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Some Common Fixed Point Results in Complex valued Metric Spaces Satisfying (E.A.) and (CLR)-Property

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Abstract: *We prove some common fixed point results for two pairs of weakly compatible mappings satisfying a general contractive condition without using the completeness of the space and continuity of maps.*

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1. Introduction

The famous Banach contraction principle states that if (X,d) is a complete metric space and $T: X \to X$ *is* a contraction mapping (i.e., $d(Tx,Ty) \leq k\, d(x,y)$, for all $x, y \in X$, where k is a nonnegative number such that $k < 1$), then T has a unique fixed point. This result is one of the cornerstones in the development of nonlinear analysis.

Recently, Azam et al. [1] introduced the complex valued metric spaces and obtained sufficient conditions for the existence of common fixed points of a pair of contractive type mapping.

Aamri and Moutawakil [5] introduced the notion of (E.A.) - property.

Sintunavrat and P. Kumam [7] introduced the notion of CLR-property. Many researchers have obtained fixed point, common fixed point, coupled fixed point and coupled common fixed point results in partially ordered metric spaces, complex valued metric spaces and other spaces. We prove some common fixed point theorems for two pairs of weakly compatible mappings satisfying a contractive condition with (E.A.) and (CLR)-property in complex valued metric space. The proved results generalize and extend some of the results in the literature.

2. Preliminaries

Let C be the set of complex numbers and let z_1 , $z_2 \in C$. Define a partial order $\lesssim$ on C as follows:

$z_1 \lesssim z_2$ if and only if $Re(z_1) \leq Re(z_2)$, $Im(z_1) \leq Im(z_2)$. It follows that $z_1 \lesssim z_2$ if one of the following conditions is satisfied:

(i) $Re(z_1) = Re(z_2)$, $Im(z_1) < Im(z_2)$,

(ii) $Re(z_1) < Re(z_2)$, $Im(z_1) = Im(z_2)$,

(iii) $Re(z_1) < Re(z_2)$, $Im(z_1) < Im(z_2)$,

(iv) $Re(z_1) = Re(z_2)$, $Im(z_1) = Im(z_2)$.

In particular, we will write $z_1 \lnsim z_2$ if one of (i), (ii) and (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied.

Definition2.1. Let X be a non-empty set. Suppose that the mapping $d: X \times X \to C$ satisfies:

(i) $0 \precsim d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$;

(iii) $d(x,y) \precsim d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Example 1. Let $X = C$. Define a mapping $d: X \times X \to C$ by $d(z_1, z_2) = e^{ik}|z_1 - z_2|$, Where $k \in [0, \pi/2]$. Then , (X,d) is called a complex valued metric space.

A point $x \in X$ is called an interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in C$ such that $B(x,r) = \{y \in X: d(x,y) \prec r\} \subseteq A$. A subset A in X is called open whenever each point of A is an interior point of A. The family $F = \{B(x,r): x \in X, 0 \prec r\}$ is a sub-basis for a Hausdorff topology τ on X. A point $x \in X$ is called a limit point of A whenever for every $0 \prec r \in C$, $B(x,r) \cap (A \backslash X) \neq \phi$.

A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in C$, with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) \prec c$, then x is called the limit point of $\{x_n\}$ and we write $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

If for every $c \in C$, with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x_{n+m}) \prec c$, then $\{x_n\}$ is called a Cauchy sequence in (X,d) is called a complete complex valued metric space.

Lemma2.2. Let (X,d) be a complex valued metric space and $\{x_n\}$ is a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma2.3. Let (X,d) be a complex valued metric space and $\{x_n\}$ is a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.

Definition2.4. Let f and g be self-maps on a set X, if $w = fx = gx$ for some x in X, then x is called coincidence point of f and g, w is called a point of coincidence of f and g, w is called a point of coincidence of f and g.

Definition2.5. Let f and g be two self-maps defined on a set X, then f and g are said to be weakly compatible if they commute at coincidence points.

Definition 2.6. Let f and g be two self-mappings of a complex valued metric space (X,d). We say that f and g satisfy the (E.A.)-property if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$, for some $t \in X$.

Definition 2.7. Let f and g be two self-mappings of a complex valued metric space (X,d). We say that f and g satisfy the (CLR_f) property if there exist a sequence $\{x_n\}$ in X such that$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = fx$.

