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The Effect of the Thickness of the Porous Material on the Parallel Plate Channel Flow of Jeffrey Fluid when the Walls are Provided with Non-Erodible Porous Lining

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Abstract: *The flow of a Jeffrey fluid in a parallel plate channel is investigated when the walls are provided with non-erodible porous lining. The flow in the porous and free flow regions are described using Darcy law and Jeffrey model respectively. The velocity in the free flow region is obtained and the effect of thickness of porous lining on the flow parameters is discussed in detail with graphs.*

Keywords: *Parallel Plate channel, Jeffrey fluid, porous lining*

1. INTRODUCTION

The study of flow through and past porous media has attracted the attention of many research workers because of its potential application in industrial, physical bio and hydrological problems. Most of the available studies on the flow past porous media concern themselves with natural permeable beds and hence do not take into account the thickness of the porous medium involved. Flows past porous media with finite thickness are of relevance in many industrial applications like lubrication and solar energy equipments.

Beavers and Joseph (1967) postulated a slip condition at the permeable boundary and verified experimentally. A rigorous theoretical justification for the existence of slip velocity at the permeable surface was given by Saffman (1971). Rajasekhara (1974) has investigated plane Couette flow in the presence of a pressure gradient and found slight deviation between his theoretical and experimental results. Chanabbasappa et al. (1976) investigated analytically The Effect of the Thickness of the porous material on the parallel plate channel flow when the walls are provided with Non- Erodible porous lining. Chikh et al. (1995) performed an analytical study on fully developed forced convection in an annular duct partially filled with a porous medium. Some valuable studies on the forced convection heat transfer in a parallel plate channel partially filled with porous media (with different positions of porous layers such as porous layers attached to walls and located in the center of channel) are made by Kuznetsov (1996, 1997, 1999, 2000). Brinkman–Forcheimer extended Darcy equation was analytically solved to determine velocity field, and the temperature field. Morosuk (2005) investigated entropy generation due to flow in a pipe and parallel plate channel partially filled with porous medium. The outer surface of the pipe was assumed to be in isothermal condition. The porous layer was inserted at the core of the pipe or attached on the inner surface of the pipe. The effects of the porous layer and the flow are discussed.

Aydın and Avci (2006) investigated analytically to predict laminar heat convection in a Couette–Poiseuille flow between two parallel plates with a simultaneous pressure gradient and an axial movement of the upper plate. The problem of natural convection from a vertical wavy plate embedded in porous media for power law fluids in presence of magnetic field is studied by Mahdy et al. (2009) and mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media is also considered by Mahdy et al. (2009). Krishna Gopal Singha (2009) investigated analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Yang et al. (2009) performed a theoretical study to investigate fully developed forced convection in a tube whose core was partially filled with a porous medium. In their study, Darcy law was used to simulate fluid motion in the porous layer.

Kuznetsov and Nield (2010) investigated a fully developed forced convection in a parallel plate channel partially occupied by a bidisperse porous medium. Satyamurty and Bhargavi (2010) performed a study on heat and fluid flow in a partially porous channel in which the inserted porous layer was attached to one wall while the other wall was in contact with clear fluid. Both walls were hold symmetrically at constant temperatures. Although an analytical solution was found for motion of fluid in the channel, the heat transfer equation was solved numerically. Vajravelu et al. (2011) studied the influence of the heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum Sreenadh et al. (2012) investigated the MHD free convective flow of a Jeffrey fluid between coaxial cylinders.

The object of this paper is to develop a theoretical model for analyzing the non-Newtonian fluid flow and heat transfer in a parallel plate channel, when one or two parallel walls are lined with a non-erodible porous material. The analysis makes use of the velocity slip boundary condition of Beavers and Joseph (1967) with suitable modifications to take into account the finite thickness of the porous layer involved. The influence of the non dimensional parameters representing the thickness and the permeability of the porous medium on the velocity field in the channel has been studied.

