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A Few e-labeling Graphs

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Abstract: *Let $G(V,E)$ be a graph of order n and size m. A e-labeling of G is a one-to-one function $f:V(G)\rightarrow\{0,1,2,...,m\}$ that induces a labeling $f^{+}:E(G)\rightarrow\{1,2,3,...,m^{2}\}$ of the edges of G defined by $f^{+}(uv)=|\,[f(u)]^{2}-[f(v)]^{2}\,|$ for every edge uv of G. The value of a e-labeling is denoted by e-val$(f)=\sum_{uv\in E}f^{+}(uv)$. The maximum value of a e-labeling of G is defined by e-val$_{max}$ (G) = max{ e-val (f) : f is a e-labeling of G }, while the minimum value of a e-labeling of G is defined by e-val$_{min}$$(G)$ = min{e-val(f) : f is a e-labeling of G }. In this paper, we investigate the e-val$_{min}$$(G)$ and e-val$_{max}$$(G)$ of fan f_{n-1} and $(n-1)$-star $B_{n-1,n-1}$.*

Keywords: *e-labeling , maximum value , minimum value.*

Mathematical subject classification (2010) 05C78

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected graphs. Let $G(V,E)$ be a graph with $p=|V(G)|$ vertices and $q=|E(G)|$ edges. Graph labeling, where the vertices are assigned values subject to certain conditions. By graph labeling we mean the vertices and edges are assigned real values or subsets of a set are subject to certain conditions. A detailed survey of graph labeling can be found in [3]. Terms not defined here are used in the sense of Harary in [2]. The concept of e-labeling was first introduced in [5] and some results on e-labeling of graphs are discussed in [5]. In this paper we investigate some more graphs for e-labeling. of We use the following definitions in the subsequent sections.

Definition 1.1[5]: Let $G(V,E)$ be a graph of order n and size m. A e-labeling of G is a one-to-one function $f:V(G)\rightarrow\{0,1,...,m\}$ that induces a labeling $f^{+}:E(G)\rightarrow\{1,2,3,...,m^{2}\}$ of the edges of G defined by $f^{+}(uv)=|\,[f(u)]^{2}-[f(v)]^{2}\,|$ for every edge uv of G. The value of a e-labeling is denoted by e-val$(f)=\sum_{uv\in E}f^{+}(uv)$. The maximum value of a e-labeling of G is defined by e-val$_{max}$ (G) = max{ e-val (f) : f is a e-labeling of G }, while the minimum value of a e-labeling of G is defined by e-val$_{min}$$(G)$ = min{e-val(f) : f is a e-labeling of G }.

Definition 1.2 [1]: For a graph G of order n and size m, a γ-labeling of G is a one-to-one function $f:V(G)\rightarrow\{0,1,...,m\}$ that induces a labeling $f^{'}:E(G)\rightarrow\{1,2,3,...,m\}$ of the edges of G defined by $f^{'}(uv)=|\,[f(u)]-[f(v)]|$ for each edge uv of G. Each γ-labeling f of a graph G of order n and size m is assigned a value denoted by val(f) and defined by val(f) = $\sum_{uv\in E} f^{'}(uv)$. The maximum value of a γ-labeling of graph G is defined by val$_{max}$(G) = max{ val(f) : f is a γ-labeling of G } , while the minimum value of a γ-labeling of G is defined by val$_{min}$(G)= min{ val(f) : f is a γ-labeling of G } .

Definition 1.3.[3]: The fan $f_n(n \geq 2)$ is obtained by joining all nodes of P_n to a further node called the center and contains $n+1$ nodes and $2n-1$ edges.

Definition 1.4.[3]: The n-bistar graph $B_{n,n}$ is the graph obtained from two copies of $K_{1,n}$ by joining the vertices of maximum degree by an edge.

2. Main Results

Theorem 2.1: Let f_{n-1} be a fan graph. For every even integer $n \geq 4$,

$$\text{e-val}_{\min}(f_{n-1}) = \frac{n^3 + 3n^2 - 8n + 4}{4}.$$

Proof: Let f_{n-1} be a fan graph. Let $V(f_{n-1}) = \{v_i : 1 \leq i \leq n\}$.