3. Main Results

Theorem 3.1. Let (X,d) be a Complex valued metric space. Suppose that the mapping f, g, h and k are four self-maps of X satisfying the following conditions:

(i) $f(X) \subseteq h(X), g(X) \subseteq k(X)$,

(ii) for all $x, y \in X$ and $0 < \lambda < 1$,

$$d(fx, gy) \leq \lambda u_{x,y}(f,g,h,k), \tag{3.1}$$

Where

$$u_{x,y}(f,g,h,k) \in \left\{d(kx,hy), d(kx,fx), d(hy,gy), \frac{d(kx,gy)+d(hy,fx)}{2}, \frac{d(fx,kx)+d(gy,hy)}{2}\right\}$$

(iii) the pairs $\{f,k\}$ and $\{g,h\}$ are weakly compatible;

(iv) One of the pairs $\{f,k\}$ or $\{g,h\}$ satisfy (E.A.)-property.

If $k(X)$or $h(X)$ is a closed subset of X. Then f, g, h and k have a unique common fixed point.

Proof: Suppose that the pair $\{f,k\}$ satisfy (E.A.) property so there exists a sequence $\{x_n\}$ in X, such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = t$, for some $t \in X$.

Further, Since $f(X) \subseteq h(X)$, there exists a sequence $\{y_n\}$ in X, such that $fx_n = hy_n$.

Now, we claim that $\lim_{n\to\infty} gy_n = t$. For this, put $x = x_n, y = y_n$ in (3.1),

We have

$$d(fx_n, gy_n) \leq \lambda u_{x_n y_n}(f,g,h,k)$$

Where

$$u_{x_n y_n}(f,g,h,k) \in$$
$$\left\{d(kx_n,hy_n), d(kx_n,fx_n), d(hy_n,gy_n), \frac{d(kx_n,gy_n)+d(hy_n,fx_n)}{2}, \frac{d(fx_n,kx_n)+d(gy_n,hy_n)}{2}\right\}$$
$$= \left\{d(kx_n,fx_n), d(kx_n,fx_n), d(fx_n,gy_n), \frac{d(kx_n,gy_n)+d(fx_n,fx_n)}{2}, \frac{d(fx_n,kx_n)+d(gy_n,fx_n)}{2}\right\}$$

Letting $n \to \infty$, we have

$$u_{x_n y_n}(f,g,h,k) \in \left\{d(t,t), d(t,t), d(t,gy_n), \frac{d(t,gy_n)+d(t,t)}{2}, \frac{d(t,t)+d(gy_n,t)}{2}\right\}$$
$$= \left\{0, d(t,gy_n), \frac{d(t,gy_n)}{2}\right\}$$

There are three possibilities:

Case (i) If $u_{x_n y_n}(f,g,h,k) = 0$,

Then $|d(t,gy_n)| \leq 0$

Case (ii) If $u_{x_n y_n}(f,g,h,k) = d(t,gy_n)$,

Then $|d(t,gy_n)| \leq \lambda\, |d(t,gy_n)|$, a contradiction.

Case (iii) If $u_{x_n y_n}(f,g,h,k) = \frac{d(t,gy_n)}{2}$

Then $|d(t,gy_n)| \leq \lambda \frac{|d(t,gy_n)|}{2}$, a contradiction.

Thus in all the cases we get $\lim_{n\to\infty} gy_n = t$.

Now, suppose first that k(X) is closed subset of X, then $t = ku$, for some $u \in X$.

Subsequently, we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = \lim_{n\to\infty} hy_n = \lim_{n\to\infty} gy_n = t = ku.$$

We claim that $fu = ku$. For, putting $x = u$ and $y = y_n$ in (3.1) we have

$d(fu, gy_n) \leq \lambda u_{u,y_n}(f, g, h, k)$

Where

$u_{u,y_n}(f, g, h, k) \in$

$\left\{d(ku, hy_n), d(ku, fu), d(hy_n, gy_n), \frac{d(ku,gy_n)+d(hy_n,fu)}{2}, \frac{d(fu,kx_n)+d(gy_n,hy_n)}{2}\right\}$

$$= \left\{d(t, hy_n), d(t, fu), d(hy_n, gy_n), \frac{d(t, gy_n) + d(hy_n, fu)}{2}, \frac{d(fu, kx_n) + d(gy_n, hy_n)}{2}\right\}$$

