International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 5, May 2014, PP 515-520
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Finite Element Analysis of Heat and Mass Transfer of a MHD / Micropolar fluid over a Vertical Channel

V.V Reddy[1], L. Anand Babu[2], Narsimlu.Gitti[3]

[1]Department of Mathematics, Kamala Nehru Polytechnic College, Hyderabad
venkatreddiv@gmail.com
[2]Department of Mathematics, Osmania University, Hyderabad.
[3]Department of Mathematics, CBIT, Hyderabad.
dr.narsimlu@gmail.com

Abstract: *An analysis is presented for the problem of the fully developed natural convection under the influence of micropolar fluid flow of heat and mass transfer in a vertical channel. Asymmetric temperature and convection boundary conditions are applied to the walls of the channel. Solutions of the coupled non-linear governing equations are obtained for different values of the buoyancy ratio and various material parameters of the micropolar fluid and magnetic parameters. The resulting non dimensional boundary value problem is solved by the Galerken Finite element method using MATLAB Software. Influence of the governing parameters on the fluid flow as well as heat and solute transfers is demonstrated to be significant.*

Keywords: *Finite Element Method, MHD, Natural convection, Micropolarfluid, viscous dissipation*

1. INTRODUCTION

Problems dealing with free convection are often found in many industrial applications. Fully-developed free convection heat transfer in a vertical channel or parallel plates was considered by Bodia et al. [3] for symmetric walls temperature. Aung et al. [1, 2] and Miyatake et al. [7] studied the case of asymmetric boundary conditions which was later examined by Nelson et al. [8]. The earliest formulation of a general theory of fluid micricontinua was attributed to Eringen[5].His theory of micro fluids has opened up new areas in research in the physics of fluid flow. By Eringen's definition, a simple microfluid is a fluent medium whose properties and behaviour are affected by the local motions of the material particles contained in each of its volume elements such a fluid possesses local inertia. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium Lukaszewicz[6].

The basic idea of micropolar fluids has originated from the need to model the non-Newtonian flow of fluids containing rotating micro-constituents. Besides the usual equations for Newtonian flow, this theory introduces some new material parameters, an additional independent vector field-the microrotation-and new constitutive equations that must be solved simultaneously with the usual Newtonian flow equations. Subsequent studies showed that the model can be successfully applied to a wide range of applications including blood flow, lubricants, porous media, turbulent shear flows, and flow in capillaries, and microchannels.

The free micropolar convection for the case of asymmetricboundary conditions or asymmetric heating was investigated by Chamkha et al. [4], Thin film flow of non-Newtonian fluidsona moving beltstudied A.M. Siddiquiet al.[9], HabibisSaleh et al.[10] studied the Simulation of Micropolar Fluid Flow in a vertical Channel using HPM.As pointed out by Rawat and Bhargava [11], the study of heat and mass transfer in micropolar fluids is of importance in the fields of chemical engineering, aerospace engineering and also industrial manufacturing effects processes. Sunil et al. [12] studied the Effect of Rotation on Double-Diffusive Convection in a Magnetized Ferro fluid with Internal Angular Momentum. Recently Bala Siddulu Malga et al.[13] studied the MHD Effects on Fully Developed Natural Convection Heat and Mass transfer of a micropolar Fluid in a Vertical Channel.

2. Mathematical Model

Consider the free convection of a micropolar fluid between two vertical plates, the space between the plates being h. The flow is assumed laminar, steady and fully-developed, i.e. the transverse velocity is zero. A uniform magnetic field is applied to the flow. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in an asymmetric heating situation.

The dimensionless governing equations are:

$$(1+K)\frac{d^2u}{dy^2}+K\frac{dN}{dy}+\theta=Mu \tag{1}$$

$$\left(1+\frac{K}{2}\right)\frac{d^2N}{dy^2}-K\left(2N+\frac{du}{dy}\right)=0 \tag{2}$$

$$\frac{d^2\theta}{dy^2}+E_c\left(\frac{du}{dy}\right)^2=0 \tag{3}$$

Subject to the boundary conditions

$$u(0)=0,\ \theta(0)=\mathrm{R},\ N(0)=0, \tag{4}$$

$$u(1)=0,\ \theta(1)=\mathrm{R}, N(1)=0, \tag{5}$$

where u is dimensionless velocity, N the dimensionless microrotation, θ dimensionless temperature, K is the material parameter and $R=\ (T_1-T_o)/(T_2-T_o), T_1$ temperature of the left cooled wall and T_2 temperature of the right wall.

