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Rough Set Approximations on A Semi Bitopological View

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Abstract: *The main goal of this paper is to introduce a new bitopological approaches to rough sets. Suggested approaches depend on two topologies, generated by a general relation. The first topology is a right topology whose subbase is the family of right neighborhoods and a subbase of the other topology, left topology, is the family of left neighborhoods, with respect to that general relation. Some Pawlak's concepts are redefined, some properties are deduced and supported with proved propositions and many counter examples. We compare among suggested approaches, by using their approximations and accuracy measures. Hence, the best of them is determined. Finally, we deduce that traditional rough set model is a special case of any suggested model in this study.*

Keywords: *Rough set approximations, rough concepts, topological space, bitopological space.*

1. INTRODUCTION

Rough set theory, proposed by Pawlak in 1982 [1], is a very useful mathematical tool in classification of a collected data under equivalence relation. In Pawlak's study, any rough set is replaced by two crisp sets called lower and upper approximations of it.

Recently, many scientists have developed traditional rough set model, in many ways such as [2-13]. Especially, many interesting extensions of it have been made by using topological spaces such as [14-16].

This paper aims to introduce a bitopological space using a new bitopological near open set called semi-bi-open set (S_{bi}). This bitopological space consists of two topologies. In our study, we consider that, every topology of this bitopological space is a view of the interested problem. These two topologies are generated by only one general relation, hence, there is no contradiction between these two views. Consequently, semi-bitopological rough concepts are introduced and compared with their traditionals. We conclude relationships among traditional and proposed semi-bitopological approaches to rough sets in a diagram. Finally, we illustrate that, Pawlak's model is a special case of any proposed semi bitopological approach to rough sets.

2. PRELIMINARIES

In this section, some basic definitions are introduced. Pawlak's concepts were defined in [1] as follows.

Definition 2.1 Let X be the universe set and let E be an equivalence relation, representing our knowledge about the elements of X. Then (X,E) is called Pawlak approximation space. An equivalence class of E determined by element x is

$$[x]_E = \{x' \in X : E(x) = E(x')\}.$$

Definition 2.2 Let (X,E) be a Pawlak approximation space. Lower, upper and boundary approximations of a subset $A \subseteq X$ are defined as

$$\underline{E}(A) = \cup\{[x]_E : [x]_E \subseteq A\}, \quad \overline{E}(A) = \cup\{[x]_E : [x]_E \cap A \neq \phi\} \text{ and } BND_E(A) = \overline{E}(A) - \underline{E}(A).$$

Definition 2.3 Let (X,E) be a Pawlak approximation space. The degree of crispness of $A \subseteq X$ is determined by the accuracy measure, defined as

$$\alpha_E(A) = \frac{|\underline{E}(A)|}{|\overline{E}(A)|}, \overline{E}(A) \neq \emptyset.$$

Definition 2.4 Let (X,E) be a Pawlak approximation space and let $x \in X$, rough membership relations to a subset $A \subseteq X$ are defined as

$$x \underline{\in} A, if \quad x \in \underline{E}(A) \quad and \quad x \overline{\in} A, if \quad x \in \overline{E}(A).$$

Definition 2.5 Let (X,E) be a Pawlak approximation space and let $A, B \subseteq X$, rough inclusion relations are defined as

$$A \underset{\rightarrow}{\subset} B, if \quad \underline{E}(A) \subseteq \underline{E}(B) \quad and \quad A \overset{\rightarrow}{\subset} B, if \quad \overline{E}(A) \subseteq \overline{E}(B).$$

Topological rough approximations proposed by Wiweger [17] is the first generalization of rough set approximations based on topological structures. In his work, the lower and upper approximations are replaced by the interior and closure operators, respectively.

Definition 2.6 [17] Let (X,τ_i) be a topological space and let $A \subseteq X$. Interior and closure operators, respectively, are

$$int_i(A) = \cup\{G \in \tau_i : G \subseteq A\} \quad and \quad cl_i(A) = \cap\{G \in \tau_i^c : A \subseteq G\}.$$

A subset $A \subseteq X$ is called open set if $A \in \tau_i$ and the family of all these open sets is denoted by O_i. The complement of any open set is called closed set and the family of all closed sets is C_i.

Remark 2.1 Let (X,τ_i) be a topological space and let $A \subseteq X$. If $int_i(A) = cl_i(A)$, then A is called i-exact set, otherwise, it is called i-roughset.

Definition 2.7 [18] Let (X,τ_i) be a topological space and let $A \subseteq X$. A subset A is called

$$Semi-open(S_i-open)\ set, if \quad A \subseteq cl_i(int_i(A)).$$

The family of all S_i-open sets is denoted by OS_i. The complement of any S_i-open set is called S_i-closed set and the family of all S_i-closed sets is denoted by CS_i.

