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Application of Queuing Theory to ICC One Day Internationals

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Abstract: *In the present paper, we seek to apply queuing theory model to one day internationals(ODIs). In ODIs, one pair of batsmen open the innings. This pair of batsmen is considered as a customer being served by a cricket pitch i.e. a single server. The one down batsman is considered as a waiting customer. The utilization factor of a server can be obtained by observing the batting of a pair of batsmen in ODIs. As a result we determine the probabilities of a tied ODIs including no result (NR) and resulted ODIs.*

Keywords: *Cricket, ODIs, Batsman, Pair of batsmen, Queuing theory.*

1. Introduction

History of ODI Cricket: One day international (ODI) cricket is played between international cricket teams who are full members of the International Cricket Council (ICC) as well as top six Associate and Affiliate members. Unlike Test matches, ODIs consist of one innings per team, having a limit in the number of overs. The limit of overs is currently 50 overs per innings, although in the past this has been 55 or 60 overs. ODI cricket is List-A cricket, so statistics and records set in ODI matches also count toward List-A records. The earliest match now recognised as an ODI was played between England and Australia in January 1971, since then there have been over 3000 ODIs played by 25 teams. The frequency of matches has steadily increased because of the increase in the number of ODI-playing countries.

At the start of the 1^{st} innings of ODI, one pair of batsmen open the innings. The one down batsman is waiting in a queue i.e. in a dressing room. The cricket pitch is considered as a single server, which serves the customers (cricketers). From the innings of a pair of batsmen, we can find the service rate and arrival rate. And hence, the utilization factor of the cricket pitch. We can consider the time for which one pair is batting as the service time. At the time of falling a wicket from among the pair of batsmen, the new batsman arrives. So the arrival time is same as the service time. That means that, the service rate and arrival rate are same. Thus, we can consider the utilization factor of the cricket pitch as $\rho = 1$.

Assumptions

- We assume that, at the start of the 1^{st} innings, the one down batsman has already padded up and is waiting for his turn.
- It is assumed that, no batsman is retired hurt or retired absent, in each innings.
- We assume that, there is no rain during ODI.

2. $M/M/1$ ODI Model

This model is the single server model which assumes a finite system limit N. N is the maximum number of customers in the system. (Maximum queue length $= N - 1$.). In the context of ODI model $N = 10$ in each innings. That means, there are 10 customers to be served or 10 wickets to be taken in each innings. Arrivals occur at the rate λ customers per unit time and the service rate is μ customers per unit time.

When the number of customers in the system reaches N, no more arrivals are allowed in the system. Thus, we have,

$$\lambda_n = \begin{cases} \lambda, & n = 0,1,2,\ldots,N-1 \\ 0, & n = N, N+1,\ldots\ldots \end{cases}.$$

$$\mu_n = \mu, n = 0,1,2,\ldots\ldots$$

Letting, $\rho = \frac{\lambda}{\mu}$, the probability of n customers in the system is

$$p_n = \begin{cases} \rho^n p_0, & n \leq N \\ 0, & n > N \end{cases}.$$

The value of p_0, the probability of 0 customers in the system, is determined by the equation,

$\sum_{n=0}^{\infty} p_n = 1$, which gives

$$p_0(1 + \rho + \rho^2 + \cdots . + \rho^N) = 1.$$

$$\therefore\ p_0 = \begin{cases} \frac{1-\rho}{1-\rho^{N+1}}, & \rho \neq 1 \\ \frac{1}{N+1}, & \rho = 1 \end{cases}.$$

Hence, the probability of n customers in the system is

$$p_n = \begin{cases} \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \frac{1}{N+1}, & \rho = 1 \end{cases} \quad n = 0,1,\ldots,N.$$

The value of $\rho = \frac{\lambda}{\mu}$ need not be less than 1 in this case, as the number of customers has limit N. In the context of ODI model, we let $\rho = 1$.

