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Effect of Applied Magnetic Field on Pulsatile Flow of Blood in a Porous Channel

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Abstract: *An approximate solution is presented to the problem of pulsatile flow of blood in a porous channel in presence of applied magnetic field. The blood is assumed to be an incompressible, laminar, fully developed, Newtonian fluid. To reduce the equation of motion to an ordinary differential equation, a dimensionless variable is used and the numerical results are obtained for different values of the magnetic parameter (the Hartmann number), frequency parameter, Reynolds number and Magnetic Reynolds number using Shooting method. It is observed that when the magnetic parameter increases, the fluid velocity decreases. While the fluid velocity increases with the increase of Reynolds number as well as Magnetic Reynolds number. Magnitude of mass flux decreases with the decrease of frequency parameter.*

Keywords: *Pulsatile, Magnetic field, Blood flow, Injection, Unsteady.*

1. Introduction

Application of magnetic field has been realised as the elegant device for the control in physiological fluid flows. The flow of blood in human circulatory system can be controlled by applying appropriate magnetic field. Many researchers have shown that blood is an electrically conducting fluid [1-4].

By the Lenze's law, the Lorentz's force will act on the constituent particles of blood and this force will oppose the motion of blood and thus reduces its velocity [5]. This deaccelerated blood flow may help in the treatment of certain cardiovascular diseases and in the diseases with accelerated blood circulations such as hypertension, hemorrhages etc. So it is very essential to study the blood flow in presence of magnetic field.

In the arteries, blood flow and blood pressure are pulsatile in nature [6].The pulsatile flow of blood with micro-organisms represented by two fluid model through vessels of small exponential divergence under the effect of magnetic field has been studied by Rathod and Gayatri [7]. A similar problem on blood flow through closed rectangular channel with micro-organisms has also been studied by Rathod and Mohesh [8]. Rathod and Parveen have studied pulsatile flow of blood with micro-organisms through a uniform pipe with sector of a circle as cross-section in the presence of transverse magnetic field [9].

Bhuyan and Hazarika [10] also discussed about the blood flow with effects of slip in arterial stenosis due to presence of transverse magnetic field. Flow in a porous channel has been investigated by Wang [11] without magnetic effect and the same in presence of transverse magnetic field has been studied by Bhuyan [12]. An attempt has been made in this analysis to study the pulsatile flow of blood in a porous channel in presence of applied magnetic field. Here blood is assumed to be an incompressible Newtonian fluid.

2. Formulation of the Problem

Here we consider a fluid driven by an unsteady pressure gradient

$$\frac{\partial p}{\partial x} = A + Be^{i\omega t} \qquad (1)$$

Between two porous plates at y = 0 and y = h. Here A and B are known constants and ω is the frequency.