Definition 2.8 [19] (X,τ_1,τ_2) is called bitopological space, where τ_1 and τ_2 are two topologies, defined on a nonempty set X. In (X,τ_1,τ_2) a subset $A \subseteq X$ is called

$$S_{12}-open\ set, if \quad A \subseteq cl_2(int_1(A)).$$

The family of all S_{12}-open sets is denoted by OS_{12}. The complement of any S_{12}-open set is called S_{12}-closed set and the family of all S_{12}-closed sets is denoted by CS_{12}.

3. SEMI-BI-NEAR ROUGH SET APPROXIMATIONS

In this section, we define a new semi-bi-near open set, called S_{bi}-open set, defined on a bitopological space (X,τ_r,τ_l) which is generated by a general relation. The subbase of the first topology τ_r (right topology) is the family of right neighborhoods ($_xR = \{y \in X : xRy\}$) and the subbase of the second topology τ_l (left topology) is the family of left neighborhoods ($R_x = \{y \in X : yRx\}$), with respect to a relation R. Relationship between traditional rough set approximations and suggested semi-bi-rough set approximations is deduced.

Definition 3.1 Let (X,τ_r,τ_l) be a bitopological space and let $A \subseteq X$. A is called

$$S_{bi}-Open\,set\,if \quad A \subseteq \{cl_l int_r A \cup cl_r int_l A\}.$$

Remark 3.1 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation, then

(a) The complement of S_{bi} -open set is called S_{bi} -closed set.

(b) The family of all S_{bi} -open sets is denoted by OS_{bi} .

(c) The family of all S_{bi} -closed sets is denoted by CS_{bi} .

Proposition 3.1 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation, then

(a) $O_r \subseteq OS_{rl} \subseteq OS_{bi}$.

(b) $O_l \subseteq OS_{lr} \subseteq OS_{bi}$.

Proof

(a) $O_r = \cup\{A \subseteq X : A = int_r A\} \subseteq \cup\{A \subseteq X : A \subseteq cl_l int_r A\} = OS_{rl}$

$\subseteq \cup\{A \subseteq X : A \subseteq cl_l int_r A \cup cl_r int_l A\} = OS_{bi}$.

We can get the proof of (b) at the same way as (a).

The following example illustrates that, containments in Proposition 3.1, may be proper.

Example 3.1 Let R be a binary general relation defined on a nonempty set $X = \{a,b,c,d\}$ defined by $R = \{(a,a),(a,c),(a,d),(b,b),(b,d),(c,a),(c,b),(c,d),(d,a)\}$. Hence, the subbase of τ_r is $\{\{a,c,d\},\{b,d\},\{a,b,d\},\{a\}\}$ and the subbase of τ_l is $\{\{a,c,d\}, \{b,c\}, \{a,b,c\}, \{a\}\}$. Then, $\tau_r = \{X,\emptyset,\{a,c,d\},\{a,b,d\},\{b,d\},\{a,d\},\{a\},\{d\}\}$ and $\tau_l = \{X,\emptyset, \{a,c,d\}, \{a,b,c\}, \{b,c\}, \{a,c\}, \{a\}, \{c\}\}$. Consequently, $OS_{rl} = \{X,\emptyset,\{a\},\{d\}, \{a,d\}, \{b,d\}, \{a,b,d\},\{a,c,d\}\}$, $OS_{lr} = \{X,\emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b,c\},\{a,c,d\}\}$, $OS_{bi} = \{X,\emptyset,\{a\},\{c\},\{d\},\{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Hence, $\{c\},\{a,c\}, \{b,c\},\{a,b,c\} \notin OS_{rl}$ and $\{d\},\{a,d\}, \{b,d\},\{a,b,d\} \notin OS_{lr}$. But, $\{c\}, \{d\}, \{a,c\}, \{b,c\}, \{b,d\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\} \in OS_{bi}$.

Definition 3.2 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation. For all $i \in \{r,l\}$, topological lower (resp. topological upper) approximation of any subset $A \subseteq X$ denoted by $\downarrow^i A$ (resp. $\uparrow_i A$) and defined as follows

$$\downarrow^i A = \cup\{G \in \tau_i : G \subseteq A\} \quad and \quad \uparrow_i A = \cap\{H \in \tau_i^c : A \subseteq H\}.$$

Definition 3.3 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation. For all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$, I-interior (resp. I-closure) of any subset $A \subseteq X$, denoted by $int_I(A)$ (resp. $cl_I(A)$), are

$$int_I A = \cup\{G \in OI : G \subseteq A\} \quad and \quad cl_I A = \cap\{H \in CI : A \subseteq H\}.$$