2. Flow in a Parallel Plate Channel with Porous Lining on One Side

2.1 Mathematical Formulation

We consider the rectilinear flow of a Jeffrey fluid through a channel formed by two rigid impermeable parallel walls at $y = 0$ and $y = h$ as shown in Fig.1. The lower wall is covered with a homogenous and isotropic porous material of thickness $h'(\neq 0)$ thus dividing the flow region into two zones, Zone1 denoting the region of the free flow between the upper impermeable wall and the nominal surface $y = h'$ and Zone 2 denoting the region of flow through the porous material.

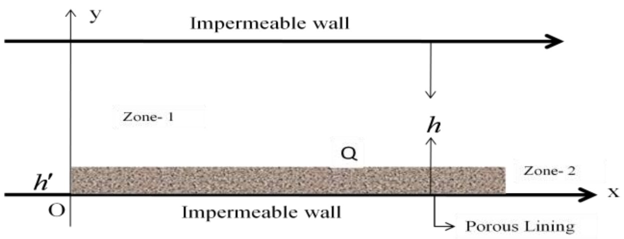


Fig.1. *Physical model (with Porous lining on one wall)*

The flow which is caused by a uniform pressure gradient in the longitudinal direction in both the zones is assumed to be fully developed and the fluid properties are all assumed to be constant. Then the flow in Zone 1 is governed by the equation

$$\frac{d^2u}{dy^2} = \frac{1+\lambda_1}{\mu}\frac{dp}{dx} \tag{1}$$

and that in Zone 2 by the Darcy law

$$Q = -\frac{k(1+\lambda_1)}{\mu}\frac{dp}{dx} \tag{2}$$

where u is the velocity, λ_1 is the Jeffrey parameter, P is the Pressure, μ is the viscosity, Q is the Darcy velocity, k is absolute permeability of the material

The boundary conditions are

$$u = 0 \text{ at } y = h \tag{3}$$

and the BJ boundary condition(Beavers and Joseph, 1967) is

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k}}(u_B - Q) \text{ at } y = h' \tag{4}$$

where α is the slip parameter We introduce the following non-dimensional quantities:

$$v = \frac{u}{\bar{u}},\quad \eta = \frac{y}{h},\ \xi = \frac{x}{h},\ \pi = \frac{p}{\frac{1}{2}\rho \bar{u}^2},\ R = \frac{\rho h \bar{u}}{\mu},\ P = \frac{-R}{2}\frac{\partial \pi}{\partial \xi},\ \sigma = \frac{h}{\sqrt{k}},\ Q' = \frac{Q}{\bar{u}},\ \varepsilon = \frac{h'}{h} \tag{5}$$

where ρ is the density, R is the Reynolds number, $\bar{u}$ is the average velocity in the channel, ε is the thickness of the porous material.

In view of (5), equations (1)-(4) reduce to

$$\frac{d^2 v}{d\eta^2} = -(1+\lambda_1)P \tag{6}$$

$$Q' = \frac{P}{\sigma^2}(1+\lambda_1) \tag{7}$$

$$v = 0 \text{ at } \eta = 1 \tag{8}$$

$$\frac{dv}{d\eta} = \alpha\sigma(v_B - Q') \text{ at } \eta = \varepsilon \tag{9}$$

where v_B is the slip velocity

2.2 Solution

Solution of (6) satistfying (8) and (9) is

$$v = (1-\eta)\left[\frac{P}{2}(1+\lambda_1)(1+\eta) - P\varepsilon(1+\lambda_1) + \frac{\alpha P}{\sigma}(1+\lambda_1) - \alpha\sigma v_B\right] \tag{10}$$

where

$$v_B = \frac{P(1+\lambda_1)(1-\varepsilon)\ \ \sigma(1-\varepsilon)+2\alpha}{2\sigma\ \ 1+\alpha\sigma(1-\varepsilon)} \tag{11}$$