3. Method of Solution

The above model is a system of second-order BVP. Equation (1) subject to the boundary conditions (4) – (5), possesses the following Finite Element solution, obtained with the help of the MATLAB software. In order to reduce the above system of differential equations to a system of dimensionless form, we may represent the velocity and microrotation, temperature and concentration by applying the Galerkin finite element method for equation (1) over a typical two-noded linear element (e) $(\ y_j\le y\le y_k)$. In equation (2)-(3), taking i=1(1) n and using boundary conditions (4) and (5), the following system of equation are obtained.

$$A_iX_i=B_i\ ,\ i=1,2,3\ .... \tag{6}$$

Where$\mathrm{A_i}$'s are matrices of order n and $\mathrm{X_i}$and$\mathrm{B_i}$'s are column matrices having n-components. The solution of above system of equations are obtained using Thomas algorithm for velocity, angular velocity and temperature. Also, numerical solutions for these equations are obtained by MATLAB program. In order to prove the convergence and stability of the method, the same MATLAB-program was run with slightly changed values of h and k, no significant change was observed in the values of u, N, θ.

4. Results and Discussion

To get the physical insight of the problem the results are discussed through graph. The computations are carried out for the governing flow of the problem with the Galerkin Finite Element Method. The obtained results are compared and these are a good agreement with the result of Habibis Saleh at al [10]. The numerical results are obtained for the velocity and microrotation has been shown graphically for different flow parameters.

The effect of magnetic field parameter M on the velocity profiles u and mocrorotation N for K=5, when R=0.5 is shown in Fig.1 and 2. Here it is observed that the velocity profiles decreases with an increase of M, microrotation profiles increases up to centre of the channel the reverse phenomenon is observed in the other part of the channel. It can be seen that the velocity profiles u increases with an increase of M, The microrotation N decreases with the increase of M, up to middle of the channel (flow direction is upward) and it is increases with increase of M, in the other part of the channel is observed.

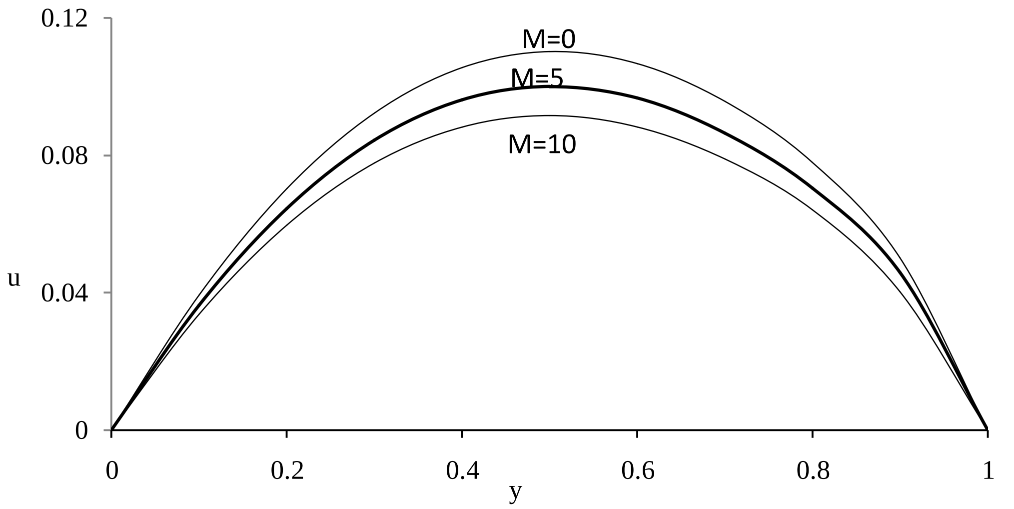

Figure1. *Effects of Magnetic parameter **M** on the velocity profiles **u** for **K=5**and **R=0.5.***