Definition 3.4 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation. For all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$, I-lower (resp. I-upper) of $A \subseteq X$, denoted by $\downarrow^I A$ (resp. $\uparrow_I A$) are

$$\downarrow^I A = int_I A \quad and \quad \uparrow_I A = cl_I A.$$

Proposition 3.2 Let (X,τ_r,τ_l) be a bitopological space, generated by a general relation, and let $A \subseteq X$, then

(a) $\downarrow^r A \subseteq \downarrow^{S_{rl}} A \subseteq \downarrow^{S_{bi}} A \subseteq A \subseteq \uparrow_{S_{bi}} A \subseteq \uparrow_{S_{rl}} A \subseteq \uparrow_r A.$

(b) $\downarrow^l A \subseteq \downarrow^{S_{lr}} A \subseteq \downarrow^{S_{bi}} A \subseteq A \subseteq \uparrow_{S_{bi}} A \subseteq \uparrow_{S_{lr}} A \subseteq \uparrow_l A.$

Proof

(a) $\downarrow^r A = \cup\{G \in \tau_r : G \subseteq A\} \subseteq \cup\{G \in OS_{rl} : G \subseteq A\} = [\downarrow^{S_{rl}} A] \subseteq \cup\{G \in OS_{bi} : G \subseteq A\} =$ $[\downarrow^{S_{bi}} A] \subseteq A \subseteq \cap\{H \in CS_{bi} : A \subseteq H\} = [\uparrow_{S_{bi}} A] \subseteq \cap\{H \in CS_{rl} : A \subseteq H\} = [\uparrow_{S_{rl}} A]$

$\subseteq \cap\{H \in \tau_i^c : A \subseteq H\} = \uparrow_r A.$

The proof of (b) is similar to the proof of (a).

The following example illustrates that, the inverse of Proposition 3.2, does not hold.

Example 3.2 According to Example 3.1, we can create Table 1, as follows

Table 1. *Comparison among proposed bitopological lowers and uppers.*

$A \subseteq X$	$\downarrow^{S_{rl}} A$	$\downarrow^{S_{lr}} A$	$\downarrow^{S_{bi}} A$	$\uparrow_{S_{bi}} A$	$\uparrow_{S_{lr}} A$	$\uparrow_{S_{rl}} A$
$\{c\}$	$\emptyset$	$\{c\}$	$\{c\}$	$\{c\}$	$\{b,c,d\}$	$\{c\}$
$\{d\}$	$\{d\}$	$\emptyset$	$\{d\}$	$\{d\}$	$\{d\}$	$\{b,c,d\}$
$\{a,c\}$	$\{a\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	X	$\{a,c\}$
$\{a,d\}$	$\{a,d\}$	$\{a\}$	$\{a,d\}$	$\{a,d\}$	$\{a,d\}$	X
$\{b,c\}$	$\emptyset$	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b,c,d\}$	$\{b,c\}$
$\{b,d\}$	$\{b,d\}$	$\emptyset$	$\{b,d\}$	$\{b,d\}$	$\{b,d\}$	$\{b,c,d\}$
$\{c,d\}$	$\{d\}$	$\{c\}$	$\{c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$
$\{a,b,c\}$	$\{a\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	X	$\{a,b,c\}$
$\{a,b,d\}$	$\{a,b,d\}$	$\{a\}$	$\{a,b,d\}$	$\{a,b,d\}$	$\{a,b,d\}$	X
$\{b,c,d\}$	$\{b,d\}$	$\{b,c\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$

From Table 1, S_{bi} -lower approximation is the greatest lower of the studied bitopological lowers and S_{bi} -upper approximation is the smallest upper of the studied bitopological uppers. So, S_{bi} -approach is the best bitopological approach in this study.

Proposition 3.3 Let (X, τ_r, τ_l) be a bitopological space generated by a general relation and let $A, E \subseteq X$. For all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$ we can prove the following properties

(a) $\downarrow^{I} X = \uparrow_{I} X = X$ and $\downarrow^{I} \varnothing = \uparrow_{I} \varnothing = \varnothing$.

(b) $\downarrow^{I} A \subseteq A \subseteq \uparrow_{I} A$.

(c) If $A \subseteq E$, then $\downarrow^{I} A \subseteq \downarrow^{I} E$ and $\uparrow_{I} A \subseteq \uparrow_{I} E$.

(d) $\downarrow^{I} A \cup E \supseteq \downarrow^{I} A \cup \downarrow^{I} E$.

(e) $\uparrow_{I} A \cap E \subseteq \uparrow_{I} A \cap \uparrow_{I} E$.

(f) $\downarrow^{I} A^{c} = [\uparrow_{I} A]^{c}$ and $\uparrow_{I} A^{c} = [\downarrow^{I} A]^{c}$, where A^{c} is the complement of A.