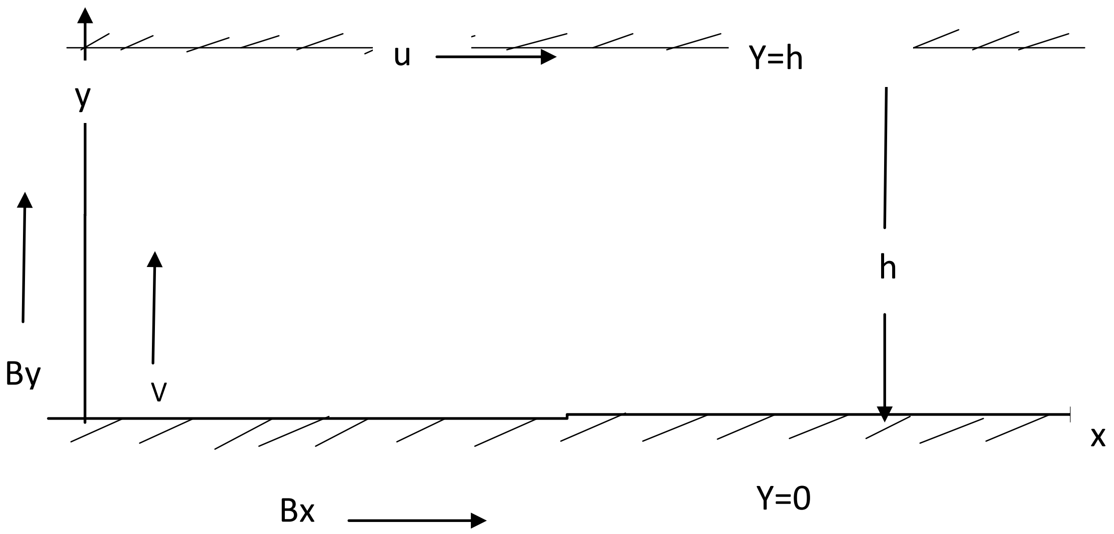

Fig1. *Geometry of the flow*

On one plate some fluid is injected with velocity V and it is sucked off at the opposite plate with the same velocity. Due to continuity, the velocity component in the y – direction will be identically equal to V everywhere. B_x and B_y are the components of magnetic field in the x and y directions respectively. So, the Navier Stokes equations under the applied magnetic field becomes –

$$\frac{\partial u}{\partial t}+V\frac{\partial u}{\partial y}= -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}{B_y}^2u+\frac{\sigma}{\rho}VB_xB_y \tag{2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y}-\frac{\sigma}{\rho}B_x\left(uB_y-VB_x\right)=0 \tag{3}$$

The magnetic induction equation reduces to

$$B_y\frac{\partial u}{\partial y}-V\frac{\partial B_x}{\partial y}+\eta_m\nabla^2B_x=0 \tag{4}$$

Where u is the velocity in the x- direction and ρ, σ, ν , η_m are the density, electrical conductivity ,kinematic viscosity and co-efficient of magnetic diffusivity respectively.

We separate (2) and (4) into a steady part denoted by a tilde and unsteady part denoted by a bar.

$$V\frac{\partial\tilde{u}}{\partial y}=-A+\nu\frac{\partial^2\tilde{u}}{\partial y^2}-\frac{\sigma}{\rho}{B_y}^2\tilde{u}+\frac{\sigma}{\rho}VB_xB_y \tag{5}$$

$$B_y\frac{\partial\tilde{u}}{\partial y}-V\frac{\partial B_x}{\partial y}+\eta_m\nabla^2B_x=0 \tag{6}$$

$$\frac{\partial\bar{u}}{\partial t}+V\frac{\partial\bar{u}}{\partial y}=-Be^{i\omega t}+\nu\frac{\partial^2\bar{u}}{\partial y^2}-\frac{\rho}{\sigma}{B_y}^2\bar{u}+\frac{\rho}{\sigma}VB_xB_y \tag{7}$$

$$B_y\frac{\partial\bar{u}}{\partial y}-V\frac{\partial B_x}{\partial y}+\eta_m\nabla^2B_x=0 \tag{8}$$

The boundary conditions are that both $\bar{u}$ and $\tilde{u}$ be zero at y = 0 and y = h and $B_x=0$ at y = 0, $B_y=1$ at y = h.

3. Solution of the Problem

Equation (5) without the magnetic terms has been studied by Berman (1958).

We introduced a new dimensionless variable

$\eta=\frac{y}{h}$ and $\tilde{u}=f(\eta)$, $B_x=g(\eta)$, $B_y=B_0$ (constant)

Then the equation (5) and (6) becomes respectively

$$f(\eta) - f'(\eta)R_e - f(\eta)M^2 + g(\eta)M^2\frac{V}{B_0} = \frac{Ah^2}{\nu} \tag{9}$$

$$g(\eta) - R_\sigma g'(\eta) + B_0\mu\sigma h f'(\eta) = 0 \tag{10}$$

Where $R_\sigma = Vh\mu_e\sigma$ (Magnetic Reynolds number)

$R_e = \frac{\rho V h}{\mu}$ (Reynolds number)

$M = B_0 h\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number), μ is the co-efficient of viscosity.

$\eta_m = \frac{1}{\mu_e\sigma}$, μ_e is the magnetic permeability.

The boundary conditions are

$f = 0$, at $\eta = 0$; $f = 0$ at $\eta = 1$

$g = g_0$ at $\eta = 0$; $g = g_c$ at $\eta = 1$

The dashes represent differentiation with respect to η. We are not interested in discussion of the steady part and so we shall not go into details here.

The unsteady equations (7) and (8) can be reduced to ordinary differential equations by introducing a non-dimensional variable-

$\eta = \frac{y}{h}$ and substituting $\overline{u} = f(\eta)e^{i\omega t}$, $B_x = g(\eta)$.