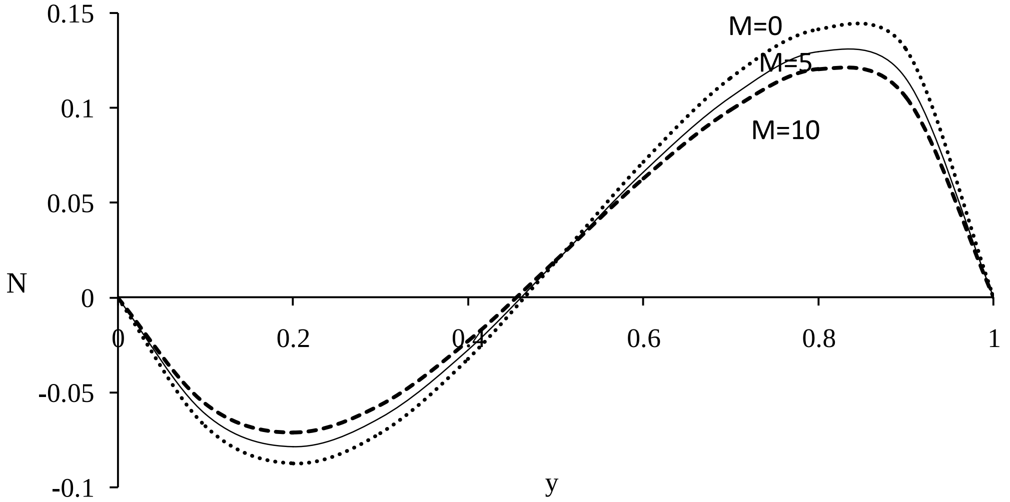

Figure 2. *Effects of Magnetic parameter **M** on the microrotation **N** for **K=5**and **R=0.5.***

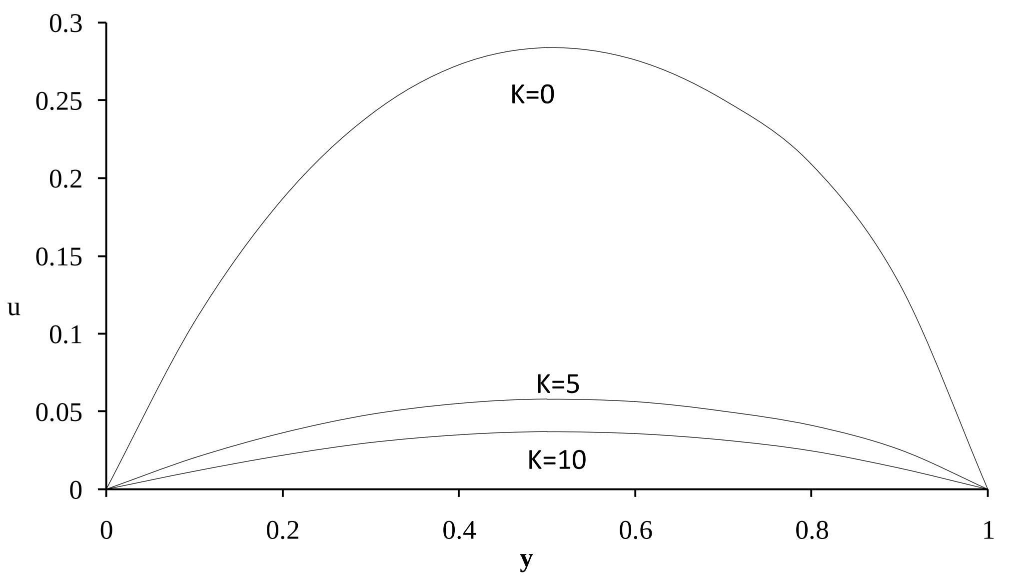

Figure 3. *The velocity profiles for R=0.5*

The Fig.3 and 4 illustrates the influence of vortex viscosity parameter K on the distribution of velocity u and microrotation N for R=0.5. It is observed that with the increasing the value of K the intensity of convective velocity u is reduced as compared to the Newtonian fluid situation (K=0). In fact it is found that as K→ ∞, u→ 0.The influence of parameter K on the microrotation N it is noticed that the variation with K of the value of N evaluated at the position half of the channel also presented in the graphs it can be seen that the intensity of N first increases with increase of K, the reverse phenomenon is observed later. The velocity and microrotation profiles for the case *R* = 0.5 and K= 0, 1, 3 are depicted in Fig.3 Clearly; good agreement is observed when we compared with Habibis Saleh at al [10].

