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Effects of Variable Viscosity and Thermal Conductivity on MHD Free Convection and Mass Transfer Flow Past a Flat Plate

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Abstract: *Two dimensional free convection and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical porous plate in the presence of heat source, thermal diffusion, large suction and under the influence of uniform magnetic field applied normal to the flow with variable viscosity and thermal conductivity is studied. Usual similarity transformations are introduced to reduce the governing partial differential equations to ordinary differential equations. The momentum, energy and concentration equations are solved numerically using shooting method.*

The results are presented in graphs and tables to observe the effect of various parameters. It is observed that velocity decreases as viscosity parameter increases.

Keywords: *Variable viscosity, variable thermal conductivity, Mass transfer.*

1. Introduction

Many transport processes occur in industrial application in which the transfer of the heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Many studies have been reported for vertical, horizontal and inclined plate in presence of a transverse magnetic field. Sanyal and Bhattacharya [2] have presented the technique to obtain the similarity solutions in a hydromagnetic flow. An extensive contribution on heat and mass transfer flow has been made by Gebhart [3] to highlight the insight on the phenomena. Gebhart and Pera [3] studied heat and mass transfer flow under various flow situations.

Therefore, several authors viz. Agrawal et al [4], Jha and Prasad [5], Singh and Yadav [8] have paid attention to the study of MHD free convection and mass transfer flows. Introducing a time dependent length scale of similarity technique, similarity solutions to study the free convection and mass transfer flow past an impulsively started vertical porous plate in a rotating fluid in presence of large suction have been studied by Sattar and Alam [6] based on perturbation technique demonstrated by Singh and Dixit [7]. Sing et al. [8] modified the work of Sattar and Alam [6] for study MHD free convection in mass transfer flow with thermal diffusion for viscous fluid flow and flow of viscous stratified liquid respectively. Subsequently, Singh and Singh [9] extended the problem of Singh [10] to analyze the effects of mass transfer on MHD flow considering constant heat flux and induced magnetic field.

In most of the studies, of this type of problems, the viscosities and the thermal conductivities of the fluid were assumed to be constant. However, it is known that these physical properties can change significantly with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are significantly changed compared to the constant property case.

Hence, the problems under consideration, the viscosity and the thermal conductivity have been assumed to be inverse linear function of temperature.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady free convection and mass transfer flow of an incompressible, electrically conducting viscous fluid past an electrically non-conducting continuously moving infinite vertical porous plate. Introducing a Cartesian co-ordinate system, x-axis is chosen along the plate in the direction of flow and y-axis normal to it. A uniform magnetic field applied normally to the flow region. The plate is maintained at a constant temperature T_w and the concentration is maintained at a constant value C_w.

The temperature of uniform flow is T_∞ and the concentration of uniform flow is C_∞. Considering the magnetic Reynold's number to be very small, the induced magnetic field is assumed to be negligible, so that $\vec{B} = (0, B_0(x), 0)$.The equation of conservation of electric charge is $\vec{\nabla}.\vec{J} = 0$, where $\vec{J} = (J_x J_y J_z)$ and the direction of propagation is assumed along y-axis so that $\vec{J}$ does not have any variation along y-axis so that the y derivative of $\vec{J}$ namely

$\frac{\partial J_y}{\partial y} = 0$ resulting in $J_y = constant$. Also the plate is electrically non-conducting , therefore the constant $J_y = 0$ everywhere in the flow. Considering the Joule heating and viscous dissipation terms to be negligible and that the magnetic field is not enough to cause Joule heating, the term due to electrical dissipation is neglected in the energy equation. The density is considered a linear function of temperature and species concentration so that the usual Boussinesq's approximation is taken as

$$\rho = \rho_0 \, [1\text{-}\{\beta(T - T_\infty) + \beta^*(c - c_\infty)\}]$$

Fluid viscosity and thermal conductivity are assumed to vary with temperature.