Letting $n \to \infty$, we have

$$u \in \left\{d(t,t), d(t, fu), d(t,t), \frac{d(t,t) + d(t, fu)}{2}, \frac{d(fu, t) + d(t,t)}{2}\right\}$$

$$= \left\{0, d(t, fu), \frac{d(t,fu)}{2}\right\}$$

There are three possibilities:

Case (i) If $u_{u,y_n}(f, g, h, k) = 0$,

Then $|d(fu, t)| \leq 0$

Case (ii) If $u_{u,y_n}(f, g, h, k) = d(t, fu)$,

Then $|d(fu, t)| \leq \lambda |d(fu, t)|$, a contradiction.

Case (iii) If $u_{u,y_n}(f, g, h, k) = \frac{d(t,fu)}{2}$

Then $|d(fu, t)| \leq \lambda \frac{|d(t,fu)|}{2}$, a contradiction.

Thus in all the cases we get $fu = ku = t$. Hence u is a coincidence point of $\{f, k\}$. Now, the weak compatibility of pair $\{f, k\}$ implies that $fku = kfu$ or $ft = kt$.

On the other hand$f(X) \subseteq h(X)$, there exists v in X such that $fu = hv$. Thus $fu = ku = hv = t$.

Let us show that v is a coincidence point of$\{g, h\}$. i.e. $gv = hv = t$. If not, then putting $x = u, y = v$ in (3.1),we have

$d(fu, gv) \leq \lambda u_{u,v}(f, g, h, k)$

Where

$u_{u,v}(f, g, h, k) \in \left\{d(ku, hv), d(ku, fu), d(hv, gv), \frac{d(ku,gv)+d(hv,fu)}{2}, \frac{d(fu,ku)+d(gv,hv)}{2}\right\}$

$$= \left\{d(t,t), d(t,t), d(t, gv), \frac{d(t, gv) + d(t,t)}{2}, \frac{d(t,t) + d(gv, t)}{2}\right\}$$

$$= \left\{0, d(t, gv), \frac{d(t,gv)}{2}\right\}$$

There are three possibilities:

Case (i) If $u_{u,v}(f, g, h, k) = 0$,

Then $|d(t, gv)| \lesssim 0$

Case (ii) If $u_{u,v}(f, g, h, k) = d(t, gv)$,

Then $|d(t, gv)| \lesssim \lambda |d(t, gv)|$, a contradiction.

Case (iii) If $u_{u,v}(f, g, h, k) = \frac{d(t,gv)}{2}$

Then $|d(t, gv)| \lesssim \lambda \frac{|d(t,gv)|}{2}$, a contradiction.

Thus in all the cases we get $gv = t$. Hence $gv = hv = t$, and v is a coincidence point of g and h.

Further the weak compatibility of pair $\{g, h\}$ implies that $ghv = hgv$ or $gt = ht$.

Therefore t is a common coincidence point of f, g, h and k. In order to show that t is a common fixed point, let us put $x = u$, and $y = t$ in (3.1) we have

$d(fu, gt) = d(t, gt) \lesssim \lambda u_{u,t}(f, g, h, k)$

Where

$$u_{u,t}(f, g, h, k) \in \left\{d(ku, ht), d(ku, fu), d(ht, gt), \frac{d(ku,gt)+d(ht,fu)}{2}, \frac{d(fu,ku)+d(gt,ht)}{2}\right\}$$

$$= \left\{d(t, gt), d(t, t), d(gt, gt), \frac{d(t,gt)+d(gt,t)}{2}, \frac{d(t,t)+d(gt,gt)}{2}\right\}$$

$$= \{0, d(t, gt)\}$$

There are two possibilities:

Case (i) If $u_{u,t}(f, g, h, k) = 0$,

Then $|d(t, gt)| \lesssim 0$

Case (ii) If $u_{u,t}(f, g, h, k) = d(t, gt)$,

Then $|d(t, gt)| \lesssim \lambda |d(t, gt)|$, a contradiction.