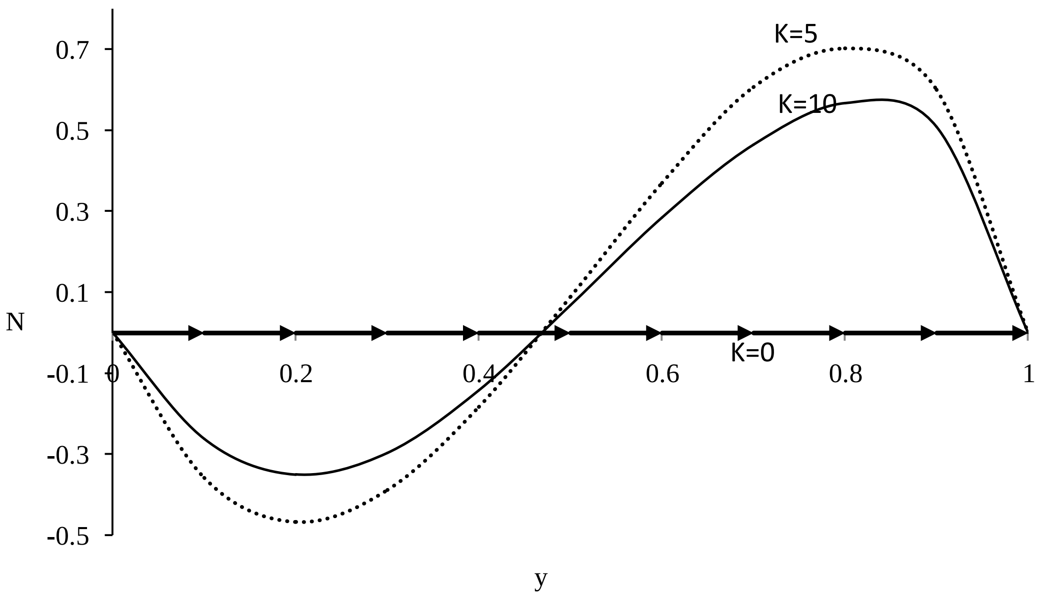

Figure 4. *The microrotation profiles for R=0.5*

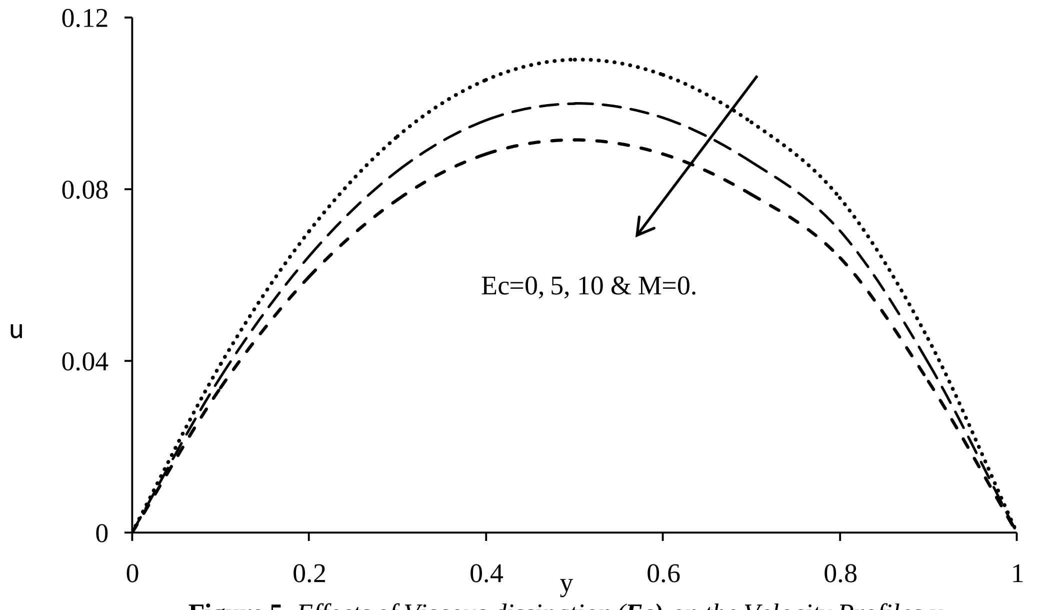

Figure 5. *Effects of Viscous dissipation (**Ec**) on the Velocity Profiles **u***