To find the quantitative effect of slip on the flow, we calculate the non-dimensional flow rate

$$M = M_1 + M_2 \tag{12}$$

where

$$\mathrm{M}_1 = \int_{\varepsilon}^{1} v\,d\eta = \frac{P}{12}(1+\lambda_1)(1-\varepsilon)^3\left[\frac{4+\alpha\sigma(1-\varepsilon)-6\alpha^2}{1+\alpha\sigma(1-\varepsilon)}\right] + \frac{\alpha P}{2\sigma}(1+\lambda_1)(1-\varepsilon)^2 \tag{13}$$

$$\equiv \frac{P}{12}A + \frac{P}{2}B$$

where $\mathrm{A} = (1+\lambda_1)(1-\varepsilon)^3\left[\frac{4+\alpha\sigma(1-\varepsilon)-6\alpha^2}{1+\alpha\sigma(1-\varepsilon)}\right]$,

$$B = \frac{\alpha}{\sigma}(1+\lambda_1)(1-\varepsilon)^2$$

and

$$M_2 = Q^{'}\varepsilon = \frac{P(1+\lambda_1)\varepsilon}{\sigma^2} \tag{14}$$

In order to bring out the effect of porous lining in the channel, we compare M with the mass flow rate M^* in the channel in the absence of lining where

$$M^* = \int_0^1 v d\eta = \frac{P(1+\lambda_1)}{3} \tag{15}$$

Then the ratio of the mass flow rate with and without porous lining is given by

$$\frac{M}{M^*} = \frac{A}{4} + \frac{3B}{2} + \frac{3\varepsilon}{\sigma^2} \tag{16}$$

3. Flow in a Parallel Plate Channel with Porous Lining on Both the Sides

3.1 Mathematical Formulation

The flow situation considered here is the same as that discussed in Section 2 except that both the bounding plates of the channel are lined by permeable material of thickness $\frac{h^{'}}{2}$ as shown in the Fig.2.

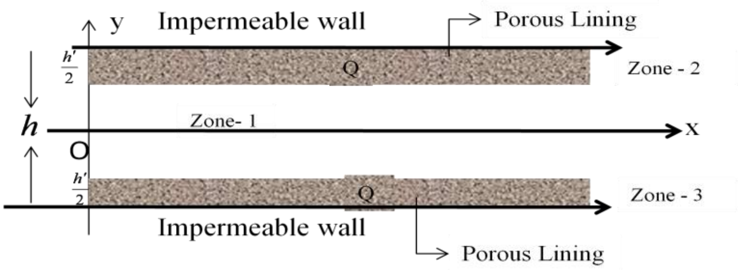


Fig. 2. *Physical model (with Porous lining on both the walls)*

In this case the boundary conditions are

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{k}}(u_B - Q) \text{ at } y = -\frac{h}{2} + \frac{h^{'}}{2} \tag{17}$$

$$u = u_B \text{ at } y = \frac{h}{2} - \frac{h^{'}}{2} \tag{18}$$

In view of (5) equations (16) and (17) reduce to

$$\frac{dv}{d\eta} = \alpha\sigma(v_B - Q^{'}) \text{ at } \eta = \frac{-1}{2} + \frac{\varepsilon}{2} \tag{19}$$

$$v = v_B \text{ at } \eta = \frac{1}{2} - \frac{\varepsilon}{2} \tag{20}$$

3.2 Solution

Solution of (6) using equations (18) and (19) is

$$v = -(1+\lambda_1)\eta^2 \frac{p}{2} + \frac{p}{2\sigma}(1+\lambda_1)\ \sigma(\varepsilon - 1) - 2\alpha\ \ \eta + \alpha\sigma v_B\left[\eta + \frac{1}{\alpha\sigma} - \frac{1}{2} + \frac{\varepsilon}{2}\right]$$

$$+(1+\lambda_1)\frac{p}{8\sigma}\left[3\sigma + 3\sigma\varepsilon^2 - (4\alpha + 6\sigma)\varepsilon + 4\alpha\right] \tag{21}$$

where

$$v_B = (1+\lambda_1)\frac{p}{2\alpha\sigma_2}\ \sigma(1-\varepsilon) + 2\alpha\ , \qquad 0 < \varepsilon < 1 \tag{22}$$

