International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 5, May 2014, PP 443-447
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Cylindrically Symmetric Massive Scalar Field In Bimetric Relativity

S. D. Deo[1] , A.A.Qureshi[2]

[1] Department of Mathematics
N.S.Science and Arts College Bhadrawati,
Dist-Chandrapur, (M.S.) 442902, India
s_deo01@yahoo.in
[2] Department of Mathematics,
Sindhu Mahavidyalaya, Nagpur-440017, India
aaq_maths@yahoo.com

Abstract: *In this paper, cylindrically symmetric space-time is studied with massive scalar field in the context of Rosen's Bimetric Theory of Relativity. Here it is shown that only vacuum model can be constructed.*

Keywords: *Cylindrically symmetric, Massive scalar field, Bimetric Relativity*

AMS Code-83C05 *(General relativity)*

1. INTRODUCTION

The bimetric theory of gravitation was proposed by Rosen [1][2](1940,1973) to modify the Einstein's general theory of relativity, by assuming two metric tensors.

In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric γ_{ij} the physical content of the theory is the same as that of the general relativity.

Thus, now the corresponding two line elements in a coordinate system x^i are –

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

and $$d\sigma^2 = \gamma_{ij} dx^i dx^j \tag{1.2}$$

where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present. Cylindrically symmetric perfect fluid distributions both static and non static have been discussed by many investigators for various matters in the context of general relativity. Heckman,O., and Schucking,E. [3], Mardar[4], Benergee,A.,and Benergee,S.[5],Krori,K.D.and Barua,J.[6],Roy and Narain [7][8],H.Baysal [9] have studied various aspects of cylindrically symmetric space-times.In addition to this cylindrically symmetric space-time is studied in bimetric relativity by Deo S.D. [10] with the matter cosmic strings and domain wall. In continuation of this study, here we have investigated cylindrically symmetric space-time with perfect fluid and Massive scalar field in bimetric relativity.

2. FIELD EQUATIONS IN BIMETRIC RELATIVITY

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi\kappa T_i^j \tag{2.1}$$

Where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj}g_{hi|\alpha}\right]_{|\beta}$ (2.2)

$N = N_\alpha^\alpha$, $\kappa = \sqrt{\frac{g}{\gamma}}$ (2.3)

and $g = \left|g_{ij}\right|$ $, \gamma = \left|\gamma_{ij}\right|$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

3. Cylindrically Symmetric Space Time With Perfect Fluid

We consider here the spherically symmetric line element of the form

$$ds^2 = A^2\left(dx^2 - dt^2\right) + B^2 dy^2 + C^2 dz^2 \tag{3.1}$$

where A, B and C are functions of x and t only.

Corresponding to equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{3.2}$$

Since γ_{ij} is the Lorentz metric i.e. (1,1,1,-1), therefore γ- covariant derivative becomes the ordinary partial derivative.

and , T_i^j the energy momentum tensor for macro matter field represented by

$$T_i^j = T_i^{j^p} = (\rho + p)u_i u^j - p g_i^j \tag{3.3}$$

Together with $g_i^j u_i u^j = 1$ where u_i is the four-velocity of the fluid having p and ρ as proper pressure and energy density respectively

Using equations (2.1) to (2.4) with (3.1) and (3.3)

We get the field equations

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) + \left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = 8\pi\kappa p \tag{3.4}$$

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right) - \frac{1}{2}\left[\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) + \left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = 8\pi\kappa p \tag{3.5}$$

$$\left(\frac{A'^2}{A^2} - \frac{A''}{A}\right) - \left(\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A}\right) + \frac{1}{2}\left[\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) - \left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) + \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = 8\pi\kappa p \tag{3.6}$$

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2} - \frac{B''}{B}\right) - \left(\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B}\right) + \left(\frac{C'^2}{C^2} - \frac{C''}{C}\right) - \left(\frac{\dot{C}^2}{C^2} - \frac{\ddot{C}}{C}\right)\right] = -8\pi\kappa\rho \tag{3.7}$$

Using equation (3.4) to (3.7), we get

$$p+\rho=0 \tag{3.8}$$

This equation of state is known as false vacuum.In view of reality conditions $p>0, \rho>0$ Equation (3.8) immediately implies that $p=0, \rho=0$ that is perfect fluid does not exists in cylindrically symmetric space-time in bimetric relativity.

4. Cylindrically Symmetric Space Time with Massive Scalar Field

We consider here the spherically symmetric line element of the form

$$ds^2=A^2\left(dx^2-dt^2\right)+B^2dy^2+C^2dz^2 \tag{4.1}$$

where A, B and C are functions of x and t only.

Corresponding to equation (4.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2=-dt^2+dx^2+dy^2+dz^2 \tag{4.2}$$

Since γ_{ij} is the Lorentz metric i.e. (1,1,1,-1), therefore γ- covariant derivative becomes the ordinary partial derivative.