Let $E(f_{n-1}) = \{v_i v_{i+1} : 1 \leq i \leq n-2 \ ; \ v_i v_n : 1 \leq i \leq n-1\}$.Then size $m = 2n-3$.

Define a e-labeling f from $V(f_{n-1})$ to $\{0,1,2,...,2n-3\}$ by $f(v_i) = i-1 \ \ if \ 1 \leq i \leq \frac{n}{2}$;

$f(v_{\frac{(n-1+2i)}{2}}) = \frac{(n+2i)}{2} \ \ if \ 1 \leq i \leq \frac{(n-2)}{2}$ and $f(v_n) = \frac{n}{2}$. Let f^+ be the induced edge labeling of f. The induced edge labels of f_{n-1} by f^+ are as follows: $f^+(v_{\frac{n}{2}} v_{\frac{(n+2)}{2}}) = 2n$;

$f^+(v_i v_{i+1}) = 2i-1 \ \ if \ 1 \leq i \leq \frac{(n-2)}{2}$; $f^+(v_{\frac{(n+2i)}{2}} v_{\frac{(n+2+2i)}{2}}) = n+1+2i \ \ if \ 1 \leq i \leq \frac{(n-4)}{2}$;

$f^+(v_i v_n) = \frac{(n-2+2i)(n+2-2i)}{4} \ \ if \ 1 \leq i \leq \frac{n}{2}$;

$f^+(v_{\frac{(n+2i)}{2}} v_n) = (n+i)i \ \ if \ 1 \leq i \leq \frac{(n-2)}{2}$.

$$\text{Then e-val}_{\min}(f_{n-1}) = \sum_{i=1}^{\left(\frac{n-2}{2}\right)} (2i-1) + \sum_{i=1}^{\left(\frac{n-4}{2}\right)} (n+1+i) + \sum_{i=1}^{\left(\frac{n}{2}\right)} \frac{(n-2+2i)(n+2-2i)}{4}$$

$$+ \sum_{i=1}^{\left(\frac{n-2}{2}\right)} (n+i)\, i + 2n = \frac{n^3 + 3n^2 - 8n + 4}{4}.$$

Theorem 2.2: Let f_{n-1} be a fan graph. For every odd integer $n \geq 5$,

$$\text{e-val}_{\min}(f_{n-1}) = \frac{n^3 + 3n^2 - 9n + 5}{4}.$$

Proof: Let f_{n-1} be a fan graph. Let $V(f_{n-1}) = \{v_i : 1 \leq i \leq n\}$.

Let $E(f_{n-1}) = \{v_i v_{i+1} : 1 \leq i \leq n-2 \ ; \ v_i v_n : 1 \leq i \leq n-1\}$.Then size $m = 2n-3$.

Define a e-labeling f from $V(f_{n-1})$ to $\{0,1,2,...,2n-3\}$ by $f(v_i) = i-1 \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$;

$f(v_{\frac{(n-1+2i)}{2}}) = \frac{(n-1+2i)}{2} \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$ and $f(v_n) = \frac{(n-1)}{2}$.

Let f^+ be the induced edge labeling of f. The induced edge labels of f_{n-1} by f^+ are as follows: $f^+(v_{\frac{(n-1)}{2}} v_{\frac{(n+1)}{2}}) = 2(n-1)$; $f^+(v_i v_{i+1}) = 2i-1 \;\; if \;\; 1 \le i \le \frac{(n-3)}{2}$;

$f^+(v_{\frac{(n-1+2i)}{2}} v_{\frac{(n+1+2i)}{2}}) = n+2i \;\; if \; 1 \le i \le \frac{(n-3)}{2}$;

$f^+(v_i v_n) = \frac{(n-3+2i)(n+1-2i)}{4} \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$;

$f^+(v_{\frac{(n-1+2i)}{2}} v_n) = (n-1+i)i \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$.