To know the effect of porous lining on both sides of the channel we can compare, as before , the total non-dimensional mass flow rate m in the channel, with M^* where

$$m = m_1 + m_2 + m_3 \tag{23}$$

$$m_1 = \int_{\frac{-1}{2}+\frac{\varepsilon}{2}}^{\frac{1}{2}-\frac{\varepsilon}{2}} v dn = (1+\lambda_1)\frac{P}{12}\left[(1-\varepsilon)^3 + (1-\varepsilon)^2\frac{6}{\alpha\sigma} + (1-\varepsilon)\frac{12}{\sigma^2}\right]$$

$$= \frac{P(1+\lambda_1)C}{12} \tag{24}$$

where $\mathrm{C} = \left[(1-\varepsilon)^3 + (1-\varepsilon)^2\frac{6}{\alpha\sigma} + (1-\varepsilon)\frac{12}{\sigma^2}\right]$

$$m_2 = m_3 = \frac{P(1+\lambda_1)}{2\sigma^2} \tag{25}$$

Then the ratio of mass flow rate with and without porous lining is given by

$$\frac{m}{M^*} = \frac{C}{4} + \frac{3}{\sigma^2} \tag{26}$$

4. Results and Discussions

The numerical values of velocity are computed from equation (10) and are depicted in Figures 3 to 7 for flow in a channel with one side porous lining. We observe from Fig. 3 that the velocity decreases with the increase in the thickness of porous layer ε We observe the same phenomenon for velocity from Fig. 4 and 5 with an increase in slip parameter α and permeability parameter σ . From Fig. 6 we observe that the velocity increases with the increase in the pressure gradient P . From Fig.7 we observe that the velocity increases with the increase in Jeffrey parameter λ_1.

The velocity for the Jeffrey fluid flow in a channel with two sides porous lining is computed numerically from equations(21) for different values of the physical parameters λ_1 , σ ,and α . From figure 8, we notice that the velocity decreases with an increase in the permeable parameter σ . This may be due to an increase in the permeability of the bounded porous layer. From figure 9 it is clear that the velocity increases with increasing in Jeffrey parameter λ_1. From Figures 10 and 11 it is found that the velocity decreases with an increase in slip parameter α and thickness of the porous layer ε. From Figure 12 it is noticed that the velocity increases with the increase in the pressure gradient P.

The expression for mass flow rate is presented and is numerically evaluated for different values of permeability parameter σ . We note that the ratio of the mass flow rates corresponding to with or without porous lining is independent of the Jeffrey parameter λ_1. From Figures 13 and14 we observed that the ratio of mass flow rate (M / M^*) covering one side porous lining decreases with the increase in the permeability parameter σ . Further the increase in the slip parameter reduces the ratio of mass flow rates. The same behavior in the ratio of (m / M^*) is observed for two sided porous lining of channel and this is clear from the figure 15 and 16.

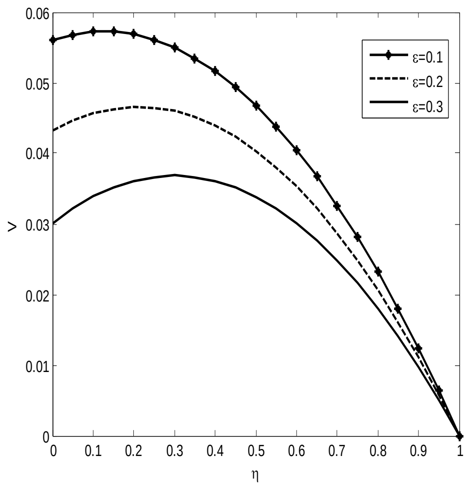


Fig. 3 Velocity distribution for various values of ε for fixed P=0.1, λ_1=0.5, α =0.1, σ =2.