With the above assumptions the equations of continuity, momentum, energy and the concentrations are

Continuity equation:
$$\frac{du}{dx} + \frac{\partial v}{dy} = 0 \tag{1}$$

Momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho g\beta(T - T_\infty) + \rho g\beta^*(C - C_\infty) - \sigma^{/}\beta_0^2(x)u \tag{2}$$

Energy equation:
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho cp}\frac{\partial}{\partial y}(K\frac{\partial T}{\partial y}) + Q(T - T_\infty) \tag{3}$$

Concentration equation:
$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} + D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions relevant to the problem are:

$$\left.\begin{array}{ll} u = U_0, v = v_0(x), T = T_w, C = C(x) & at \;\; y = 0 \\ \mathrm{u} = 0, \mathrm{v} = 0, \mathrm{T} = \mathrm{T}_\infty, \mathrm{C} = \mathrm{C}_\infty & \text{as } \; \mathrm{y} \to \infty \end{array}\right\} \tag{5}$$

where u, v are velocity components along x-axis and y-axis, g acceleration due to gravity, T the temperature, T_w the wall temperature, T_∞ the temperature of the uniform flow, K thermal conductivity, D_M the molecular diffusivity, C the concentration of species, C_∞ the concentration of species for uniform flow, $\beta_0(x)$ the uniform magnetic field, Cp the specific heat at constant pressure, Q the constant heat source (absorption type), D_T the thermal diffusivity, C_w the mean concentration, $C(x)$ variable concentration at the plate, $v_0(x)$ suction velocity, ρ the density, υ the kinematic viscosity, β the volumetric co-efficient of thermal expansion with concentration and the other symbols have their usual meaning.

For similarity solution, the plate concentration $C(x)$ is considered to be

$$C(x) = C_\infty + (C_w - C_\infty)x$$

We introduce the following local similarity variables:

$$\Psi = \sqrt{2\upsilon x U_0} f(\eta), \eta = y\sqrt{\frac{U_0}{2\upsilon x}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C(x) - C_\infty}{C_w - C_\infty}$$

Introducing the above stated similarity variables using the relations $u = \frac{\partial\psi}{\partial y}$ and

$v = -\frac{\partial\psi}{\partial x}$ and the equation of continuity (1), we obtain

$$u = U_0 f^{/}(\eta) \text{ and } \upsilon = \sqrt{\frac{\upsilon U_0}{2x}}\,(\eta f^{/} - f)$$

Here θ and φ are non-dimensional temperature and concentration.

In the light of Lai and Kulaki, we have assumed viscosity of a fluid to be an inverse linear function of temperature as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[T + \gamma(T - T_\infty)] \qquad \text{or} \qquad \frac{1}{\mu} = a(T - T_r)$$

where $a = \frac{\gamma}{\mu_\infty}$

$$T_r = T_\infty - \frac{1}{\gamma}$$

Where μ is the fluid viscosity, μ_∞ is the viscosity of free stream, T is the temperature, T_∞ is the temperature at free stream, γ is the constant based on the thermal property of the fluid.

Khound and Hazarika, in our investigation, the variation of thermal conductivity shall be considered as

$$\frac{1}{k} = \frac{1}{k_\propto}[1 + \xi(T - T_\infty)] \qquad \text{or} \qquad \frac{1}{k} = b(T - T_e)$$

where $b = \frac{\xi}{k_\propto}$ and $T_e = T_\infty - \frac{1}{\xi}$

where k and T are the thermal conductivity and temperature of the fluid, $k_\propto$ and T_∞ are the thermal conductivity and temperature at free stream, ξ is a constant based on the thermal property of the fluid.

The non-dimensional form of viscosity and thermal conductivity parameters $\theta(\eta)$ and θ_r can be written as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} \tag{8}$$

Under the above substitution equation (1) is identically satisfied and the equations (2-4) reduced to

$$f^{///} - (\frac{\theta-\theta_r}{\theta_r})ff^{//} - \frac{1}{(\theta-\theta_r)}\theta^{/}f^{//} - (\frac{\theta-\theta_r}{\theta_r})G_r\theta - (\frac{\theta-\theta_r}{\theta_r})G_m\varphi + (\frac{\theta-\theta_r}{\theta_r})Mf^{/} = 0 \tag{9}$$

$$\theta^{//} - (\frac{\theta-\theta_e}{\theta_e})P_r f\theta^{/} - \frac{1}{(\theta-\theta_e)}\theta^{/2} - (\frac{\theta-\theta_e}{\theta_e})P_r S\theta = 0 \tag{10}$$

$$\phi^{//} - 2S_c f^{/}\phi + S_c f\phi^{/} + S_0 S_c\theta^{//} = 0 \tag{11}$$

where

$P_r = \frac{\mu C_p}{K}$ (Prandtl number)