$$\text{Then e-val}_{\min}(f_{n-1}) = \sum_{i=1}^{\left(\frac{n-3}{2}\right)} (2i-1) + \sum_{i=1}^{\left(\frac{n-3}{2}\right)} (n+i) + \sum_{i=1}^{\left(\frac{n-1}{2}\right)} \frac{(n-3+2i)(n+1-2i)}{4}$$
$$+ \sum_{i=1}^{\left(\frac{n-1}{2}\right)} (n-1+i)\,i + 2(n-1) = \frac{n^3+3n^2-9n+5}{4}.$$

Example 2.3: The minimum e-labeling of fan f_7 is shown in the Figure-1.

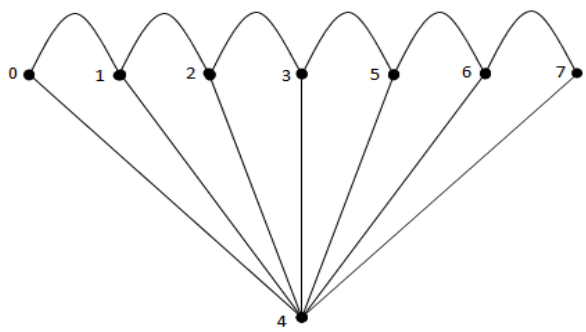

Figure-1

Remark:2.4: From the above example 2.3, observed that e-val$_{\min}$(f_7) =161.

Theorem 2.5: Let f_{n-1} be a fan graph. For every even integer $n \ge 4$,

$$\text{e-val}_{\max}(f_{n-1}) = \frac{65n^3 - 285n^2 - 418n - 204}{12}.$$

Proof: Let f_{n-1} be a fan graph. Let $V(f_{n-1}) = \{v_i : 1 \le i \le n\}$.

Let $E(f_{n-1}) = \{v_i v_{i+1} : 1 \le i \le n-2 \; ; \; v_i v_n : 1 \le i \le n-1\}$. Then size $m = 2n-3$.

Define a e-labeling f from $V(f_{n-1})$ to $\{0,1,2,...,2n-3\}$ by $f(v_{2i-1}) = i-1 \;\; if \;\; 1 \le i \le \frac{n}{2}$;

$f(v_{2i}) = 2n-3-i \;\; if \;\; 1 \le i \le \frac{(n-2)}{2}$ and $f(v_n) = 2n-3$. Let f^+ be the induced edge labeling of f. The induced edge labels of f_{n-1} by f^+ are as follows:

$f^+(v_{2i-1}v_{2i}) = (2n-4)(2n-2-2i) \;\; if \;\; 1 \le i \le \frac{(n-2)}{2}$;

$f^+(v_{2i}v_{2i+1}) = (2n-3)(2n-3-2i) \;\; if \;\; 1 \le i \le \frac{(n-2)}{2}$;

$f^+(v_{2i-1}v_n) = (2n-4+2i)(2n-2-i) \;\; if \;\; 1 \le i \le \frac{n}{2}$;

$f^+(v_{2i}v_n) = (4n-6-i)i \;\; if \;\; 1 \le i \le \frac{(n-2)}{2}$.

Then e-val$_{max}$ $(f_{n-1}) = \sum_{i=1}^{\left(\frac{n-2}{2}\right)} (2n-4)(2n-2-2i) + \sum_{i=1}^{\left(\frac{n-2}{2}\right)} (2n-3)(2n-3-2i)$

$$+ \sum_{i=1}^{\left(\frac{n}{2}\right)} (2n-4+2i)\,(2n-2-i) + \sum_{i=1}^{\left(\frac{n-2}{2}\right)} (4n-6-i)\,i$$

$$= \frac{65n^3 - 285n^2 - 418n - 204}{12}.$$

Theorem 2.6: Let f_{n-1} be a fan graph. For every odd integer $n \ge 5$,

e-val$_{max}$ $(f_{n-1}) = \frac{65n^3 - 297n^2 - 457n - 237}{12}$.