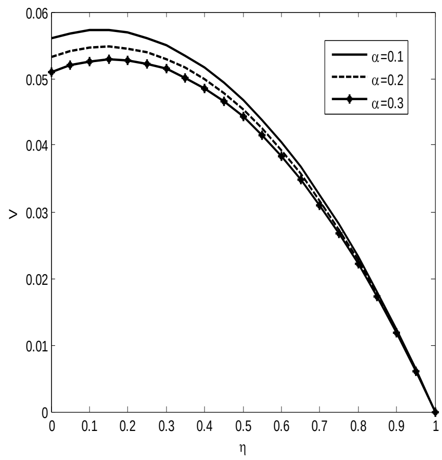


Fig. 4 *Velocity distribution for various values of* α *for fixed P=0.1,* λ_1 *=0.5,* ε *=0.1,* σ *=2.*

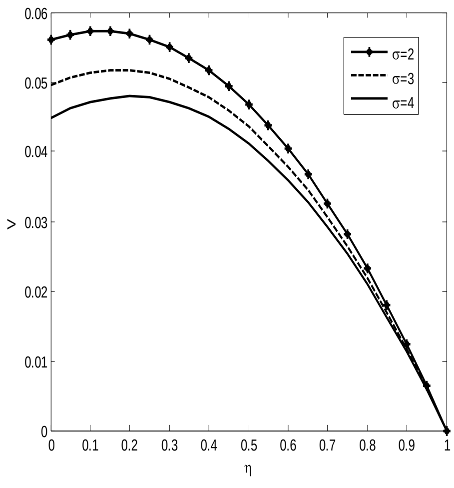
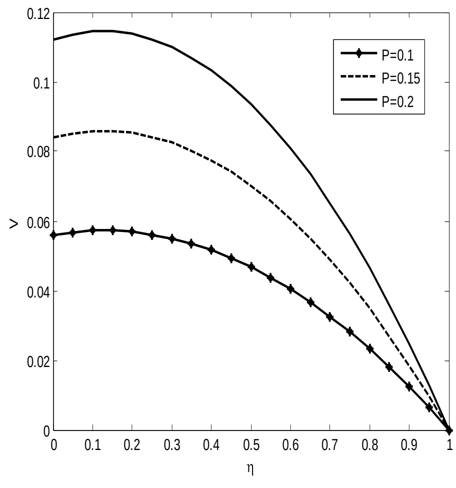


Fig.5. *Velocity distribution for various Values* σ *for fixed P=0.1,* λ_1 *=0.5,* ε *=0.1,* α *=0.1.*



F ig. 6. *Velocity distribution for various values of* P *for fixed* σ *=2,* λ_1 *=0.5,* ε *=0.1,* α *=0.1.*

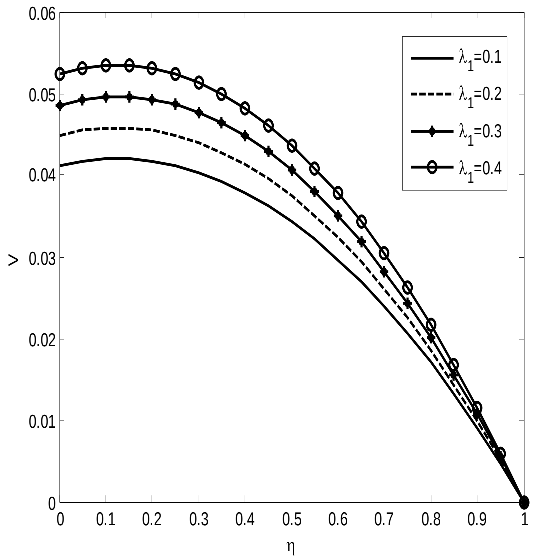


Fig. 7. *Velocity distribution for various values of* λ_1 *for fixed* σ *=2,* $p = 0.1$, ε *=0.1,* α *=0.1.*