Proof By using the properties of bitopological lower and bitopological upper approximations, defined in Definition 3.4, we get the proof, directly.

The following example illustrates that, containments in Property (c), may be proper.

Example 3.3 According to Example 3.1, if $A = \{b\}$ and $E = \{a, b\}$, then $\downarrow^{S_{bi}} A = \downarrow^{S_{rl}} A = \downarrow^{S_{lr}} A = \varnothing$, $\uparrow_{S_{rl}} A = \uparrow_{S_{lr}} A = \uparrow_{S_{bi}} A = \{b\}$, $\downarrow^{S_{bi}} E = \downarrow^{S_{rl}} E = \downarrow^{S_{lr}} E = \{a\}$,$\uparrow_{S_{rl}} E = \{a, b, c\}$, $\uparrow_{S_{lr}} E = \{a, b, d\}$ and $\uparrow_{S_{bi}} E = \{a, b\}$. Hence, $\downarrow^{S_{rl}} A \neq \downarrow^{S_{rl}} E$, $\downarrow^{S_{lr}} A \neq \downarrow^{S_{lr}} E$, $\downarrow^{S_{bi}} A \neq \downarrow^{S_{bi}} E$, $\uparrow_{S_{rl}} A \neq \uparrow_{S_{rl}} E$, $\uparrow_{S_{lr}} A \neq \uparrow_{S_{lr}} E$ and $\uparrow_{S_{bi}} A \neq \uparrow_{S_{bi}} E$.

The following example illustrates that, a containment in Property (d), may be proper.

Example 3.4 According to Example 3.1, if $A = \{b\}$ and $E = \{c, d\}$, then $\downarrow^{S_{bi}} A = \downarrow^{S_{rl}} A = \downarrow^{S_{lr}} A = \varnothing$, $\downarrow^{S_{lr}} E = \{c\}$,$\downarrow^{S_{rl}} E = \{d\}$,$\downarrow^{S_{bi}} E = \{c, d\}$, $\downarrow^{S_{rl}} \{b, c, d\} = \{b, d\}$, $\downarrow^{S_{lr}} \{b, c, d\} = \{b, c\}$ and $\downarrow^{S_{bi}} \{b, c, d\} = \{b, c, d\}$. Hence, $\downarrow^{S_{rl}} A \cup E \neq \downarrow^{S_{rl}} A \cup \downarrow^{S_{rl}} E$, $\downarrow^{S_{lr}} A \cup E \neq \downarrow^{S_{lr}} A \cup \downarrow^{S_{lr}} E$ and $\downarrow^{S_{bi}} A \cup E \neq \downarrow^{S_{bi}} A \cup \downarrow^{S_{bi}} E$.

The following example illustrates that, a containment in Property (e), may be proper.

Example 3.5 According to Example 3.1, if $A = \{a, b\}$ and $E = \{a, c, d\}$, then $\uparrow_{S_{rl}} A = \{a, b, c\}$, $\uparrow_{S_{lr}} A = \{a, b, d\}$, $\uparrow_{S_{bi}} A = \{a, b\}$, $\uparrow_{S_{rl}} E = \uparrow_{S_{lr}} E = \uparrow_{S_{bi}} E = X$, $\uparrow_{S_{rl}} \{a\} = \{a, c\}$,

$\uparrow_{S_{lr}}\{a\} = \{a,d\}$ and $\uparrow_{S_{bi}}\{a\} = \{a\}$. Hence, $\uparrow_{S_{rl}} A\cap E \neq \uparrow_{S_{rl}} A \cap \uparrow_{S_{rl}} E$, $\uparrow_{S_{lr}} A\cap E \neq \uparrow_{S_{lr}} A \cap \uparrow_{S_{lr}} E$ and $\uparrow_{S_{bi}} A\cap E \neq \uparrow_{S_{bi}} A \cap \uparrow_{S_{bi}} E$.

Proposition 3.4 Let (X,τ_r,τ_l) be a bitopological space, generated by a general relation and let $A,E\subseteq X$, for all $I\in\{S_{rl},S_{lr},S_{bi}\}$, the following properties hold.

(a) $\uparrow_I A\cup E \supseteq \uparrow_I A \cup \uparrow_I E.$

(b) $\downarrow^I A\cap E \subseteq \downarrow^I A \cap \downarrow^I E.$

Proof By using the properties of interior and closure operators, also from Definitions 3.3 and 3.4, we get the proof, directly.

The following example illustrates that, a containment in Property (a), may be proper. Let $I = S_{bi}$.