Proof: Let f_{n-1} be a fan graph. Let $V(f_{n-1}) = \{v_i : 1 \le i \le n\}$.

Let $E(f_{n-1}) = \{v_i v_{i+1} : 1 \le i \le n-2 \; ; \; v_i v_n : 1 \le i \le n-1\}$. Then size $m = 2n-3$.

Define a e-labeling f from $V(f_{n-1})$ to $\{0,1,2,...,2n-3\}$ by $f(v_n) = 2n-3$;

$f(v_{2i-1}) = i-1 \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$ and $f(v_{2i}) = 2n-3-i \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$.

Let f^+ be the induced edge labeling of f.

The induced edge labels of f_{n-1} by f^+ are as follows:

$f^+(v_{2i-1}v_{2i}) = (2n-4)(2n-2-2i) \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$;

$f^+(v_{2i}v_{2i+1}) = (2n-3)(2n-3-2i) \;\; if \;\; 1 \le i \le \frac{(n-3)}{2}$;

$f^+(v_{2i-1}v_n) = (2n-4+2i)(2n-2-i) \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$;

$f^+(v_{2i}v_n) = (4n-6-i)i \;\; if \;\; 1 \le i \le \frac{(n-1)}{2}$.

Then e-val$_{\max}(f_{n-1}) = \sum_{i=1}^{\left(\frac{n-1}{2}\right)} (2n-4)(2n-2-2i) + \sum_{i=1}^{\left(\frac{n-3}{2}\right)} (2n-3)(2n-3-2i)$

$$+ \sum_{i=1}^{\left(\frac{n-1}{2}\right)} (2n-4+2i)(2n-2-i) + \sum_{i=1}^{\left(\frac{n-1}{2}\right)} (4n-6-i)\, i$$

$$= \frac{65n^3 - 297n^2 - 457n - 237}{12}.$$

Example 2.7: The maximum e-labeling of fan f_6 is shown in the Figure-2.

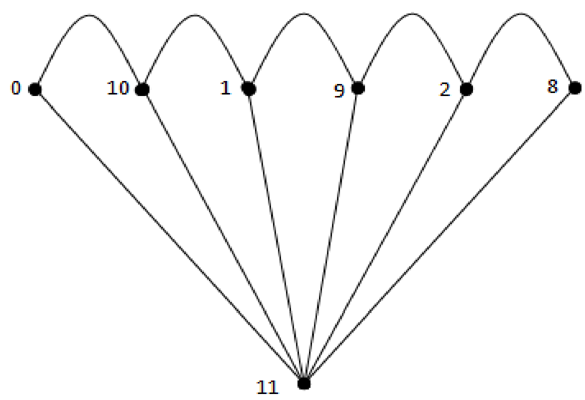

Figure-2

Remark:2.8: From the above example 2.7, observed that e-val$_{\max}$(f_6) =892.

Theorem 2.9: Let $B_{n-1,n-1}$ be a $(n-1)$-bistar. Then for every integer $n \geq 3$,

e-val$_{\min}(B_{n-1,n-1}) = 2n^3 - 2n^2 + 3n - 1$.

Proof: Let $B_{n-1,n-1}$ be a $(n-1)$-bistar. Let $V(B_{n-1,n-1}) = \{u_i, v_i : 1 \leq i \leq n\}$.

Let $E(B_{n-1,n-1}) = \{u_i u_n : 1 \leq i \leq n-1 ;\ v_i v_n : 1 \leq i \leq n-1 ;\ u_n v_n\}$. Then size $m = 2n-1$.

Define a e-labeling f from $V(B_{n-1,n-1})$ to $\{0,1,2,...,2n-1\}$ by $f(u_i) = i-1$ *if* $1 \leq i \leq n$;

$f(v_i) = n+i$ *if* $1 \leq i \leq (n-1)$; $f(v_n) = n$. Let f^+ be the induced edge labeling of f.