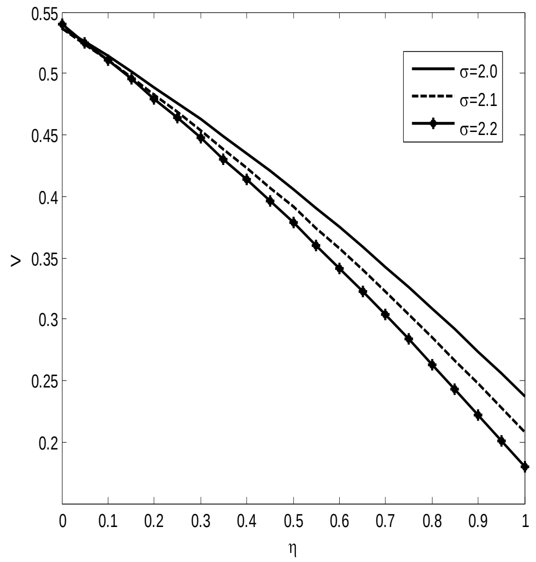


Fig.8. *Velocity distribution for various values of* σ *for fixed P=0.1,* λ_1 *=0.5,* ε *=0.1,* α *=0.1.*

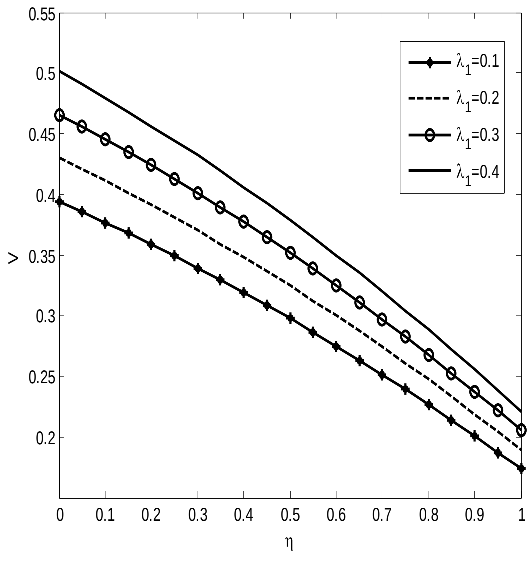


Fig.9. *Velocity distribution for various values of* λ_1 *for fixed* σ *=2,* $p = 0.1$, ε *=0.1,* α *=0.*

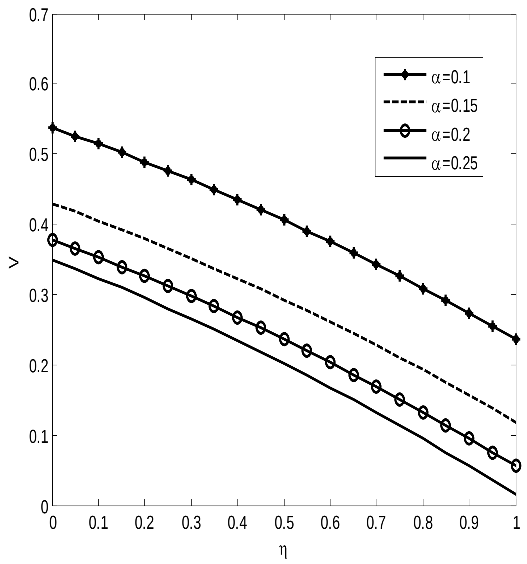


Fig. 10. *Velocity distribution for various values of* α *for fixed P=0.1,* $\lambda_1 = 0.5$, ε *=0.1,* σ *=2.*

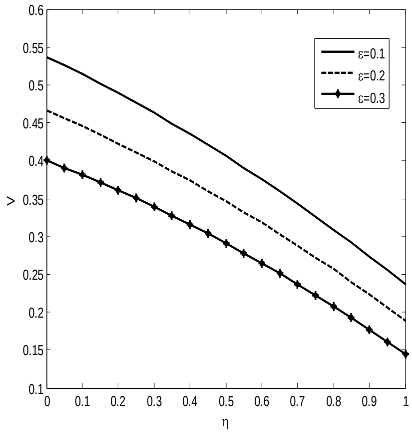


Fig. 11. *Velocity distribution for various values of* ε *for fixed P=0.1,* $\lambda_1 = 0.5$, α *=0.1,* σ *=2.*