The expected number of customers in the system is determined by

$$\begin{aligned} L_s &= \sum_{n=0}^{N} n p_n \\ &= p_0(\rho + 2\rho^2 + 3\rho^3 + \cdots + N\rho^N) \\ &= p_0(1 + 2 + 3 + \cdots + N) \\ &= \frac{1}{N+1}\left[\frac{N(N+1)}{2}\right] \end{aligned}$$

$$\therefore\ L_s = \frac{N}{2}.$$

This suggests that, the expected number of customers (wickets) in each innings is

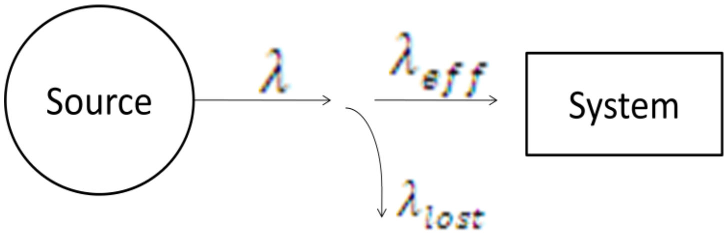
$$L_s = \frac{10}{2} = 5.$$

The relationship between L_s and W_s (also L_q and W_q) is known as Little's formula and is given as

$$L_s = \lambda_{eff} W_s \text{ and } L_q = \lambda_{eff} W_q.$$

The parameter λ_{eff} is the effective arrival rate at the system. It is equal to the arrival rate λ, when all arriving customers can join the system. If some customers can not join because the system is full then $\lambda_{eff} < \lambda$.

In the context of ODI model, if $\rho = 1$, then the effective arrival rate λ_{eff}, rather than the arrival rate λ, is the rate that matters. The effective arrival rate λ_{eff} can be computed by observing the schematic diagram given below, where customers arrive from the source at the rate λ customers per hour.



Relationship between λ, λ_{eff} *and* λ_{lost}

An arriving customer may enter the system or will be lost with rates λ_{eff} or λ_{lost}, which means that,

$$\lambda = \lambda_{eff} + \lambda_{lost}.$$

A customer will be lost from the system, if N customers are already in the system. This means that, the proportion of customers that will not be able to enter the system is p_N. Thus,

$$\lambda_{lost} = \lambda p_N.$$

$$\therefore\ \lambda_{eff} = \lambda - \lambda_{lost} = \lambda - \lambda p_N = \lambda(1 - p_N).$$

In the context of ODI model, $\lambda_{eff} = \lambda\left(1 - \frac{1}{11}\right) = \left(\frac{10}{11}\right)\lambda = \left(\frac{10}{11}\right)\mu.$

$$\therefore\ \lambda_{eff} < \mu.$$

Obviously, $\lambda_{eff} < \lambda$, since $\lambda = \mu$.

By definition, $\begin{pmatrix}Expected\ waiting\\ time\ in\ system\end{pmatrix}$=$\begin{pmatrix}Expected\ waiting\\ time\ in\ queue\end{pmatrix} + \begin{pmatrix}Expected\ service\\ time\end{pmatrix}$.

$$\therefore\ W_s = W_q + \frac{1}{\mu}.$$

Multiplying this formula by λ_{eff}, we can determine relationship between L_s and L_q , using Little's formula, that is

$$\lambda_{eff} W_s = \lambda_{eff} W_q + \frac{\lambda_{eff}}{\mu}$$

$$\therefore\ L_s = L_q + \frac{\lambda_{eff}}{\mu}.$$

In the context of ODI model, $L_q = L_s - \frac{\lambda_{eff}}{\mu}$

$$= L_s - \frac{\left(\frac{10}{11}\right)\mu}{\mu}$$

$$= 5 - \frac{10}{11}$$

$$\therefore\ L_q = 4.09.$$

That is, the expected number of customers (wickets) in the queue $= 4.09$. That means that, almost 4 batsmen are waiting in the queue in each innings.

By definition, the difference between the average number in the system L_s and the average number in the queue L_q must be equal to the average number of busy servers $\bar{c}$. Therefore, we have,

$$\bar{c} = L_s - L_q = \frac{\lambda_{eff}}{\mu}.$$

Also, it follows that, $\begin{pmatrix} facility \\ utilization \end{pmatrix} = \frac{\bar{c}}{c}$.

In the context of ODI model, the average number of busy servers is

$\bar{c} = L_s - L_q = \frac{\lambda_{eff}}{\mu} = \frac{\left(\frac{10}{11}\right)\mu}{\mu} = \frac{10}{11} = 0.909090$. That means that, 1 busy server.

It follows that, $\begin{pmatrix} facility \\ utilization \end{pmatrix} = \frac{\bar{c}}{c} = \frac{0.909090}{1} = 0.909090$. That means that, 1 is the utilization factor.

