μ(v_1v_2) = 0.4,μ(v_1v_3) = 0.2, μ(v_2v_3) = 0.2, μ(v_3v_4) =0.5, μ(v_3v_5) = 0.2, μ(v_3v_6) = 0.2, μ(v_5v_7) = 0.5, μ(v_6, v_7) = 0.2. Then the vertex block fuzzy graph of fuzzy graph G:(V ,σ, μ) is a fuzzy graph with vertex set V(B_v(F(G)) = V(F(G)) ∪ {SBF(G)} = { $v_1, v_2, v_3, v_4, v_5, v_6, v_7, B_1, B_2, B_3$ / B_1, B_2, B_3 are block in fuzzy graph G:(V ,σ, μ)} and $E(B_v(F(G))) = \{uB_i$ / u is a vertex in B_i, $1< i < 3\}$ with membership grades of vertices σ (v_1) = 1, σ(v_2) = 0.8, σ(v_3) = 0.7, σ(v_4) = 0.8, σ(v_5) = 0.8, σ(v_6) = 0.8, σ(v_7) = 0.8, σ(B_1) = 1, σ(B_2) = 0.8, σ(B_3) = 0.8, and as μ(v_1B_1) =0.8, μ(v_2B_1) = 1, μ(v_3B_1)= 0.7,

$\mu(v_3B_2) = 0.7$, $\mu(v_5B_2) = 0.8$, $\mu(v_6B_2) = 0.8$, $\mu(v_7B_2) = 08$, $\mu(v_3B_3) = 0.7$, $\mu(v_4B_3) = 0.8$ is a vertex block fuzzy graph $B_v(F(G))$.

3.3 Note: (i) The crisp graph of vertex block fuzzy graph is a vertex block graph.

(ii) Since $\mu(u_iB_j)$ = min { $\sigma(u_i)$ and $\sigma(B_j)$ / for all u_i, B_j}, vertex block fuzzy graph $B_v(F(G))$

is a complete fuzzy sub graph of Semitotal block fuzzy graph $T_{STBF}(G)$.

(iii) Vertex block fuzzy graph is a spanning fuzzy subgraph of semitotal block fuzzy graph.

3.4 Theorem: $T_{STBF}(G) = G:(V,\sigma,\mu) \oplus B_v(STBF(G))$

Proof: Let $G:(V,\sigma,\mu)$ fuzzy graph and $B_v(F(G))$ vertex block fuzzy graph . Then fuzzy graph G and vertex block fuzzy graph both are sub graphs of semitotal block fuzzy graph implies that $G \cup B_v(F(G)) \subseteq T_{STBF}(G)$. By the definition of $T_{STBF}(G)$, every edge in $T_{STBF}(G)$ is either in G or in $B_v(F(G))$. So $T_{STBF}(G) \subseteq G \cup B_v(F(G))$. Therefore, $T_{STBF}(G) = G \cup B_v(F(G))$. By the definition of vertex block fuzzy graph $B_v(F(G))$, no edge of fuzzy graph G is in $B_v(F(G))$. $E(G:(V,\sigma,\mu) \oplus B_v(STBF(G)))=E[(G:(V,\sigma,\mu)\backslash E(B_v(STBF(G))] \cup [E(B_v(STBF(G)\backslash E[(G:(V,\sigma,\mu)]=$

$E[(G:(V,\sigma,\mu) \cup E(B_v(STBF(G))]$. Thus $(G:(V,\sigma,\mu) \oplus (B_v(STBF(G) = G:(V,\sigma,\mu) \cup (B_v(STBF(G)$.

Thus $T_{STBF}(G) = G:(V,\sigma,\mu) \oplus B_v(STBF(G))$

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