Also T_i^j the energy momentum tensor for massive meson is given by

$$T_i^j=T_{i\ meson}^j=V_{,i}V^{,j}-\frac{1}{2}g_i^j\left(V_{,k}V^{,k}-m^2V^2\right) \tag{4.3}$$

together with $\sigma=g_i^{\ j}V_{;i}^{j}+m^2V,\quad v_4v^4=1$ where m is the mass parameter and σ is the source density of the meson field. Here afterwards the suffix (,) and semicolon (;) after a field variable represent ordinary and covariant differentiation with respect to x^i and g_i^j resp.

Using equations (2.1) to (2.4) and (4.1) and (4.3), we get

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=4\pi\kappa\left(v_4^{\ 2}-m^2v^2\right) \tag{4.4}$$

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)-\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=4\pi\kappa\left(v_4^{\ 2}-m^2v^2\right) \tag{4.5}$$

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)+\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)-\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)+\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=4\pi\kappa\left(v_4^{\ 2}-m^2v^2\right) \tag{4.6}$$

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-4\pi\kappa\left(v_4^{\ 2}+m^2v^2\right) \tag{4.7}$$

And overhead primes and dots stand for ordinary differentiation with respect to coordinate x and t respectively.

Using (4.3) - (4.7), we get

$$8\pi\kappa V_4^{\ 2}=0$$

ie $V_4 = 0$ ie V = constant (4.8)

Thus for the space-time (3.1) the massive scalar field with or without mass parameter does not survive in Bimetric theory of relativity.In both cases source density becomes constant.

5. Coupling of Perfect fluid and massive scalar Field

The energy momentum tensor for a mixture of macro and micro fields representing perfect fluid and scalar meson field together is given by

$$T_i^{\,j} = T_i^{\,j^p} + T_i^{\,j}{}_{meson} \tag{5.1}$$

Where $T_i^{\,j^p}$ *and* $T_i^{\,j}{}_{meson}$ are already defined.

By the use of co-moving co-ordinate system,the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-p-4\pi\kappa\left(v_4^{\,2}-m^2v^2\right) \tag{5.2}$$

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)-\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-p-4\pi\kappa\left(v_4^{\,2}-m^2v^2\right) \tag{5.3}$$

$$\left(\frac{A'^2}{A^2}-\frac{A''}{A}\right)-\left(\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}\right)+\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)-\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)+\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=-p-4\pi\kappa\left(v_4^{\,2}-m^2v^2\right) \tag{5.4}$$

$$\frac{1}{2}\left[\left(\frac{B'^2}{B^2}-\frac{B''}{B}\right)-\left(\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}\right)+\left(\frac{C'^2}{C^2}-\frac{C''}{C}\right)-\left(\frac{\dot{C}^2}{C^2}-\frac{\ddot{C}}{C}\right)\right]=\rho+4\pi\kappa\left(v_4^{\,2}+m^2v^2\right) \tag{5.5}$$

using (5.2) and (5.5), we obtain

$$p+\rho+8\pi\kappa V_4^{\,2}=0 \tag{5.6}$$

In view of the reality conditions i.e. $p > 0, \rho > 0$, the above equation implies that

$p = 0, \rho = 0$ and V= constant.

6. Conclusion

In cylindrically symmetric space-time, there is nil contribution of perfect fluid and massive scalar field in bimetric theory of relativity respectively. It is observed that the matter fields like perfect fluid and massive scalar field cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists.

Acknowledgment

The authors are thankful to Dr. K. J. Cherian, Principal, Sindhu Mahavidyalaya Nagpur and Dr. R. D. Giri, Prof. Emeritus,P.G.T.D. (Mathematics), R. T. M. Nagpur University, Nagpur, India for their constant inspiration.

References

[1] Rosen N., General Relativity and Flat space, I. Phys. Rev.57, 147 (1940)
[2] Rosen N., A bimetric theory of gravitation, I Gen. Rela. Grav. 04, 435-47 (1973)

[3] Heckmann,O.,Schucking E., Gravitation, An introduction to current research, John Wiley, New York,P.438 (1962)

[4] Mardar Gravitational waves in general relativity II, The reflection of cylindrical waves.Proc.R.Soc.,246,133 (1958)

[5] Benergee A.,Benergee S., Stationary distribution of dust and Electromagnetic fields in general relativity.J.Phys.A.Ser.2,1, 188 (1968)

[6] Krori K.D.,Barua J., Magnetic energy and cylindrically symmetric distribution of matter in equilibrium in general relativity (1974)

[7] Roy S.R. and Narain S., Some plane symmetric models of perfect fluid distribution in general relativity . Indian J. Pure Appyl.Phy.,10, 763 (1979)

[8] Roy S.R. and Narain S., Cylindrically symmetric non-static perfect fluid distribution in general relativity with pressure equal to density. Indian J. Pure and Applied Phy.,20,709 (1981)

[9] Baysal H. Strings, Cosmological models in cylindrically symmetric inhomogeneous universe in general relativity. Turk. J.Phys. 25, 283 (2001).

[10] Deo S.D., Cylindrically symmetric inhomogeneous cosmic strings and domain walls in bimetric relativity, International journal of mathematical archive-2(1),:121- 123 (2011)