With these substitutions equation (7) and (8) becomes respectively-

$$f(\eta) - R_e f'(\eta) - M^2 f(\eta) - {M_1}^2 i f(\eta) + M^2\frac{V}{B_0}g(\eta)e^{-i\omega t} = \frac{Bh^2}{\nu} \tag{11}$$

$$g(\eta) - R_\sigma g'(\eta) + R_\sigma\frac{B_0}{V}f'(\eta)e^{i\omega t} = 0 \tag{12}$$

Where $M_1 = \sqrt{\frac{\omega}{\nu}}h$ (Non- dimensional frequency parameter)

We put $f = \overline{u}e^{-i\omega t} = (\overline{u}_1 + i\overline{u}_2)(\cos\omega t - i\sin\omega t) = v + iw$

where $\overline{u} = \overline{u}_1 + i\,\overline{u}_2$ and $v = \overline{u}_1 \, cos\, \omega t + \overline{u}_2 \, sin\, \omega t$, $w = \overline{u}_2 \, cos\, \omega t - \overline{u}_2 \, sin\, \omega t$.

and $g(\eta) = B_x = B_{x_1} + i\,B_{x_2}$.

On putting the values of $f,\ f', f$ and g, g', g in (11) and (12) respectively, equating real and imaginary parts and after a few steps of calculation we get the following ordinary differential equations.

$$\overline{u}_1 - R_e\overline{u}_1' - M^2\overline{u}_1 + {M_1}^2\overline{u}_2 + M^2\frac{V}{B_0}B_{x_1} = \frac{Bh^2}{\nu}\,cos\,\omega t \tag{13}$$

$$\overline{u}_2 - R_e\overline{u}_2' - M^2\overline{u}_2 - {M_1}^2\overline{u}_1 + M^2\frac{V}{B_0}B_{x_2} = \frac{Bh^2}{\nu}\,sin\,\omega t \tag{14}$$

And $B_{x_1} - R_\sigma B_{x_1}' + R_\sigma\frac{B_0}{V}\overline{u}_1{}' = 0$ (15)

$B_{x_2} - R_\sigma B_{x_2}' + R_\sigma\frac{B_0}{V}\overline{u}_2{}' = 0$ (16)

The boundary conditions are

$\overline{u}_1 = 0, \overline{u}_2 = 0$ at η = 0 and η = 1.

$B_{x_1} = B_{n_1}, B_{x_2} = 0$ at η = 0; $B_{x_1} = B_{n_2}, B_{x_2} = 0$ at η = 1.

Equations (13), (14), (15) and (16) are solved numerically using Shooting method for $\overline{u}_{1,}\ \overline{u}_2$ and consequently the real part of $'f'$ can be computed.

4. Results and Discussion

The problem under consideration is reduced to a boundary value problem given by (13) to (16). This problem is solved numerically using shooting method. Numerical calculations have been done for various combinations of parameters i.e. the magnetic parameter (Hartmann number, M), Reynolds number R_e, the frequency parameter M1, and Magnetic Reynolds number R_σ. The velocity profiles are computed for the various parameters.

Numerical results are shown graphically by using the following parameters values R_e=0.05, V=0.10, M1 =0.10, B =1.00, M = 1.25, R_σ = 0.70, A = 0.10, ωt = 0.05. It has been observed that the effect of magnetic parameter M, Reynolds number R_e, Magnetic Reynolds number R_σ, Frequency parameter M1 on the velocity field and effect of frequency parameter M1 on mass flux is very prominent.

In order to analyze the flow field insensibly, figure (2) exhibits the velocity profile for different values of Reynolds number at M1=0.1. It is seen that as the Reynolds number increases, initially the flow velocity decreases but from η = 0.4 to η = 1 the velocity increases.

The effect of magnetic parameter M on velocity field is shown in figure (3) for M =0.1. When the frequency parameter M1 is small, it is seen that the velocity profiles are almost parabolic, equally distributed over the boundary layer region and observed that velocity decreases as the magnetic parameter M increases.

In figures (4) and (5), velocity field for different values of magnetic parameters are observed at R_e=0.1 and R_e = 1.3 respectively. In both the cases the fluid velocity decreases with the increase of magnetic parameter M and it is seen that for higher values of R_e , the fluid velocity is slightly shifted to the boundary layer. In figure (6), similar effect of magnetic parameter M is seen for higher values of R_e=1.3 and frequency parameter M1= 0.99.