Example 3.6 According to Example 3.1, if $A=\{a,c\}$ and $E=\{d\}$, then $\uparrow_{S_{bi}} A=\{a,c\}$, $\uparrow_{S_{bi}} E = \{d\}$ and $\uparrow_{S_{bi}} A\cup E = X$. Hence, $\uparrow_{S_{bi}} A\cup E \neq \uparrow_{S_{bi}} A \cup \uparrow_{S_{bi}} E$.

The following example illustrates that, a containment in Property (b), may be proper. Let $I = S_{bi}$.

Example 3.7 According to Example 3.1, if $A=\{b,c\}$ and $E=\{b,d\}$, then $\downarrow^{S_{bi}} A = \{b,c\}$, $\downarrow^{S_{bi}} E = \{b,d\}$ and $\downarrow^{S_{bi}} A\cap E = \emptyset$. Hence, $\downarrow^{S_{bi}} A\cap E \neq \downarrow^{S_{bi}} A \cap \downarrow^{S_{bi}} E$.

Proposition 3.5 Let (X,τ_r,τ_l) be a bitopological space, generated by a general relation and let $A\subseteq X$, for all $I\in\{S_{rl},S_{lr},S_{bi}\}$, the following properties hold

(a) $\downarrow^I\downarrow^I A = \downarrow^I A.$

(b) $\uparrow_I\uparrow_I A = \uparrow_I A.$

Proof

(a) Let $Y = \downarrow^I A$ and let $u\in Y$. But, $Y = \downarrow^I A = \cup\{G\in OI : G\subseteq A\}$. Then, for all $G\subseteq A$, we have $G\subseteq Y$, hence, $u\in \downarrow^I Y$, it follows that, $Y \subseteq \downarrow^I Y$. On the other hand, from Proposition 3.3, we can deduce that, $\downarrow^I Y \subseteq Y$. Consequently, $\downarrow^I Y = Y$. Thus, $\downarrow^I\downarrow^I A = \downarrow^I A$.

(b) From Proposition 3.3, we have, $\uparrow_I A^c = [\downarrow^I A]^c$. Then, $\uparrow_I A = [\downarrow^I A^c]^c$ and then, $\uparrow_I\uparrow_I A = \uparrow_I [\downarrow^I A^c]^c = [\downarrow^I ([\downarrow^I A^c]^c)^c]^c = [\downarrow^I\downarrow^I A^c]^c$, from Property (a) of Proposition 3.5, we have, $\downarrow^I\downarrow^I A^c = \downarrow^I A^c$. Hence, $[\downarrow^I\downarrow^I A^c]^c = [\downarrow^I A^c]^c = \uparrow_I A$. Thus, $\uparrow_I\uparrow_I A = \uparrow_I A$.

Proposition 3.6 Let (X,τ_r,τ_l) be a bitopological space, generated by a general relation and let $A\subseteq X$, for all $I\in\{S_{rl},S_{lr},S_{bi}\}$, the following properties do not hold

(a) $\uparrow_I\downarrow^I A = \downarrow^I A.$

(b) $\downarrow^{I}\uparrow_{I} A = \uparrow_{I} A$.

The following example proves Property (a), of Proposition 3.6, at $I = S_{rl}$.

Example 3.8 According to Example 3.1, if $A = \{a, c, d\}$, then $\downarrow^{S_{rl}} A = \{a, c, d\}$ and $\uparrow_{S_{rl}} \downarrow^{S_{rl}} A = X$. Hence, $\uparrow_{S_{rl}} \downarrow^{S_{rl}} A \neq \downarrow^{S_{rl}} A$.

The following example proves Property (b) of Proposition 3.6, at $I = S_{lr}$.

Example 3.9 According to Example 3.1, if $A = \{a, b\}$, then $\uparrow_{S_{lr}} A = \{a, b, d\}$ and $\downarrow^{S_{lr}} \uparrow_{S_{lr}} A = \{a\}$. Hence, $\downarrow^{S_{lr}} \uparrow_{S_{lr}} A \neq \uparrow_{S_{lr}} A$.

Lima 3.1 Let (X, τ_r, τ_l) be a bitopological space, generated by a general relation, and let $A, E \subseteq X$. For all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$ we can prove the following property

$$[cl_I A]^c = int_I A^c.$$

Proof

$$[cl_I A]^c = X - \cap\{H \in CI : A \subseteq H\} = \cup\{(X - H) \in OI : (X - A) \supseteq (X - H)\}$$

$$= \cup\{G \in OI : G \subseteq A^c\} = int_I A^c.$$

Proposition 3.7 Let (X, τ_r, τ_l) be a bitopological space, generated by a general relation, and let $A, E \subseteq X$. For all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$ we can prove that

$$\downarrow^{I} A - E \subseteq \downarrow^{I} A - \downarrow^{I} E.$$

Proof

Where $A - E = A \cap E^c$, then $\downarrow^{I} A - E = \downarrow^{I} A \cap E^c = int_I A \cap E^c = int_I A \cap int_I E^c$. By using Lema 3.1, we get $int_I A \cap int_I E^c = int_I A \cap [cl_I E]^c = int_I A - cl_I E$. But, $int_I E \subseteq cl_I E$. Consequently, $int_I A - cl_I E \subseteq int_I A - int_I E = \downarrow^{I} A - \downarrow^{I} E$. Hence, $\downarrow^{I} A - E \subseteq \downarrow^{I} A - \downarrow^{I} E.$

The following example illustrates that, a containment in Proposition 3.7, may be proper, at $I = S_{bi}$.

