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Fuzzy Difference Equations in Finance

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Abstract: *Fuzzy difference equations initially introduced by Kendel and Byatt [1,2]. An important effort to study of equations has been made by Buckley[3]. In this work we apply the method of fuzzy difference equations to study same problems in finance. As a source of different cases of finance equations we use the work by Chrysafis, Papadopolous, Papaschinopoulos [4,13,16,17]. To illustrate the applicability of the method we give numerical examples.*

Keywords: *Difference Equations, Interval Arithmetic, Interest Rate.*

1. INTRODUCTION

Fuzzy difference equation has been growing rapidly developed for the many years. To solve the difference equation is by finding a sequence that satisfies the equation. Chrysafis, Papadopolous, Papaschinopoulos [1,14,15,16] in their study about the fuzzy difference equation of finance. The fuzzy difference equations initially are introduced by Kendel and Byatt [1,2,18,19]. Also, the fuzzy difference equations and their applications are studied by Buckley [5-10,20]. In this paper, we will review the fuzzy difference equations and numerical solution of their application in finance.

2. PRELIMINARIES

2.1 Fuzzy Numbers

The most popular kind of fuzzy numbers is triangular fuzzy numbers. Following representation is interpreted as membership function [11]

$$\mu_u(x) = \begin{cases} \dfrac{x-a}{b-a}, & a < x \le b \\ \dfrac{c-x}{c-b}, & b < x < c \\ 0, & otherwise \end{cases}$$

$\propto$-cuts of u can be found as

$$[u]_\propto = [a + (b-a)\propto, c - (c-b)\propto]$$

2.2 Fuzzy Sets

Fuzzy set is sets whose elements have membership function. Fuzzy sets were introduced by Lotfi A. Zadeh in 1965[1]. If A is a function from $R^+ = (0, \infty)$ is to interval [0,1] then A is valued as a fuzzy set.

$A$is convex, if for every $t \in [0,1]$, and x_1, x_2we have $A(tx_1 + (1-t)x_2) \geq \min\{A(x_1), A(x_2)\}$.$A$is normalized if there exists an $x \in R^+$such that $A(x) = 1$, if A is a fuzzy set by a cut, $a \in [0,1]$ we mean the sets $[A]_a = \{x \in R^+ : A(x) \geq a\}$.. It is known that A is a fuzzy number, if the following conditions hold;

i) A is normal

ii) A is convex fuzzy set

iii) A is upper semi continuous

iv) The support of A, supp $A = \overline{U_{a \in [0,1]}[A]_a} = \overline{\{x : A(x) > 0\}}$ is compact. Then the a-cuts of A are closed intervals.

Let A be fuzzy number with $[A] = [A_{l,a}, A_{r,a}]$. We define a norm on the fuzzy numbers space as follows:

$$\|A\| = supmax\{|A_{l,a}|, |A_{r,a}|\}$$

Where sup is taken for all $a \in [0,1]$ let $x_n$be sequence of positive fuzzy numbers such that,

$$[x_n]_a = [L_{n,a}, R_{n,a}], \quad a \in [0,1], \quad n = 0,1,..$$

And let x be positive fuzzy number such that

$$[x]_a = [L_a, R_a], \quad a \in [0,1]$$

2.3 Fuzzy Operations

If $A$and $B$are fuzzy numbers like,

$$\tilde{A} = [a_1, b_1]$$
$$\tilde{B} = [a_2, b_2]$$

We can describe the multiplication of two fuzzy number as

$$\tilde{A}.\tilde{B} = [\gamma, \beta]$$

$$\begin{cases} \gamma = min\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\} \\ \beta = max\{a_1a_2, a_1b_2, b_1a_2, b_1b_2\} \end{cases}$$

We can describe the division of two fuzzy number as [12]

$$\tilde{A}/\tilde{B} = [\gamma, \beta]$$

$$\begin{cases} \gamma = min\left\{\frac{a_1}{b_2}, \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{b_1}{a_2}\right\} \\ \beta = max\left\{\frac{a_1}{b_2}, \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{b_1}{a_2}\right\} \end{cases}$$

2.4 Difference Equations

Let the price $P(n)$ of a product after n years and the supply $S(n)$ after n years. The price $P(n)$ is related to the supply $S(n)$by $P(n) = a - bS(n)$, where a and b positive constants. In the price-supply relation, the price lows because of a large supply in a given year.