3. Calculation of Probabilities

Before the start of the ODI match, a coin is tossed to decide which team will bat first. We define the events of batting in the 1st innings and batting in the 2nd innings.

Let B_1 be the event that, the team bat in the 1st innings and B_2 be the event that, the team bat in the 2nd innings.

$$\therefore P(B_1) = P(B_2) = \frac{1}{2}.$$

Also, we define the event that, the 1st innings is completed, irrespective of the number of wickets falling in the innings and the events of team having 1st innings and 2nd innings win.

Let I be the event that, the 1st innings is completed, irrespective of the number of wickets falling in the innings.

$$\therefore P(I) = P(n \leq 10 \; in \; the \; 1^{st} \; innings)$$
$$= \frac{1}{11} + \frac{1}{11} + \cdots + \frac{1}{11} \; (11 \; times)$$
$$\therefore P(I) = 1.$$

If there are less than 10 wickets falling in the 1st innings, then also $P(I)$ is taken as 1, since it is irrespective of the number of wickets falling.

Let I_1 be the event that, team having 1st innings win and I_2 be the event that, team having 2nd innings win.

Then, the probability of team having 2nd innings win is

$$P(I_2) = P(B_2)P\big(n \leq 8 \; in \; the \; 2^{nd} \; innings\big)$$
$$= \frac{1}{2}\left(\frac{1}{11} + \frac{1}{11} + \cdots + \frac{1}{11}\right) (9 \; times)$$
$$= \frac{1}{2}\left(\frac{9}{11}\right)$$
$$\therefore P(I_2) = \frac{9}{22}.$$

The probability of team having 1st innings win is

$$P(I_1) = P(B_1)P(I) + P(B_2)P\big(n = 10 \; in \; the \; 2^{nd} \; innings\big)$$
$$= \frac{1}{2}(1) + \frac{1}{2}\left(\frac{1}{11}\right)$$
$$= \frac{1}{2} + \frac{1}{22}$$
$$\therefore P(I_1) = \frac{12}{22}.$$

Now, we define the event that, the ODI match is tied or have no result (NR).

Let T be the event that, the ODI match is resulting in a tie or having no result (NR). Then, the probability of tied ODI match is

$$P(T) = P(B_2)P\big(n = 9 \; in \; the \; 2^{nd} \; innings\big)$$

$$= \frac{1}{2}\left(\frac{1}{11}\right)$$

$$\therefore P(T) = \frac{1}{22} = 0.045454.$$

We observe that, the total probability is 1. That is, the sum of probabilities of team having 1st innings win, team having 2nd innings win and a tied match including no result (NR), is 1. Hence, the probability of winning an ODI match by any one of the two teams is

$$P(I_1) + P(I_2) = \frac{12}{22} + \frac{9}{12} = \frac{21}{22} = 0.954545.$$

In other words, we can say that, $4.54\,\%$ ODIs result in a tie or having no result (NR). And $95.45\,\%$ ODIs result in a win (by any one of the two teams). That means that, out of 100 ODIs 4.54 ODIs result in a tie or having no result (NR) and 95.45 ODIs result in a win (by any one of the two teams).

Also, the probability of zero batsmen on the pitch in single innings is $p_0 = \frac{1}{11}$, that is, the proportion of zero batsmen in single innings is $\frac{1}{11}$.

Actual ODI Records: (As on January-2013)

Teams	Matches	Won	Lost	Tied	NR	Tied+NR %
England	595	290	278	7	20	4.54
Australia	811	500	275	9	27	4.44
West Indies	690	356	304	6	24	4.35
India	817	405	371	6	35	5.02
New Zealand	635	272	325	5	33	5.98
Pakistan	784	421	340	6	17	2.93
South Africa	482	299	165	5	13	3.73
Sri Lanka	681	318	330	4	29	4.85
Zimbabwe	410	107	289	5	9	3.41
Bangladesh	267	75	190	0	2	0.75
Kenya	148	41	102	0	5	3.38
Ireland	74	34	35	1	4	6.76
Total	**6394**	**3061**	**3061**	**54**	**218**	**4.25**

- Excluding the records of teams, whose span for ODIs is either one, two, four or few years or having no tied matches, like USA, UAE, Namibia, Honk Kong, Africa XI or Asia XI. (Excluding the records of teams having very short duration of international cricket.)
- Including the matches which are abandoned without a ball being bowled.