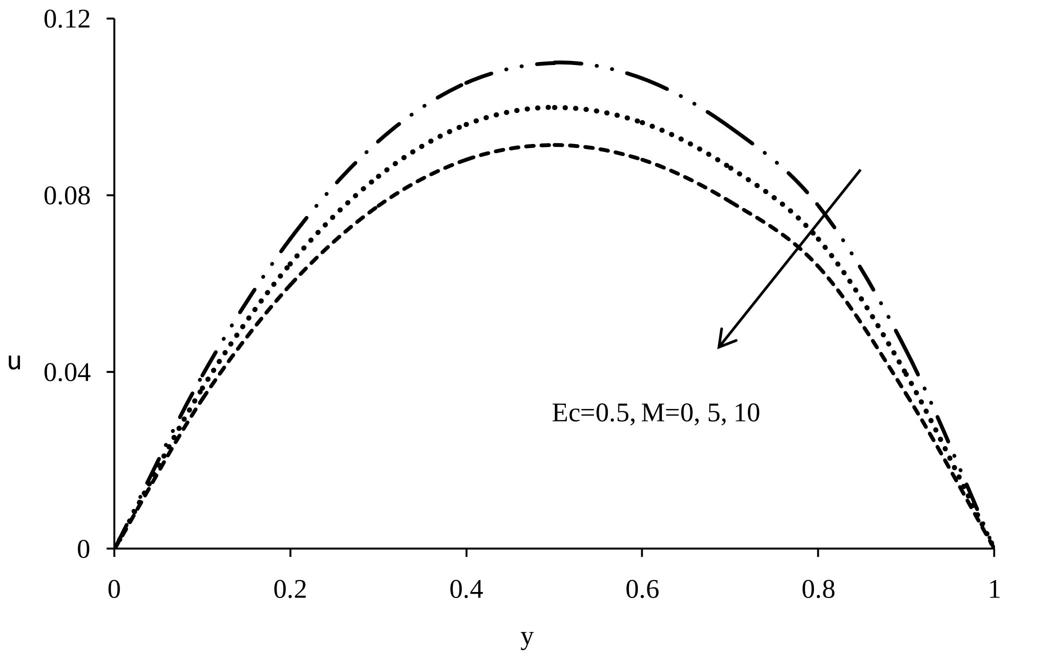

Figure 6. *Effects of Viscous dissipation (Ec) and Manetic Parameter M on the Velocity Profiles* ***u***

It is seen from the Fig. 5 and 6 for the values of magnetic parameter M = 0, 5, 10, the velocity decreasing at a point and then cross the side and increasing with magnetic parameter M. This is because of the velocity profiles having lower peak values for higher values of magnetic parameter M tend to decreases comparatively slower along y-direction than velocity profiles with higher peak values for lower values of magnetic parameter M. We may conclude that for increasing values in M; the Lorentz force, which opposes the flow, there is a fall in velocity maximum due to the retarding effect of the magnetic force in the region. As a result the momentum boundary layer thickness becomes larger and the separation of the boundary layer will occur earlier. Here, it is observed that the increase in the viscous dissipation (Ec) decreases the velocity.