Thus, we consider the following difference finance equation by

$$\begin{cases} P(n+1) = bP(n) + a \\ P(n_0) = P_0 \end{cases} n = 0,1,2,..$$

If we solve the difference equation

$$P(1) = bP(n_0) + a$$
$$P(2) = b(bP(n_0) + a) + a$$

$$P(2) = b^2P(n_0) + a(b + 1)$$
$$P(3) = b^3P(n_0) + a\left(b^2 + b + 1\right)$$
$$\vdots$$
$$P(n+1) = b^{n+1}.P(n_0) + a.\left(b^n + b^{n-1} + \cdots + b^1 + b^0\right)$$

Then we can easily prove that

$$\left(b^n + b^{n-1} + \cdots + b^1 + b^0\right) = \left[\frac{1 - b^{n+1}}{1 - b}\right]$$

$n = 1,2, ...$is the solution of the difference equation

$$P(n+1) = b^{n+1}.P(n_0) + a.\left[\frac{1 - b^{n+1}}{1 - b}\right]$$

2.5 Fuzzy Difference Equations

If,$P_0$and a are fuzzy numbers, and b is constant, and the difference equation is

$$\tilde{P}(n+1) = b\tilde{P}(n) + \tilde{a}$$
$$\tilde{P}(n_0) = \tilde{P}_0$$

To solve this equation first we have to find a solution for fuzzy difference equation

$$\tilde{P}(n+1) = b^{n+1}.\tilde{P}(n_0) + \tilde{a}\left[\frac{1 - b^{n+1}}{1 - b}\right]$$

And upper and lower solution is

$$\underline{P}(n+1) = b^{n+1}\underline{P}(n_0) + \underline{a}\left[\frac{1 - b^{n+1}}{1 - b}\right]$$
$$\overline{P}(n+1) = b^{n+1}.\overline{P}(n_0) + \overline{a}\left[\frac{1 - b^{n+1}}{1 - b}\right]$$

k, a, b are fuzzy numbers, and the difference equation is

$$\tilde{P}(n+1) = \tilde{b}\tilde{P}(n) + \tilde{a}$$
$$\tilde{P}(n_0) = \tilde{k}P_0$$

If$\tilde{b}.\tilde{k} = \tilde{c}$, differential equation turns to

$$\tilde{P}(n+1) = \tilde{c}P(n) + \tilde{a}$$

To solve this equation first we have to find a solution for fuzzy difference equation

$$\tilde{P}(n) = \tilde{c}^n.P(n_0) + \tilde{a}\left[\frac{1 - \tilde{c}^n}{1 - \tilde{c}}\right]$$

And upper and lower solution is

$$\underline{P}(n) = \underline{c}^nP(n_0) + \underline{a}\left[\frac{1 - \underline{c}^n}{1 - \underline{c}}\right]$$
$$\overline{P}(n) = \overline{c}^n.P(n_0) + \overline{a}\left[\frac{1 - \overline{c}^n}{1 - \overline{c}}\right]$$

3. NUMERIC EXAMPLES

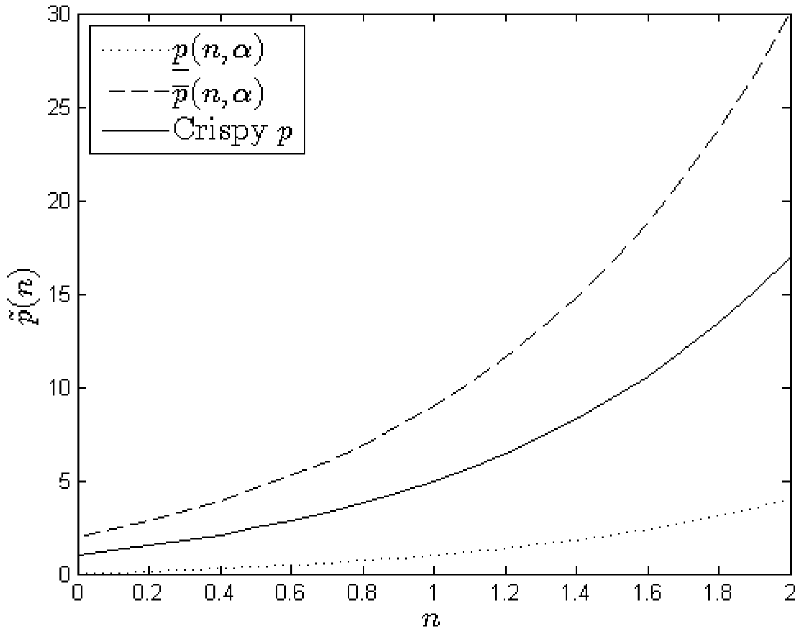
Example 3.1: Let us consider the following fuzzy difference equation [21]To ensure a high-quality product, diagrams and lettering must be either computer-drafted or drawn using India ink.