Example 3.10 According to Example 3.1, if $A = \{a, b, c\}$ and $E = \{a, c\}$, then $\downarrow^{S_{bi}} A = \{a, b, c\}$, $\downarrow^{S_{bi}} E = \{a, c\}$ and $\downarrow^{S_{bi}} A - E = \varnothing$. Hence, $\downarrow^{S_{bi}} A - E \neq \downarrow^{S_{bi}} A - \downarrow^{S_{bi}} E$.

Proposition 3.8 In a bitopological space (X, τ_r, τ_l), generated by a general relation, for any two subsets $A, E \subseteq X$, the following property may be not satisfied for all $I \in \{S_{rl}, S_{lr}, S_{bi}\}$

$$\uparrow_I A - E = \uparrow_I A - \uparrow_I E.$$

Proposition 3.8 is proved by the following example, at $I = S_{bi}$.

Example 3.11 According to Example 3.1, if $A = \{c, d\}$ and $E = \{c\}$, then $\uparrow_{S_{bi}} A = \{b, c, d\}$, $\uparrow_{S_{bi}} E = \{c\}$ and $\uparrow_{S_{bi}} A - E = \{d\}$. Hence, $\uparrow_{S_{bi}} A - E \neq \uparrow_{S_{bi}} A - \uparrow_{S_{bi}} E$.

Definition 3.5 In a bitopological space (X, τ_r, τ_l) generated by a general relation, for all $I \in \{r, l, S_{rl}, S_{lr}, S_{bi}\}$ a subset $A \subseteq X$ is called

(a) I-Totally definable (I-exact), if $\downarrow^I A = \uparrow_I A = A$.

(b) I-Internally definable, if $\downarrow^I A = A$ and $\uparrow_I A \neq A$.

(c) I-Externally definable, if $\downarrow^I A \neq A$ and $\uparrow_I A = A$.

(d) I-Rough, if $\downarrow^I A \neq A$ and $\uparrow_I A \neq A$.

The following example illustrates Definition 3.5, at $I = S_{bi}$.

Example 3.12 From Example 3.1, we get the following results: $\{b\}$, $\{a, b\}$ are S_{bi}-externally definable sets, $\{c, d\}$, $\{a, c, d\}$ are S_{bi}-internally definable sets and $\{a\}$, $\{c\}$, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{b, c, d\}$ are S_{bi}-exact sets.

Proposition 3.9 In a bitopological space (X, τ_r, τ_l), generated by a general relation, for any subset $A \subseteq X$, the following properties are satisfied

(a) A is r-exact $\rightarrow$ A is S_{rl}-exact $\rightarrow$ A is S_{bi}-exact.

(b) A is l-exact $\rightarrow$ A is S_{lr}-exact $\rightarrow$ A is S_{bi}-exact.

(c) A is S_{bi}-rough $\rightarrow$ A is S_{rl}-rough $\rightarrow$ A is r-rough.

(d) A is S_{bi}-rough $\rightarrow$ A is S_{lr}-rough $\rightarrow$ A is l-rough.

Proof

(a) Let $A \subseteq X$ is r-exact set, then $\downarrow^r A = \uparrow_r A = A$. From Proposition 3.2, we have, $\downarrow^r A \subseteq \downarrow^{S_{rl}} A \subseteq \downarrow^{S_{bi}} A$ and $\uparrow_{S_{bi}} A \subseteq \uparrow_{S_{rl}} A \subseteq \uparrow_r A$. Therefor, $\downarrow^{S_{rl}} A = \downarrow^{S_{bi}} A = \uparrow^{S_{bi}} A = \uparrow_{S_{rl}} A = A$. Consequently, A is S_{rl}-exact set and A is S_{bi}-exact set.

We can get the proof of (b), (c) and (d) at the same way as (a).

4. Semi-Bi-Rough Concepts

In this section, Pawlak's concepts are redefined and studied. The relationships among suggested semi-bi-rough concepts and their traditionals rough concepts. A comparison among all these approaches by using their accuracy measures is given. Finally, we conclude the relationship among all studied approaches to rough sets in a diagram.