Thus $gt = t$. Hence $ft = kt = gt = ht = t$. Similarly, the property (E.A.) of the pair $\{g, h\}$ will give the similar result.

For uniqueness of common fixed point, let us assume that w be another common fixed point of f, g, h and k. Then putting $x = w, y = t$ in (3.1) we have

$d(w, t) = d(fw, gt) \lesssim \lambda u_{w,t}(f, g, h, k)$

Where

$$u_{w,t}(f, g, h, k) \in \left\{d(kw, ht), d(kw, fw), d(ht, gt), \frac{d(kw,gt)+d(ht,fw)}{2}, \frac{d(fw,kw)+d(gt,ht)}{2}\right\}$$

$$= \left\{d(w, t), d(w, w), d(t, t), \frac{d(w,t)+d(t,w)}{2}, \frac{d(w,w)+d(t,t)}{2}\right\}$$

$$= \{0, d(w, t)\}$$

There are two possibilities:

Case (i) If $u_{w,t}(f, g, h, k) = 0$,

Then $|d(w, t)| \lesssim 0$

Case (ii) If $u_{w,t}(f,g,h,k) = d(w,t)$,

Then $|d(w,t)| \leq \lambda |d(w,t)|$, a contradiction.

Thus $w = t$. Hence $ft = gt = ht = kt = t$, and t is the unique common fixed point of f, g, h and k. This completes the proof.

Corollary3.2. Let (X,d) be any Complex valued metric space. Suppose that the mapping f and k be two self-maps of X satisfying the following conditions:

(i) $f(X) \subseteq h(X)$,

(ii) for all $x, y \in X$ and $0 < \lambda < 1$,

$$d(fx,fy) \leq \lambda u_{x,y}(f,h),$$

where

$$u_{x,y}(f,h) \in \left\{d(hx,hy), d(hx,fx), d(hy,fy), \frac{d(hx,fy)+d(hy,fx)}{2}, \frac{d(fx,hx)+d(fy,hy)}{2}\right\}$$

(iii) The pair $\{f,h\}$ is weakly compatible.

(iv) The pair $\{f,h\}$ satisfy (E.A.) property.

If $h(X)$ is a closed subset of X. Then f and h have a unique common fixed point.

Proof: If $g = f$ and $k = h$ in theorem 3.1, we get the result.

Theorem 4.1. Let (X,d) be a Complex valued metric space. Suppose that the mapping f, g, h and k are four self-maps of X satisfying the following conditions:

(i) $f(X) \subseteq h(X), g(X) \subseteq k(X)$,

(ii) for all $x, y \in X$ and $0 < \lambda < 1$,

$$d(fx,gy) \leq \lambda u_{x,y}(f,g,h,k), \tag{4.1}$$

Where

$$u_{x,y}(f,g,h,k) \in \left\{d(kx,hy), d(kx,fx), d(hy,gy), \frac{d(kx,gy)+d(hy,fx)}{2}, \frac{d(fx,kx)+d(gy,hy)}{2}\right\}$$

(iii) The pairs $\{f,k\}$ and $\{g,h\}$ are weakly compatible.

(iv) If the pair $\{f,k\}$ satisfy (CLR_f) property, or the pair $\{g,h\}$ satisfy (CLR_g) property.

Then mappings f, g, h and k have a unique common fixed point.

Proof: First suppose that the pair $\{f,k\}$ satisfy (CLR_f) property,then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} kx_n = fx$

for some $x \in X$. Further, since $f(X) \subseteq h(X)$, we have $fx = hu$, for some $u \in X$. We claim that $hu = gu = t$ (say). If not, then putting $x = x_n$ and $y = u$ in (ii) we have

$$d(fx_n, gu) \leq \lambda u_{x_n u}(f,g,h,k)$$

Where

$$u_{x_n u}(f,g,h,k) \in \left\{d(kx_n,hu), d(kx_n,fx_n), d(hu,gu), \frac{d(kx_n,gu)+d(hu,fx_n)}{2}, \frac{d(fx_n,kx_n)+d(gu,hu)}{2}\right\}$$

Letting $n \to \infty$, we have

$u_{x_n,u}(f,g,h,k) \in$
$\left\{d(kx_n,fx), d(kx_n,fx_n), d(fx,gu), \frac{d(kx_n,gu)+d(fx,fx_n)}{2}, \frac{d(fx_n,fx_n)+d(gu,fx)}{2}\right\}$
$$= \left\{0, d(fx,gu), \frac{d(fx,gu)}{2}\right\}$$

There are three possibilities:

Case (i) If $u_{x_n,u}(f,g,h,k) = 0$,

Then $|d(fx,gu)| \leq 0$

Case (ii) If $u_{x_n,u}(f,g,h,k) = d(fx,gu)$,

Then $|d(fx,gu)| \leq \lambda |d(fx,gu)|$, a contradiction.