$$\begin{cases} \tilde{P}(n+1) = 3.\tilde{P}(n) + \tilde{2} \\ \tilde{P}(n_0) = \check{k}(1+{n_0}^2) \end{cases} \qquad n = 0,1,2,\ldots$$

According to chapter 2.5,$\tilde{2} = [1{+}\propto ,3{-}\propto]$and $\tilde{k} = [\propto ,2{-}\propto]$are the triangular fuzzy numbers having ∝-level sets. Now if we substitute these ∝-cut sets in (4.1) we get lower and upper solution of $\tilde{P}(n)$

$$\underline{P}(n,\propto) = 3^n(\propto)P(n_0) + \left[\frac{1-3^n}{1-3}\right].(1{+}\propto)$$

$$\overline{P}(n,\propto) = 3^n(2{-}\propto)P(n_0) + \left[\frac{1-3^n}{1-3}\right].(3{-}\propto)$$



Example 3.2: Consider the fuzzy difference equation [21]

$$\tilde{P}(n+1) = \tilde{2}\tilde{P}(n) + \tilde{0}$$

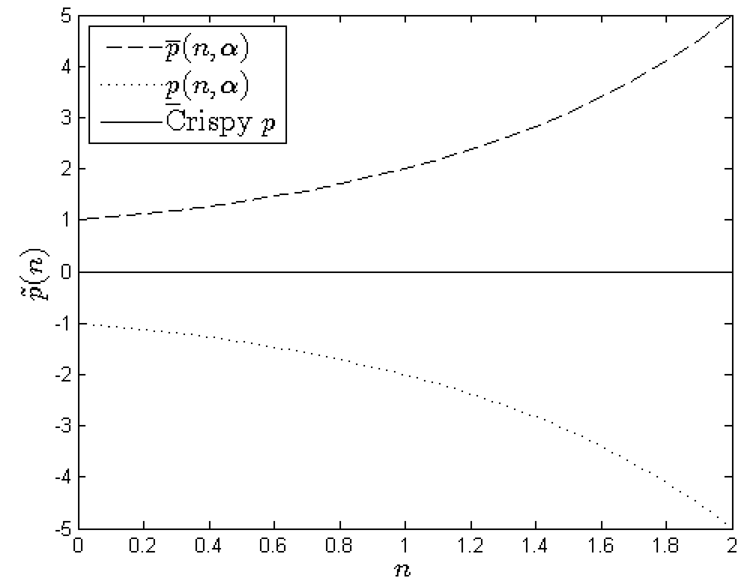
with the initial condition

$$\begin{cases} \tilde{P}(n_0) = \tilde{k}.n_0(1+{n_0}^2) \\ \tilde{k} = [\propto -1,1{-}\propto] \\ \tilde{2} = [1{+}\propto ,3{-}\propto] \\ \tilde{0} = [\propto -1,1{-}\propto] \end{cases}$$

According to chapter 2.6 the solution is

$$\underline{P}(n,\propto) = (3{-}\propto)^n(\alpha-1)P(n_0) + (\propto -1).\left[\frac{1-(4\propto -\propto^2+3)^n}{1-(3-4\propto +\propto^2)}\right]$$

$$\overline{P}(n,\propto) = (3{-}\propto)^n(1{-}\propto)P(n_0) + (1{-}\propto).\left[\frac{1-(4\propto -\propto^2+3)^n}{1-(3-4\propto +\propto^2)}\right]$$