Effect of frequency parameter M1 on flow velocity are shown in figures (7) and (8). It is observed that for smaller value of magnetic parameter M (M = 0.10), the fluid velocity increases uniformly with the increase of the values of frequency parameter M1 (from 0 to 1.2), while at M1=1.6, the velocity increases tremendously. Again for higher values of Magnetic parameter M (M=1.25), the velocity field decreases with the increase of frequency parameter M1and for higher values of M1, back flow of the fluid is observed near the boundary at η =1.

In figures (9) and (10), effect of magnetic Reynolds number on the flow velocity is observed for different values of frequency parameter. For the smaller values of M1=0.1, the velocity profile increases for the increases of magnetic Reynolds number. Similarly for the higher values of M1=0.99, as the magnetic Reynolds number increases, fluid velocity also increases and at smaller value of R_σ=0, back flow is seen near the boundary at η =1.

The effect of ωt on fluid flow is observed in figure (11). It is seen that at smaller values of M1=0.1 the fluid velocity increases when ωt changes from 0^0 to 180^0.

The instantaneous mass flux Q may be obtained by integrating the expression for velocity across the channel. Figure (12) shows the comparison of magnitude of mass flux at M=1.25 for different values of frequency parameter M1. For fixed M1, initially mass flux shows a constant value and later as Reynolds number increases from R_e=0.3 the mass flux gradually increases. It has been observed that mass flux increases with the increase of the frequency parameter M1.

5. Conclusion

It is seen that the fluid velocity is greatly affected due to the presence of the magnetic field.

1. Hartmann number enhance the fluid velocity across the channel. The fluid velocity decreases with the increases of the magnetic parameter.
2. Reynolds number accelerates the fluid velocity across the channel.
3. Frequency parameter has the tendency to accelerate the fluid velocity.
4. Magnetic Reynolds number also accelerates the flow velocity.
5. Mass flux shows higher values with the higher values of frequency parameter.

Thus the mathematical expressions may help medical practitioners to control the blood flow of a patient by applying a suitable magnetic field.