Case (iii) If $u_{x_n,u}(f,g,h,k) = \frac{d(fx,gu)}{2}$

Then $|d(fx,gu)| \leq \lambda \frac{|d(fx,gu)|}{2}$, a contradiction.

Thus in all cases, we have $fx = gu$ whence $hu = gu = t$(say). It shows that u is a coincidence point of $\{g,h\}$.Also the weak compatibility of $\{g,h\}$ implies that $ghu = hgu$ or $gt = ht$. Further, since $g(X) \subseteq k(X)$, so there exist an $v \in X$, such that $gu = kv$. We claim that $fv = t$. if not, then from (ii), we have

$d(fv,gu) \leq \lambda u_{v,u}(f,g,h,k)$

Where
$u_{v,u}(f,g,h,k) \in \left\{d(kv,hu), d(kv,fv), d(hu,gu), \frac{d(kv,gu)+d(hu,fv)}{2}, \frac{d(fv,kv)+d(gu,hu)}{2}\right\}$

$$= \left\{d(gu,gu), d(gu,fv), d(gu,gu), \frac{d(gu,gu)+d(gu,fv)}{2}, \frac{d(fv,gu)+d(gu,gu)}{2}\right\}$$
$$= \left\{0, d(t,fv) \frac{d(t,fv)}{2}\right\}$$

There are three possibilities:

Case (i) If $u_{v,u}(f,g,h,k) = 0$,

Then $|d(fv,t)| \leq 0$

Case (ii) If $u_{v,u}(f,g,h,k) = d(fx,gu)$,

Then $|d(fv,t)| \leq \lambda |d(t,fv)|$, a contradiction.

Case (iii) If $u_{v,u}(f,g,h,k) = \frac{d(fx,gu)}{2}$

Then $|d(fv,t)| \leq \lambda \frac{|d(t,fv)|}{2}$, a contradiction.

Therefore $fv = t$ whence $fv = kv = t$ and v is a coincidence point of $\{f,k\}$. Also the weak compatibility of $\{f,k\}$ implies that $fkv = kfv$ or $ft = kt$. Therefore t is a coincidence point of f, g, h and k.

In order to show that t is a common fixed point of f, g, h and k. Let us put $x = v, y = t$ in (4.1), we have

$$d(t, gt) = d(fv, gt) \leq \lambda u_{v,t}(f, g, h, k)$$

Where

$$u_{v,t}(f, g, h, k) \in \left\{d(kv, ht), d(kv, fv), d(ht, gt), \frac{d(kv,gt)+d(ht,fv)}{2}, \frac{d(fv,kv)+d(gt,ht)}{2}\right\}$$

$$= \left\{d(t, gt), d(t, t), d(gt, gt), \frac{d(t, gt) + d(gt, t)}{2}, \frac{d(t, t) + d(gt, gt)}{2}\right\}$$

$$= \{0, d(t, gt)\}$$

There are two possibilities:

Case (i) If $u_{v,t}(f, g, h, k) = 0$,

Then $|d(t, gt)| \leq 0$

Case (ii) If $u_{v,t}(f, g, h, k) = d(t, gt)$,

Then $|d(t, gt)| \leq \lambda\, |d(t, gt)|$, a contradiction.

Therefore $gt = t$. Similarly $ft = t$. Hence t is a common fixed point of f, g, h and k. The uniqueness of common fixed point of t follows easily. Similarly, the property (CLR$_g$) of the pair $\{g, h\}$ will give the similar result.

4. Conclusion

We prove some common fixed point results for two pairs of weakly compatible mappings satisfying a general contractive condition which generalize the result of various authors present in fixed point theory literature.

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