Example 3.3: Let us consider the situation for a deposit in a bank. Bank has a limit to make deposit stable. If the amount of deposit is under this limit, it will get some cuts. $P(n)$is the deposit and $\tilde{z}$ is the cutting rate during the time which deposit in a bank account. To find a final amount of the deposit after 2 year, we have following fuzzy difference equation: [21]

$$\tilde{P}(n+1) = \tilde{P}(n) - \tilde{z}, \qquad z > 0$$

$$\tilde{P}(n_0) = 1000€$$

$$\tilde{z} = [0.025 + 0.002\alpha, 0.029 - 0.002\alpha]$$

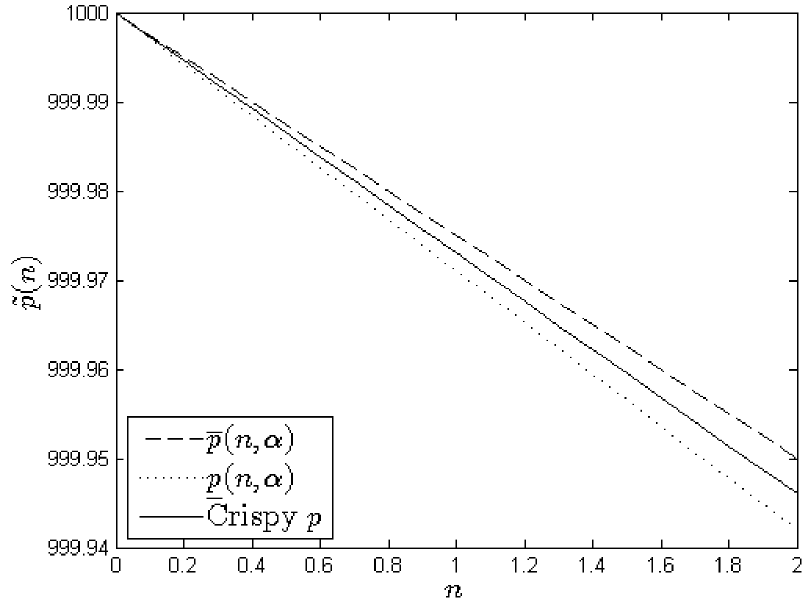
$$\tilde{k} = [\alpha, 2 - \alpha]$$

Solution for this fuzzy difference equation is as follows

$$\tilde{P}(n) = \tilde{P}(n_0) - n\tilde{z}$$

$$\overline{P}(n) = \overline{P}(n_0) - n\overline{z}$$

$$\underline{P}(n) = \overline{P}(n_0) - n\underline{z}$$



Example 3.4 Here, deposit in a bank account is denoted by $P(n)$ in order to get interest. $P_0$is the original amount of the deposit and, $\tilde{I}$ is the uncertain interest rate. This system is usually implemented in financial actions for the deposit which getting interest by years. To find a final amount of the deposit after 2 year, we have following fuzzy difference equation: [21]

$$\tilde{P}(n+1) = P(n) + \tilde{I}\tilde{P}(n_0)$$

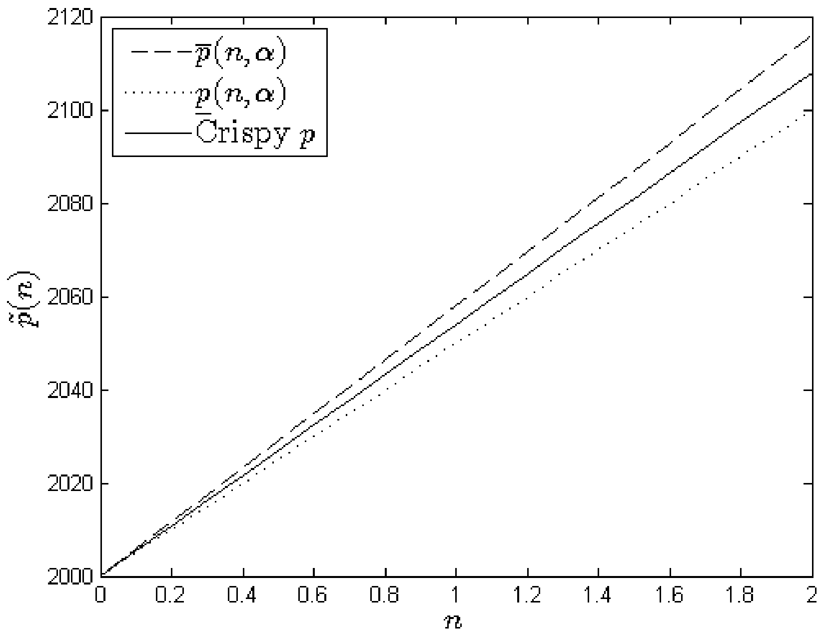
$$\tilde{P}(n_0) = 2000€$$

$$\tilde{I} = [0.025 + 0.002\alpha, 0.029 - 0.002\alpha]$$

Solution of this fuzzy difference equation is as follows

$$\tilde{P}(n) = \tilde{P}(n_0) + n\tilde{I}\tilde{P}(n_0)$$
$$\overline{P}(n) = \overline{P}(n_0) + n\overline{I}\overline{P}(n_0)$$