The induced edge labels of f_{n-1} by f^+ are as follows: $f^+(u_n v_n) = 2n-1$;

$f^+(u_i u_n) = (n-2+i)(n-i)$ *if* $1 \leq i \leq (n-1)$; $f^+(v_i v_n) = (2n+i)i$ *if* $1 \leq i \leq (n-1)$.

Then e-val$_{\min}(B_{n-1,n-1}) = \sum_{i=1}^{(n-1)} (n-2+i)(n-i) + \sum_{i=1}^{(n-1)} (2n+i)i + (2n-1)$

$$= 2n^3 - 2n^2 + 3n - 1.$$

Example 2.10: The minimum e-labeling of 4-bistar $B_{4,4}$ is shown in the Figure-3.

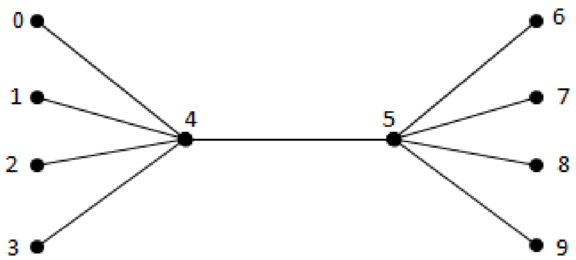

Figure-3

Remark:2.11: From the above example 2.10, observed that e-val$_{\min}$($B_{4,4}$) =189.

Theorem 2.12: Let $B_{n-1,n-1}$ be a $(n-1)$-bistar. Then for every integer $n \geq 3$,

$$\text{e-val}_{\max}(B_{n-1,n-1}) = \frac{16n^3 - 30n^2 + 26n - 9}{3}.$$

Proof: Let $B_{n-1,n-1}$ be a $(n-1)$-bistar. Let $V(B_{n-1,n-1}) = \{u_i, v_i : 1 \leq i \leq n\}$.

Let $E(B_{n-1,n-1}) = \{u_i u_n : 1 \leq i \leq n-1 \text{ ; } v_i v_n : 1 \leq i \leq n-1 \text{ ; } u_n v_n\}$.

Then size $m = 2n-1$. Define a e-labeling f from $V(B_{n-1,n-1})$ to $\{0,1,2,...,2n-1\}$ by $f(u_i) = 2i-2$ *if* $1 \leq i \leq n$; $f(v_i) = 2i-1$ *if* $1 \leq i \leq n$.

Let f^+ be the induced edge labeling of f.

The induced edge labels of f_{n-1} by f^+ are as follows:

$f^+(u_n v_n) = 4n-3$;

$f^+(u_i u_n) = 4(n-2+i)(n-i)$ *if* $1 \leq i \leq (n-1)$;

$f^+(v_i v_n) = 4(n-1+i)(n-i)$ *if* $1 \leq i \leq (n-1)$.

Then $\text{e-val}_{\max}(B_{n-1,n-1}) = \sum_{i=1}^{(n-1)} 4(n-2+i)(n-i) + \sum_{i=1}^{(n-1)} 4(n-1+i)(n-i) + (4n-3)$

$$= \frac{16n^3 - 30n^2 + 26n - 9}{3}.$$

Example 2.13: The maximum e-labeling of 5-bistar $B_{5,5}$ is shown in the Figure-4.

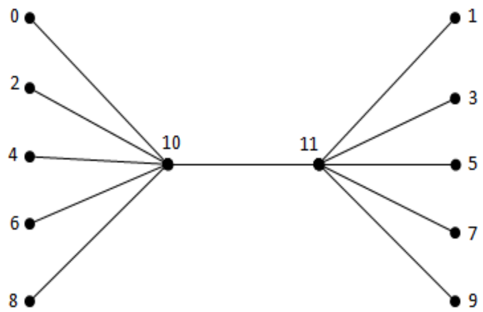

Figure-4

Remark: 2.14: From the above example 2.13, observed that e-val$_{max}$($B_{5,5}$) =841.

3. Conclusion

In this paper, we have investigated the maximum and the minimum values of e-labeling of fan and n-bistar graphs. We have planned to investigate the maximum and the minimum values of e-labeling of cycle related graphs in the next paper.

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