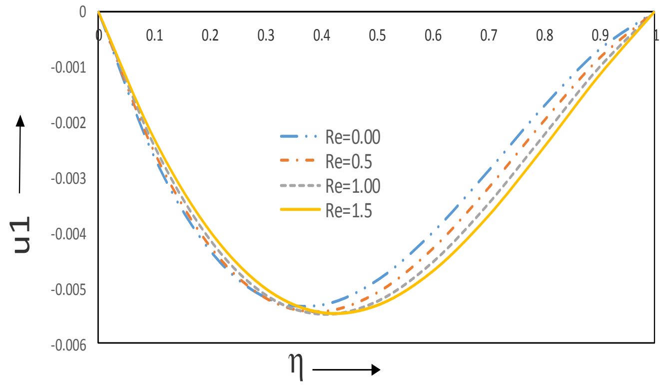

Fig2. *Effect of Reynolds Number R_e at M1=0.1*

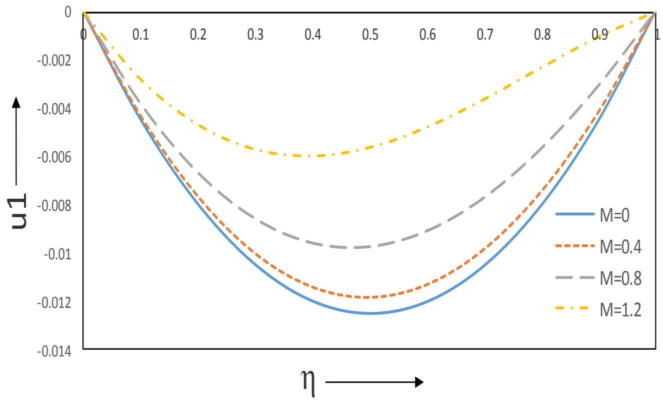

Fig3. *Effect of magnetic parameter M at M1=0.1*

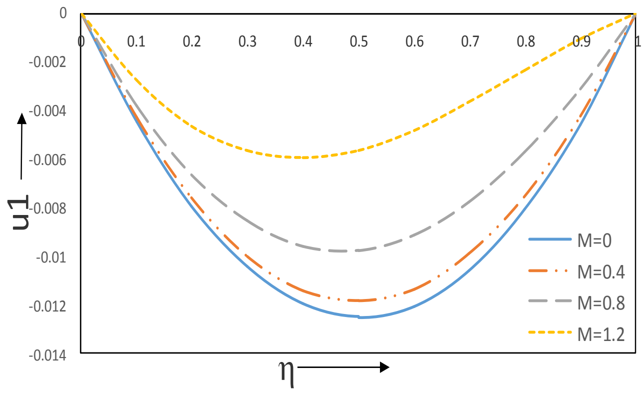

Fig4. *Effect of mssagnetic parameter (M) at Re=0.1*

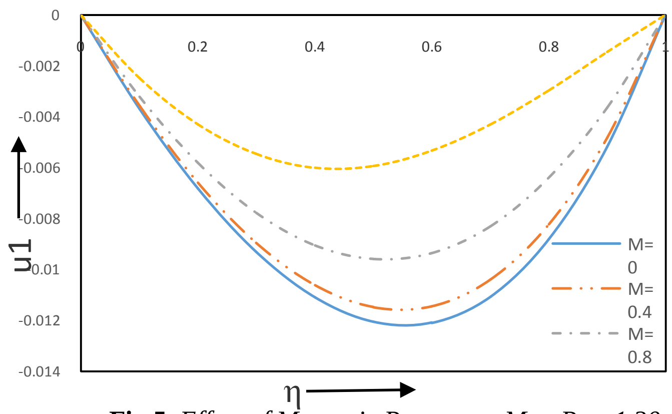

Fig 5. *Effect of Magnetic Parameter M at Re= 1.30*

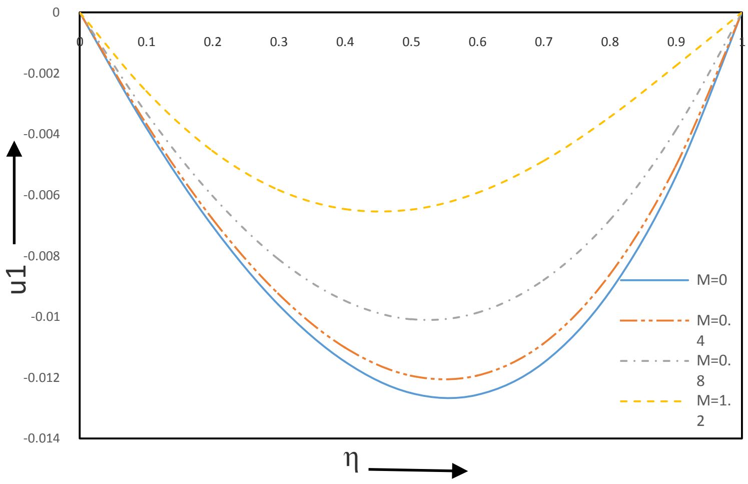

Fig6. *Effect of magnetic parameter (M) at M1=0.99, R_e=1.30*

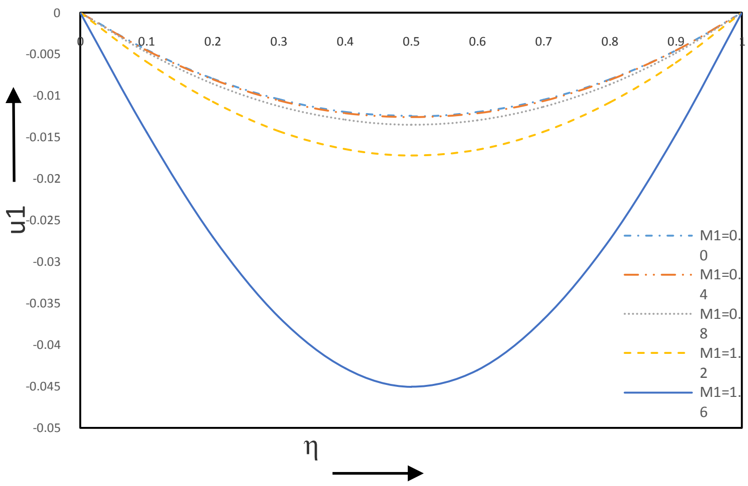

Fig7. *Effect of Frequency parameter M1at M=0.10*

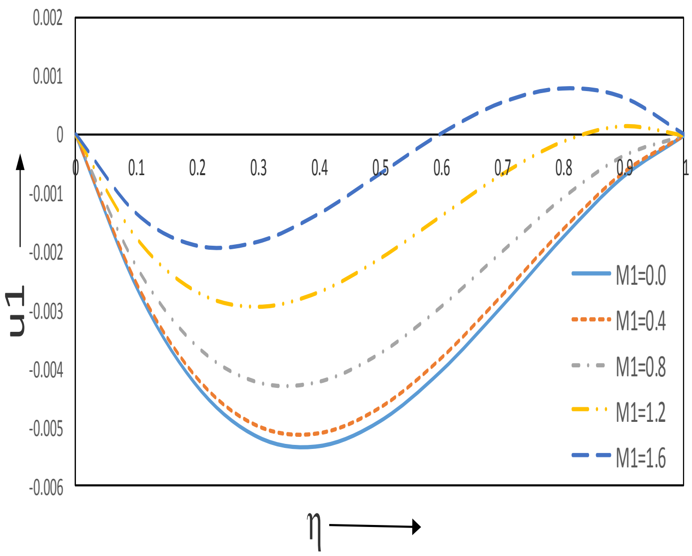

Fig8. *Effect of Frequency parameter (M1) at M=1.25*

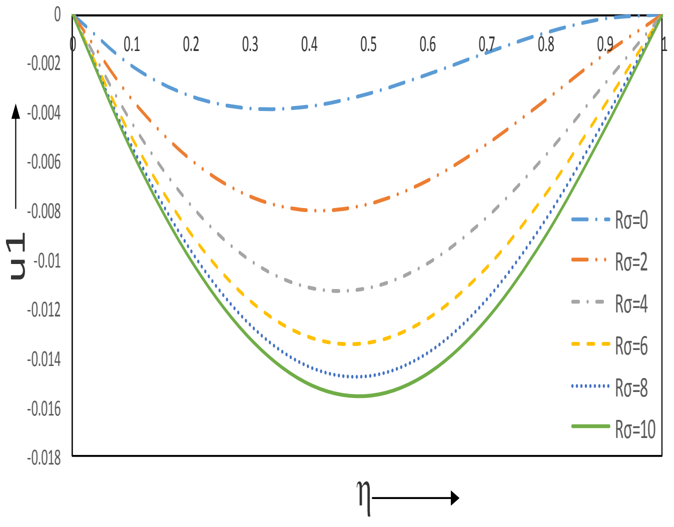

Fig9. *Effect of Magnetic Reynolds number R_σ at M1=0.10*

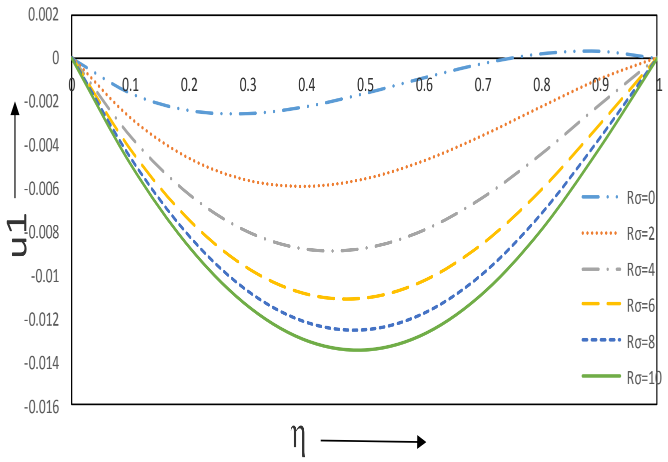

Fig10. *Effect of Magnetic Reynolds number R_σ at M1=0.99.*

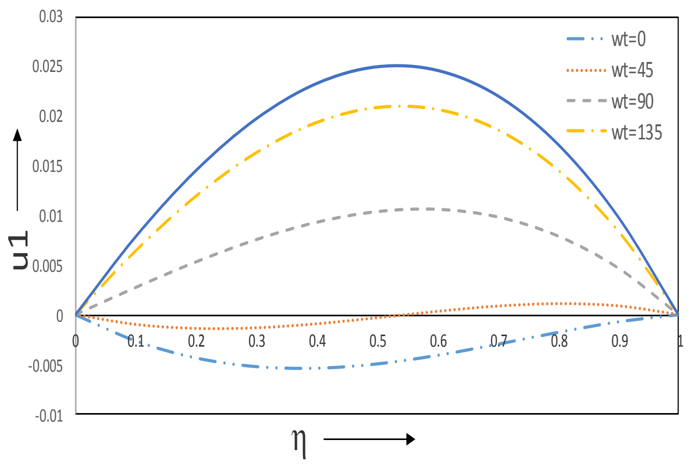

Fig11. *Effect of wt on flow velocity at M1=0.10*

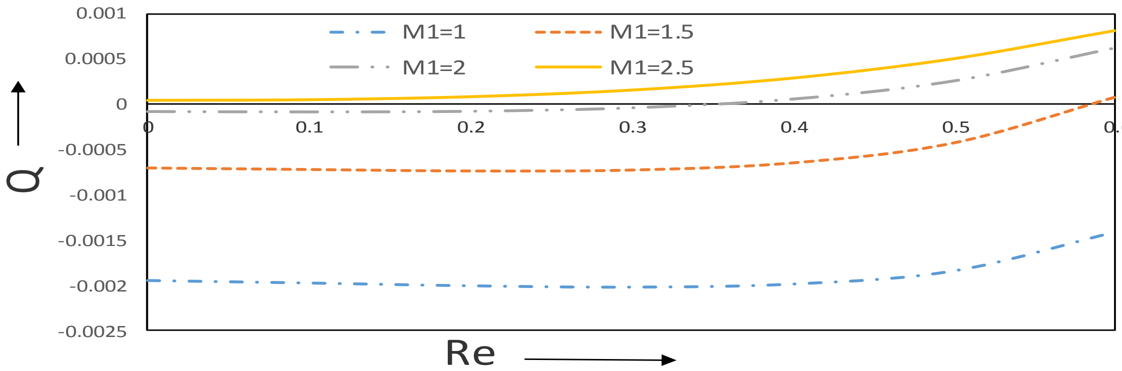

Fig12. *Mass flux for different values of frequency parameter M1 at M=1.25.*