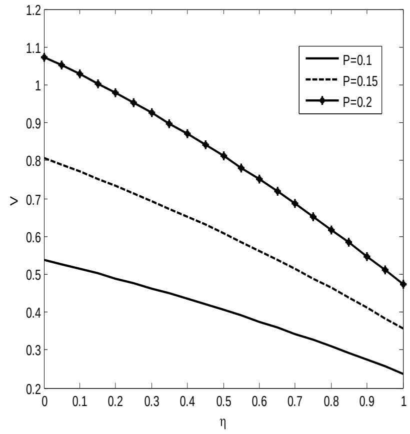


Fig. 12. *Velocity distribution for various values of P for fixed* σ *=2,* $\lambda_1 = 0.5$, ε *=0.1,* α *=0.1.*

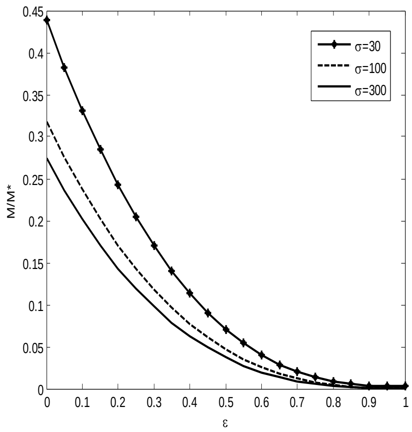


Fig. 13. *Variation of* M/M^* *with* ε *for fixed* $\alpha = 0.1$.

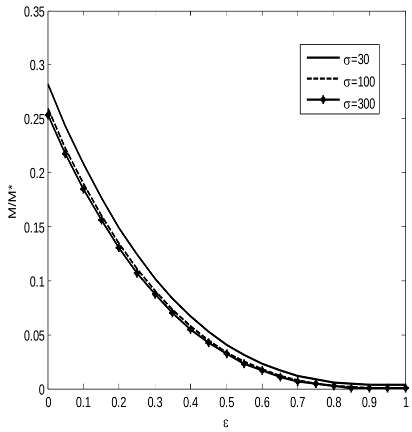


Fig. 14. *Variou of* M/M^* *with* ε *for fixed* $\alpha = 0.8$.

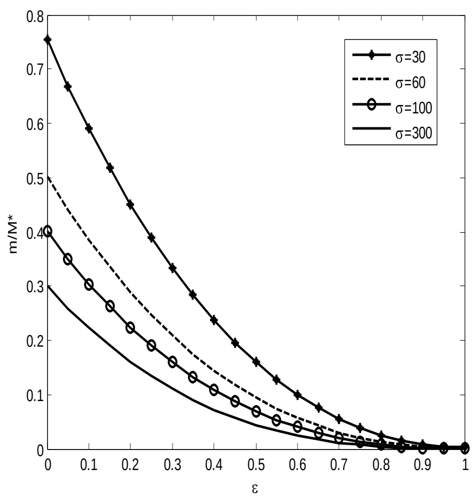


Fig. 15. *Variation of* m/M^* *with* ε *for fixed* $\alpha = 0.1$

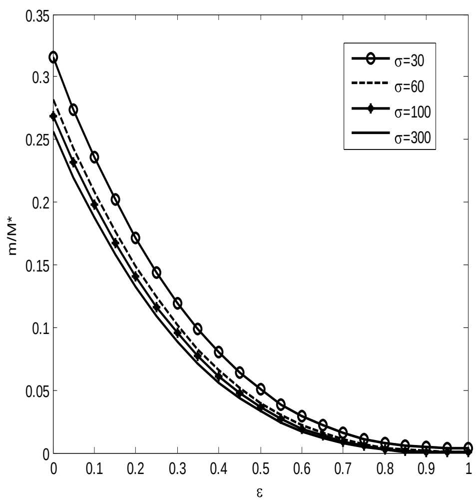


Fig.16. *Variation of* m/M^* *with* ε *for fixed* $\alpha = 0.1$

5. Conclusions

The velocity decreases with the increase in the thickness of porous layer or slip parameter or permeability parameter (with porous lining on one wall and both the walls). The velocity increases with the increase in the pressure gradient or Jeffrey parameter (with porous lining on one wall and both the walls). The ratio of mass flow rate decreases with the increase in the permeability parameter (with porous lining on one wall and both the walls).

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