From this table, we observe that, $4.25\,\%$ of ODIs have been tied or having no result (NR), which is almost equal to $4.54\,\%$. We see that, there is no much difference between these percentages. Thus, we can verify our results.

Table-1

Total ODIs Played	Number of Wins (by any one of the two teams)	Number of Tied ODIs including NR	Actual Probability of Tied+NR ODIs
3197	3061	136	$\frac{136}{3197} = 0.042539$

At the start of the ODI match, we have no knowledge of a tied match or an ODI match having no result. That's why we consider two events that the team batting first win and the team batting second win, with their probabilities $\frac{12}{22}$ and $\frac{9}{22}$ respectively. The probability of team batting first and team batting second is $\frac{1}{2}$ each.

The expected value of a variable x is given by $E(x) = \sum_{i=1}^{n} x_i p(x_i)$. Therefore, the expected probability of the two events is $E(x) = \left(\frac{1}{2}\right)\left(\frac{12}{22}\right) + \left(\frac{1}{2}\right)\left(\frac{9}{22}\right)$

$$= \frac{12}{44} + \frac{9}{44}$$

$$= \frac{21}{44}$$

$$= 0.4772.$$

We now verify this probability by actual record of ODI matches as follows:

Actual ODI Records: (As on 7 June-2013)

We consider the top ten ODI teams in the world only.

Teams	Matches	Won	Lost	Tied	NR	Won %
Australia	453	282	152	4	15	62.25
Bangladesh	134	31	103	0	0	23.13
England	288	122	155	3	8	42.36
India	390	177	191	4	18	45.38
New Zealand	323	133	177	2	11	41.18
Pakistan	422	226	177	4	15	53.55
South Africa	236	146	82	2	6	61.86
Sri Lanka	337	159	162	2	14	47.18
West Indies	311	151	142	3	15	48.55
Zimbabwe	210	57	148	3	2	27.14
Total	**3104**	**1484**	**1489**	**27**	**104**	**47.81**

From this table, we observe that, $47.81\ \%$ of ODIs have been won by team batting first, if we consider the top ten ODI teams in the world. The expected percentage is $47.72\ \%$. We see that, there is no much difference between these percentages. Thus, we can verify our results.

Table-2

Total ODIs Played	Number of Wins by team batting first	Number of Wins by team batting second	Actual Probability of team batting first	Actual Probability of team batting second
3104	1484	1489	0.4781	0.4797

From the above table, we can see that the expected probability match with the actual probabilities.

4. Conclusion

As a result, we conclude that, the probability of the ODI match resulting in a win by any one of the two teams, is $\frac{21}{22}$. That means that, out of 22 ODI matches, 21 ODIs result in a win by any one of the two teams. The probability of the ODI match resulting in a tie or having no result (NR), is $\frac{1}{22}$. That means that, out of 22 ODI matches, 1 ODI results in a tie or have no result (NR). In other words, we can say that, 4.54 % ODIs result in a tie or having no result (NR). And 95.45 % ODIs result in a win (by any one of the two teams).

Also, the expected probability of the team batting first win or the team batting second win is 0.4772. That means that, the expected percentage of the team batting first win or the team batting second win is 47.72 %.

Also, the probability of zero batsmen on the pitch in single innings is $\frac{1}{11}$. That means that, out of 11 innings, in 1 innings there is no batsman to be served by a pitch.

References

[1] Rust K., "Using Little's Law to Estimate Cycle Time and Cost", Proceedings of the 2008 Winter Simulation Conference, IEEE Press, Dec. 2008, doi:10.1109/WSC.2008.4736323.

[2] Taha H.A., Operations Research-An Introduction. 8th Edition, ISBN 0131889230. Pearson Education, 2007.

[3] Worthington D and Wall A (1999). Using the discrete time modelling approach to evaluate the time-dependent behaviour of queuing systems. J Opl Res Soc 50: 777-888.

[4] Stevenson WJ (1996). Production/Operations Management, 6th edn. Irwin, McGraw-Hill, USA.

[5] Tijms HC (1986). Stochastic Modelling and Analysis. A Computational Approach. Wiley: Chichester.

[6] Cooper RB (1972). Introduction to Queuing Theory. McMillan: New York.

[7] Little J.D.C., "A Proof for the Queuing Formula: L = λW", Operations Research, vol. 9(3), 1961, pp. 383-387, doi:10.2307/167570.

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