$S_c = \frac{\upsilon}{D_M X}$ (Schmidt number)

$G_r = \frac{g\beta(T_w - T_\infty)2x}{U_0^2}$ (Grashof number)

$M = \frac{\sigma' B_0^2(x)2x}{\rho U_0}$ (Magnetic parameter)

$S_c = \frac{T_w - T_\infty}{C_w - C_\infty}$ (Soret number)

$G_m = \frac{g\beta^*(C_w - C_\infty)2x}{U_0^2}$ (modified Grashof number)

$S = \frac{2xQ}{U_0}$ (Heat source parameter)

The boundary conditions (5) transformed to

$f = f_w, f^/ = 1, \theta = 1, \phi = 1$ at $\eta = o$

$f^/ = 0, \theta = 0, \phi = 0$ as $\eta = \infty$

where $f_w = -\upsilon_0(x)\sqrt{\frac{2x}{\upsilon U_0}}$ and primes denotes the derivatives with respect to η.

Here $f_w > 0$ denotes the injection and $f_w < 0$ the suction.

Skin friction co-efficient, Nusselt no. and Sherwood no. are respectively proportional to $f^{//}(0)$, $\theta^/(0)$ & $\phi^/(o)$.

3. METHOD OF SOLUTION

To solve the boundary value problems (2)-(4) the Runge-kutta shooting method is applied .In this method the BVP are converted to initial value problems by estimating *the missing initial values to a desired degree of accuracy by an iterative scheme.* Hazarika [1] showed that through there is no guarantee of convergence of the iterative scheme, if the initial guesses for the missing initial values are on opposite sides of the true values. The convergence is rapid and agrees well with other methods.

Using shooting method, the missing initial values viz. f''(0), θ'(0), φ'(0) etc are estimated for various combination of parameters and consequently the problem is solved

Table1. ***Missing values of*** *f''(0), θ'(0), φ'(0) for various values of θ_r and θe=-10, Gr=10, Gm=15,M=1, Pr=.7, Sc=.22, S=1.0, So=.30, f_w=5.0.*

θ_r	f''(0)	θ'(0)	φ'(0)
-10	-0.47464	-3.77563	-9.30525
-9	-0.47817	-3.77547	-9.30481
-8	-0.48256	-3.77527	-9.30426
-7	-0.48817	-3.77501	-9.30356
-6	-0.4956	-3.77468	-9.30263
-5	-0.50589	-3.77422	-9.30135
-4	-0.52109	-3.77354	-9.29946
-3	-0.54584	-3.77244	-9.2964
-2	-0.59335	-3.77039	-9.29059

Table2. ***Missing values of*** *f'' (0), θ'(0), φ'(0) for various values of θ_e and θr=-10, Gr=10, Gm=15,M=1, Pr=.7, Sc=.22, S=1.0, So=.30, f_w=5.0.*

θ_e	f''(0)	θ'(0)	φ'(0)
-10	-0.47464	-3.77563	-9.30525
-9	-0.49023	-3.8141	-9.29145

-8	-0.50944	-3.86221	-9.27423
-7	-0.5337	-3.92408	-9.25211
-6	-0.56528	-4.00661	-9.22267
-5	-0.60811	-4.12223	-9.18156
-4	-0.66952	-4.29584	-9.12005
-3	-0.76497	-4.58581	-9.01791
-2	-0.9339	-5.16924	-8.81419

4. Results and Discussion

Here the systems of differential equations governed by the boundary conditions are solved numerically by using fourth order Runge-Kutta Shooting method. The whole numerical scheme can be programmed and applied easily. It is experienced that the convergence of the iteration process is quite rapid. The estimated values of the missing initial values are arranged in different tables for various values of the viscosity parameters .The following figures explain the velocity and temperature distribution against different dimensionless parameters.

From figure 1, we can conclude that with the increase of viscosity (θ_r), velocity decreases.