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REFERENCES

[1] Sharma, G. C., Jain, M. and Kumar, Anil, MHD Flow in a Stenosed Artery, Conference Proceeding, Int. Conference on Mathematical Modelling, January, 29-31, (2001).

[2] Kollin, A. ,Electro magnetic flow meter: principle of method and its application to blood flow measurement.,Proc. Soc. Exp. Bio med., 35, pp.33, (1936).

[3] Korchevskii, E. M. and Marochnik, L.S. , Magnetohydrodynamic version of movement of blood., Biophysics, 10, p.411-413, (1965).

[4] Vardanyan, V. A. ,Effect of magnetic field on blood flow, Biofizika, 18(3), pp. 491-496, (1973).

[5] Sud, V. K. and Sekhon, G. S. ,Blood flow through the human arterial system in the presence of a steady magnetic field, Phys. Med. Biol. 34(7), pp.795-805, (1989).

[6] Roston, S. Blood Pressure and the Cardiovascular System, Annals N. Y. Acad. Sci., 96, pp. 962,(1962).

[7] Rathod, V.P. and Gayatri. ,A study of magnetic field on two layer model blood flow with micro-organisms through vessels of small exponential divergence, Applied Science Periodical, 2(1), pp.50-58, (2000).

[8] Rathod, V. P. and Mahesh, K.B. ,Pulsatile blood flow through closed rectangular channel with the effect of micro-organisms and magnetic field, The Mathematics Education, 32(1), pp.32-41, (1998).

[9] Rathod, V. P. and Parveen, S. R.,Pulsatile flow of blood with micro-organisms through a uniform pipe with sector of a circle as cross-section in the presence of transverse magnetic field, The Mathematics Education, 31(3), Sept., (1997).

[10] Bhuyan, B. C. and Hazarika, G.C., Blood flow with effects of slip in arterial stenosis due to presence of transverse magnetic field, Proc. WMVC-2001, Conference, March, p. 17-19,(2001).

[11] Wang, C. Y., Pulsatile Flow in a Porous channel, Journal of Applied Mechanics, 38, pp.553-555, (1971).

[12] Bhuyan, B. C. and Hazarika, G.C., “Effect of Magnetic field on Pulsatile Flow of Blood in a porous channel.” Bio- Science Research Bulletin, 172),pp 105-112, (2001).

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