Definition 4.1 In a bitopological space (X,τ_r,τ_l) generated by a general relation, we can determine the degree of crispness of any subset $A\subseteq X$, by using a bitopological accuracy measure denoted by $C_I A$, for all $I\in\{r,l,S_{rl},S_{lr},S_{bi}\}$ and it is defined as

$$C_I A=\frac{\downarrow^I A}{\uparrow_I A},\quad A\neq\phi.$$

Proposition 4.1 Let (X,τ_r,τ_l) be a bitopological space and let $A\subseteq X$, then

(a) $0\leq C_r A\leq C_{S_{rl}} A\leq C_{S_{bi}} A\leq 1.$

(b) $0\leq C_l A\leq C_{S_{lr}} A\leq C_{S_{bi}} A\leq 1.$

Proof By using Proposition 3.2, we get the proof directly.

The following example studies a comparison among suggested semi-bitopological approaches, by using their accuracy measures.

Example 4.1 From Example 3.1, we can create Table 2, as follows

Table 2. *Comparison among studied approaches by using their accuracy measures.*

	$\{a\}$	$\{c\}$	$\{d\}$	$\{a,c\}$	$\{a,d\}$	$\{b,c\}$	$\{b,d\}$	$\{c,d\}$	$\{a,b,c\}$	$\{a,b,d\}$	$\{b,c,d\}$
$C_{S_{rl}} A$	1/2	0	1/3	1/2	1/2	0	2/3	1/3	1/3	3/4	2/3
$C_{S_{lr}} A$	1/2	1/3	0	1/2	1/2	2/3	0	1/3	3/4	1/3	2/3
$C_{S_{bi}} A$	1	1	1	1	1	1	1	2/3	1	1	1

From Proposition 4.1, we can deduce that, the best approach to rough sets, in this study, is S_{bi}-approach. Also, from Table 2, we notice that, by using S_{bi}-set approximations, many subsets of X become crisp, although they are not S_{rl}-exact sets or S_{lr}-exact sets.

Definition 4.2 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation and let $A\subseteq X$. For all $I\in\{r,l,S_{rl},S_{lr},\ S_{bi}\}$ I-rough membership relations, denoted by $\underline{\in}_I$ and $\overline{\in}_I$, are defined as

$$x\underline{\in}_I A\quad if\quad x\in\downarrow^I A\quad and\quad x\overline{\in}_I A\quad if\quad x\in\uparrow_I A.$$

Proposition 4.2 In a bitopological space (X,τ_r,τ_l) generated by a general relation. For any subset $A\subseteq X$ and for all $I\in\{r,l,S_{rl},S_{lr},\ S_{bi}\}$, we can prove that,

(a) $x\underline{\in}_r A\Rightarrow x\underline{\in}_{S_{rl}} A\Rightarrow x\underline{\in}_{S_{bi}} A\Rightarrow x\in A\Rightarrow x\overline{\in}_{S_{bi}} A\Rightarrow x\overline{\in}_{S_{rl}} A\Rightarrow x\overline{\in}_r A.$

(b) $x\underline{\in}_l A\Rightarrow x\underline{\in}_{S_{lr}} A\Rightarrow x\underline{\in}_{S_{bi}} A\Rightarrow x\in A\Rightarrow x\overline{\in}_{S_{bi}} A\Rightarrow x\overline{\in}_{S_{lr}} A\Rightarrow x\overline{\in}_l A.$

Proof From Proposition 3.2, we get the proof, directly.

The following Example illustrates that, the inverse of Property (a), does not hold.

Example 4.2 According to Example 3.1, if $A=\{a,d\}$, $B=\{a,b\}$, $C=\{a,c,d\}$ and $E=\{b,c\}$, then $\downarrow^{S_{rl}} E=\emptyset$, $\downarrow^{S_{bi}} E=\{b,c\}$, $\downarrow^{S_{bi}} B=\{a\}$, $\uparrow_{S_{bi}} C=X$, $\uparrow_{S_{bi}} B=\{a,b\}$, $\uparrow_{S_{rl}} B=\{a,b,c\}$, $\uparrow_{S_{lr}} B=\{a,b,d\}$, $\downarrow^{S_{lr}} A=\{a\}$ and $\downarrow^{S_{bi}} A=\{a,d\}$. Hence, $c \underline{\notin}_{S_{rl}} E$ but $c \underline{\in}_{S_{bi}} E$, $d \underline{\notin}_{S_{lr}} A$ but $d \underline{\in}_{S_{bi}} A$, $b \underline{\notin}_{S_{bi}} B$ but $b\in B$, $b\notin C$ but $b \overline{\in}_{S_{bi}} C$, $c \overline{\notin}_{S_{bi}} B$ but $c \overline{\in}_{S_{rl}} B$ and $d \overline{\notin}_{S_{bi}} B$ but $d \overline{\in}_{S_{lr}} B$.