$$\underline{P}(n) = \underline{P}(n_0) + n\underline{I}\underline{P}(n_0)$$



4. Conclusions

We proved solution to difference equation of finance by using fuzzy interval arithmetic operations. We also study the time value of money via fuzzy difference equations and different numerical examples are shown by graphically. Further research we propose the study of nonlinear fuzzy difference finance equation.

Acknowledgements

An acknowledgement section may be presented after the conclusion, if desired.

References

[1] A. Kandel, W. J. Byatt, Proc. Int. Conf. Cybernatics and Society, Tokyo, Novenber, 1213 1216(1978)

[2] A. Kandel, W. J. Byatt, Proc. IEEE, 66, 1619-1639(1978)

[3] J.J. Buckley, The fuzzy mathematics of finance, Fuzzy Sets and Systems 21 (3) 257 273(1987)

[4] K. A. Chrysafis, B. K. Papadopoulos, and G. Papaschinopoulos, Fuzzy Sets and Systems, 159, 3259-3270(2008)

[5] L.A. Zadeh, Information and Control 8, 338-353(1965)

[6] Q.Zhang, L.Yang, D.Liao,.Engineering and Technology, 75(2011)

[7] G. Papaschinopoulos, G. Stefanidou and P.Efraimidis, Information Sciences, 177, 3855-3870(2007)

[8] J.J.Buckly, Fuzzy Sets and Systems, 21 (3), 257-273(1987)

[9] ZehuaLv, Hai Jin and Pingpeng Yuan, IEEE Computer and Informaton Technology(2009)

[10] M. Darus and A. Wahab, Review on Fuzzy difference equation, Malasysian Journal of Fundemental& Applied Sciences, 8(4), 172-176(2012)

[11] Shang Gao and Zaiyue Zhang, Multiplication Operation on Fuzzy Numbers, Journal Of Software, Vol. 4, No. 4(2009)

[12] DemetOzgur, Master Thesis, Celal Bayar University, Manisa, Turkey(2008)

[13] M. Kwapisz, On difference equations arising in mathematics of finance, Nonlinear Anal. Theory, Methods Appl. 30 (2) 1207–1218(1997)

[14] D. Dubois, E. Kerre, R.Mesiar, H. Prade, Fuzzy interval analysis, in: D. Dubois, H. Prade (Eds.), Fundamentals of Fuzzy Sets, The Handbooks of Fuzzy Sets Series, Kluwer, Boston, Mass, pp. 483–581 (2000)

[15] E.Y. Deeba, A. De Korvin,Analysis by fuzzy difference equations of a model of CO2 level in the blood, Appl. Math. Lett. 12 33–40(1999)

[16] G. Papaschinopoulos, B.K. Papadopoulos, On the fuzzy difference equation x_(n+1)=A+x/x_(n-m) , Fuzzy Sets and Systems 129 (1) 73–81(2002)

[17] G. Papaschinopoulos, B.K. Papadopoulos, On the fuzzy difference equation x_(n+1)=A+B/x_n , Soft Comput. 6 (6) 456–461 (2002)

[18] G. Papaschinopoulos, G. Stefanidou, Boundedness and asymptotic behaviour of the solutions of a fuzzy difference equation, Fuzzy Sets and Systems 140 (3) 523–539 (2003)

[19] H. Sedaghat, A class of nonlinear second order difference equations from macroeconomics, Nonlinear Anal. Theory, Methods & Appl. 29 (5) 593–603(1997)

[20] S. Elaydi, I. Gyory, G. Ladas, Advances in Difference Equations, Veszprem, Hungary(1995)

[21] Konstantinos A. Chrysafis, Basil K. Papadopoulos, G. Papaschinopoulos, On the fuzzy difference equations of finance, Fuzzy Sets and Systems 159 3259–3270(2008)

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