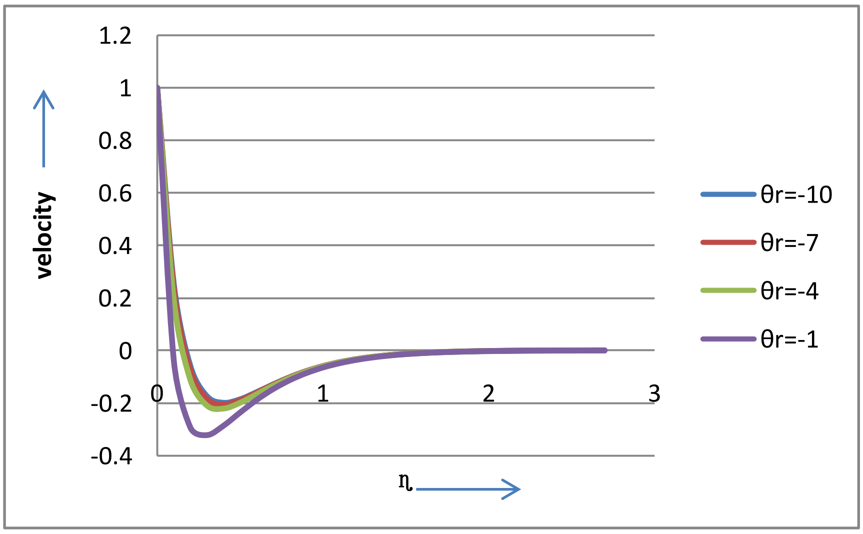

Fig 1. *Variation of velocity with θr*

From figure 2, we can conclude that with the increase of thermal conductivity (θ_e), velocity increases.

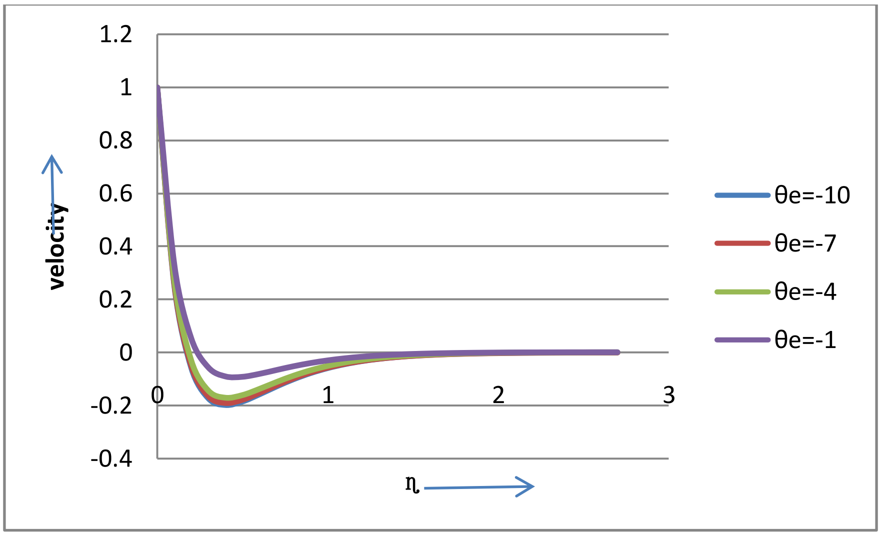

Fig 2. *Variation of velocity with θe*

From figure 3, we can conclude that with the increase of thermal conductivity (θ_e), temperature decreases.

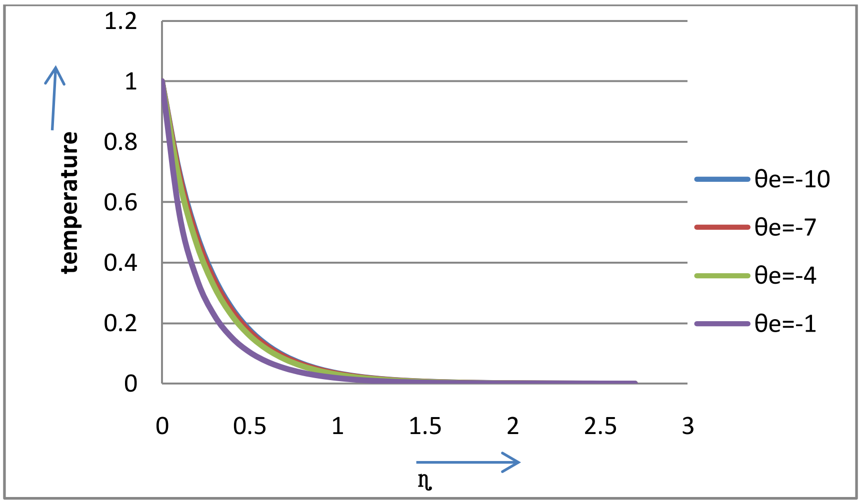

Fig 3. *Variation of temperature with θe*

From figure 4, we can conclude that with the increase of heat source parameter (S), velocity increases.