Definition 4.3 Let (X,τ_r,τ_l) be a bitopological space generated by a general relation and let $A,E\subseteq X$. For all $I\in\{r,l,S_{rl},S_{lr},\ S_{bi}\}$ I-rough inclusion relations, denoted by $\underset{\rightarrow I}{\subset}$ and $\overset{\rightarrow}{\subset}_I$, are

$$A\underset{\rightarrow I}{\subset} E \quad if \quad \downarrow^{I} A\subseteq\downarrow^{I} E \quad and \quad A\overset{\rightarrow}{\subset}_I E \quad if \quad \uparrow_I A\subseteq\uparrow_I E.$$

The following example illustrates Definition 4.3, at $I=S_{bi}$.

Example 4.3 According to Example 3.1, if $A=\{a,b\}$, $C=\{a,c\}$, $D=\{b,d\}$ and $E=\{c,d\}$, then $\downarrow^{S_{bi}} A=\{a\}$, $\downarrow^{S_{bi}} C=\{a,c\}$, $\uparrow_{S_{bi}} D=\{b,d\}$ and $\uparrow_{S_{bi}} E=\{b,c,d\}$. Hence, $A\underset{\rightarrow S_{bi}}{\subset} C$ and $D\overset{\rightarrow}{\subset}_{S_{bi}} E$. Although, $A\not\subseteq C$ and $D\not\subseteq E$.

Remark 4.1 From our study, we can conclude the relationship among suggested approaches to rough sets of $A\subseteq X$ in Diagram 1, as follows

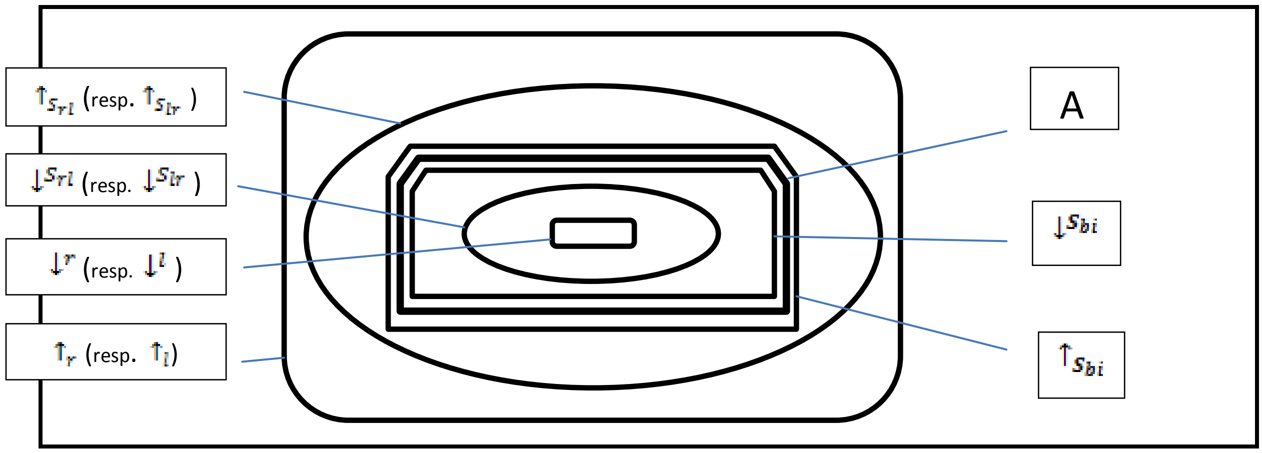

Diagram 1: *Relationship among studied approaches to rough sets.*

From Diagram 1, we can deduce that, any suggested semi bitopological approach to rough sets is better than its traditional. In addition, S_{bi} -approach is the best model of proposed models in this study.

5. Conclusion

In this paper, proposed semi bitopological approaches, depend on a general relation. If we repleace this relation by an equivalence relation R, then we get $_xR=R_x=[x]_R$ and then $\tau_r=\tau_l$, generated by R. It followes that, (X,τ_r,τ_l) becomes Pawlak space (X,R). Therefor $\downarrow^{I} A=\underline{R}(A)$ and

$\uparrow_I A = \overline{R}(A)$, for all $I \in \{r, l, S_{rl}, S_{lr}, S_{bi}\}$. It means that, in this special case, all properties and concepts of any suggested approach must be returned to their traditionals.

In addition, as we proved, S_{bi}-approach to rough sets is the best suggested approach in this study. Hence, by using this model, any vague concept has a big chance to be a precise concept.

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