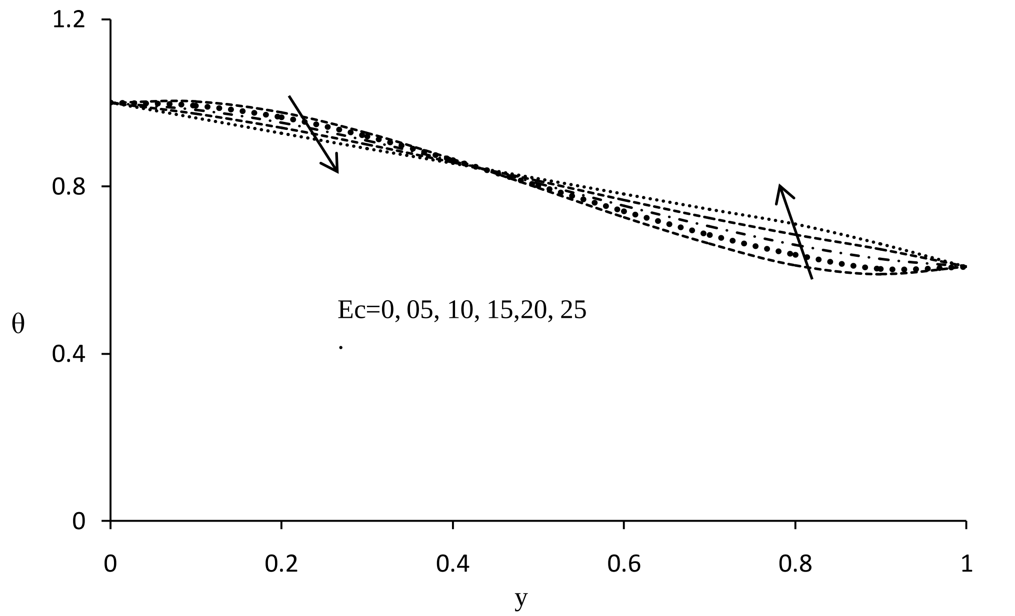

Figure 7. *Effects of Viscous dissipation(**Ec**) on the Temperature Profiles θ*

It is also observed from Fig.7 that as the viscous dissipation parameter (Ec) increases, the temperature profiles increases. The increase in the viscous dissipation cools the fluid. The temperature profile for various values of the viscous dissipation parameter (Ec) while the other parameters are kept constant. It is found that increase in viscous dissipation parameter (Ec) leads to a corresponding increase in the temperature profile. It is also seen that the temperature decreases at certain portion of the channel and then increases this could be due to the dissipation effect and the harmonic pressure term.

It is known that the viscous dissipation produces heat due to drag between the fluid particles and this extra heat causes an increase of the initial fluid temperature (see Fig. 7). This increase of temperature causes an increase of the buoyant force. The increase of the buoyant force causes an increase of the fluid velocity. The bigger fluid velocities cause bigger drag between the fluid particles and consequently bigger viscous heating of the fluid. The new increase of fluid temperature influences the buoyant force and this procedure goes on. There is a continuous interaction between the viscous heating and the buoyant force. This mechanism produces different results in the upward and downward flow. In the upward flow where the fluid is warmer than the ambient the extra viscous heat is added to the initial heat (the warm fluid becomes warmer) and the fluid velocity increases. In the downward flow the fluid is cooler than the ambient and the viscous heating causes an increase in the initial fluid temperature (the cold fluid becomes warmer).

References

[1] W. Aung, Fully developed laminar free convection between vertical plated heated Asymmetrically, *International Communications in Heat and Mass Transfer*, Vol. 15,1972, pp. 1577–1580.

[2] W. Aung, L.S. Fletcher, V. Sernas, Developed laminar free convection between vertical plateswith asymetric heating, *International Communications in Heat and Mass Transfer*, Vol. 15, 1972, pp. 2293–2308.

[3] J.R. Bodia, J.F. Osterle, The development of free convection between heated vertical plates,ASME *Journal of Heat Transfer*, Vol. 84, 1962,pp. 40–44.

[4] A.J. Chamkha, T. Grosan, I. Pop, Fully developed free convection of a micropolar fluid in avertical chanel, *International Communications in Heat and Mass Transfer*, Vol. 29, 2002, pp.1119–1127.

[5] A.C. Eringen, Theory of micropolar fluids, *Journal of Mathematics and Mechanics*, Vol.16,1966, pp. 1–18.

[6] G. Lukaszewich, *Micropolar fluids: theory and applications*, Birkhauser: Basel,1999.

[7] O. Miyatake, T. Fujii, Natural convection heat transfer between vertical parallel plates atunequal uniform temperatures, *Heat Transfer Japanese Research*, Vol. 2, 1973, pp. 79–88.

[8] D.J. Nelson, B.D. Wood, Combined heat and mass transfer natural convection between verticalparallel plates, *International Journal of Heat and MassTransfer*, Vol. 32, 1973, pp. 1779–1787.

[9] A.M. Siddiqui, M. Ahmed, Q.K. Ghori, Thin film flow of non-Newtonian fluids on a movingbelt, *Chaos, Solitons and Fractals*, Vol. 33, No. 3, 2007, pp. 1006-1016.

[10] HabibisSaleh,IshakHashim, Simulation of Micropolar Fluid Flow in a Vertical Channel usingHomotopy-Perturbation Method,*Applied Computing Conference* (ACC '08), Istanbul, Turkey, May 27-30, 2008,pp.378-381.

[11] S. Rawat, R. Bhargava, Finite element study of natural convection heat and mass transferin a micropolar fluid-saturated porous regime withSoret/Dufour effect, Int. J.Appl.Math. Mech. 5 (2009), pp. 58–71.

[12] Sunil, P. Chand, A. Mahajanand P. Sharma, Effect of Rotation on Double-Diffusive Convection in aMagnetized Ferrofluid with Internal Angular Momentum Journal of Applied Fluid Mechanics, 4(4), (2011), pp. 43-52.

[13] Bala Siddulu Malga,N.Kishan, MHD Effects on Fully Developed Natural Convection Heat and Mass Transferof a Micropolar Fluid in a Vertical Channel, Elixir Appl. Math. 63 (2013) pp:18555-18568.