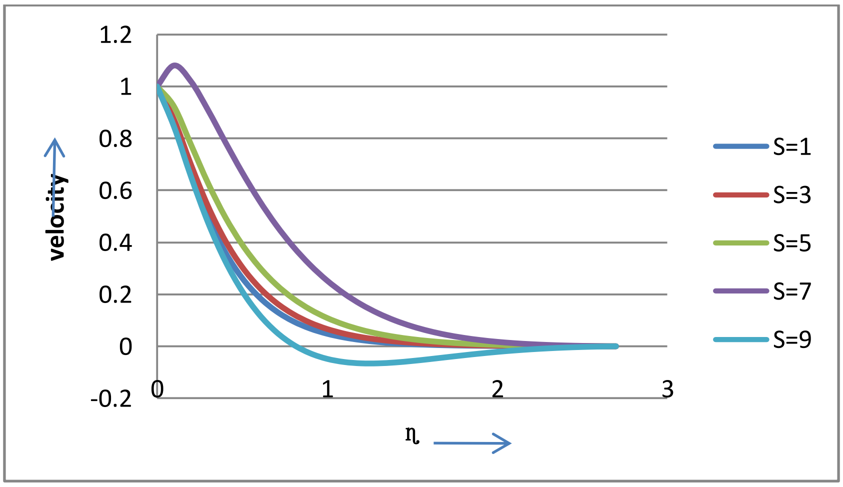

Fig 4. *Variation of velocity with S*

From figure 5, we can conclude that with the increase of Hartmann numbers (M), velocity decreases.

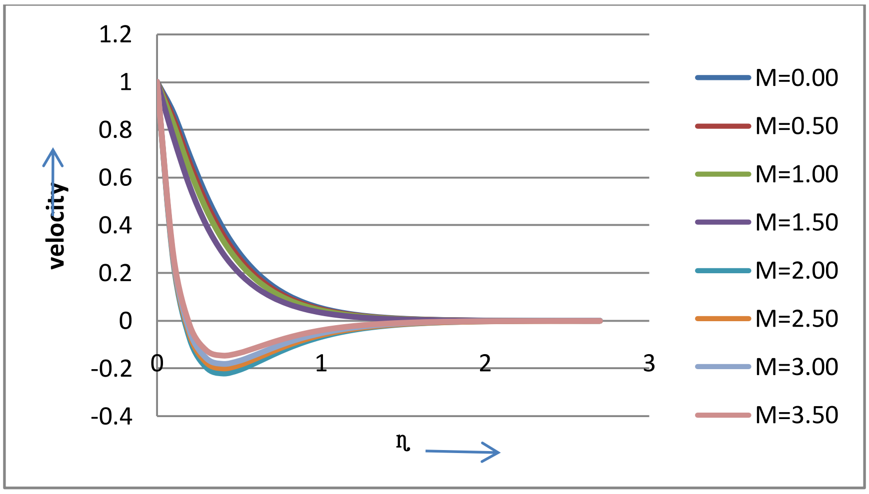

Fig 5. *Variation of velocity with M*

Table -1 and Table-2 give the missing values of f''(0), θ'(0), φ'(0) for different values of θr and θe and for Gr=10, Gm=15,M=1, Pr=.7, Sc=.22, S=1.0, So=.30, f_w=5.0.

5. CONCLUSION

From the studies made on this paper we conclude as under.

1. Viscosity decelerates the fluid velocity.
2. Thermal conductivity enhances the fluid velocity and decrease the fluid temperature.
3. Heat source parameter enhance the fluid velocity.
4. Magnetic field decelerates the fluid due to Lorentz Force.
5. Sheer stress as well as the rate of heat flow increases due to increase of viscocity and thermal conductivity.
6. Mass transfer rate decreases for increase of viscosity and thermal conductivity.

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