A Peculiar Case In Game Theory- A Computational Study

Kanduri Venkata Lakshmi Narasimhacharyulu	I.Pothuraju
Associate Professor	Assistant Professor
Department of Mathematics,	Department of Mathematics
Bapatla Engineering College,	Bapatla Engineering College,
Bapatla,India.	Bapatla, Andhra Pradesh, India.
kvlna@yahoo.com	*im_praju@yahoo.com*

Abstract*: A peculiar case of game theory problem is investigated with the help of Brown's Algorithm in this article. This game problem has the dominance nature for both rows and columns. It is constructed with the strategy of increasing in an action B1 to B15 of player B with the addition of successive natural numbers according to the influences of action A1 to A15 of Player A. Some worthy conclusions are invented by computing maximum number of possible iterations in the classical Java program. The errors are also computed for each computation and the Lower bounds and Upper bounds are also computed.*

Keywords: *Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound*

AMS Classification: **91A05,91A18,91A43, 91A90**

1. INTRODUCTION

The scientific computations of effective and efficient algorithms have played a vital role in the development of operation research. Sometimes the computations of few methods are so complex and are involved huge computations. Practically it is difficult to gain the optimum solutions. In such cases computer oriented programs help us to get the necessary solutions. These language programs have become the essential and integral part of operations research. In these days of scientific development, OR methodology and computer methodology are both growing up simultaneously to open new eras in the field of OR.

K.V.L.N.Acharyulu et.al[1-4] investigated some special cases of game theory in their earlier research work. McKinsey [10] explained the theory of Games in 1952. Raiffa, R. D [9] discussed the nature of games and possible decisions in 1958.Afterwards Dresher, M [8] explicated the strategies and various real applications of game theory in 1961.Later, Rapoport [7], Levin and Desjardins [6] opened innovative ideas in the field of game theory. Billy E.Gillett [5] discussed the constructive concepts and explained how to solve the large size of problems in the games by using Brown's algorithm.

The authors examined a 15x15 game problem which is a peculiar case of game theory and solved it with the help of Brown's Algorithm. The considered game has the dominance nature for both the rows and columns. The principle is considered for constructing this model by increasing in an action B1 to B15 of player B with the addition of successive natural numbers according to the influences of action A1 to A15 of Player A. few fruitful results are obtained by computing maximum number of possible iterations. The errors are also traced and tabulated in a table. Lower bounds and Upper bounds are computed in each computation for investigating the nature of the game. The possible iterations have been considered to obtain the best optimum mixed strategies for the players. The iterations are calculated from 50 the iteration to 500th iteration. The authors utilized Brown's algorithm with the aid of programming language of Java for this scientific study. The effects among the actions of Player A and the actions of Player B are identified.

2. BASIC FORMATION OF 15x15 GAME

A peculiar game is created with 15 rows and 15 columns of player A & Player B with all 15 possible opposing actions on one and another. One player chooses only one single action from his/her set possible actions. It consists of fifteen possible actions of A i.e A1,A2,A3,A4, A5,A6,A7,A8,A9,A10, A11,A12,A13,A14,A15 which will effect on the other fifteen possible actions of player B i.e B1,B2,B3, B4,B5, B6,B7,B8,B9,B10,B11, B12,B13, B14,B15.The pay off matrix of this game is having the size of 15x15 which is given below in matrix form.

Player B

$$
\text{Player A} \begin{bmatrix}
1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 & 19 & 21 & 23 & 25 & 27 & 29 \\
31 & 33 & 35 & 37 & 39 & 41 & 43 & 45 & 47 & 49 & 51 & 53 & 55 & 57 & 59 \\
61 & 63 & 65 & 67 & 69 & 71 & 73 & 75 & 77 & 79 & 81 & 83 & 85 & 87 & 89 \\
91 & 93 & 95 & 97 & 99 & 101 & 103 & 105 & 107 & 109 & 111 & 113 & 115 & 117 & 119 \\
121 & 123 & 125 & 127 & 129 & 131 & 133 & 135 & 137 & 139 & 141 & 143 & 145 & 147 & 149 \\
151 & 153 & 155 & 157 & 159 & 161 & 163 & 165 & 167 & 169 & 171 & 173 & 175 & 177 & 179 \\
181 & 183 & 185 & 187 & 189 & 191 & 193 & 195 & 197 & 199 & 201 & 203 & 205 & 207 & 209 \\
211 & 213 & 215 & 217 & 219 & 221 & 223 & 225 & 227 & 229 & 231 & 233 & 235 & 237 & 239 \\
241 & 243 & 245 & 247 & 249 & 251 & 253 & 255 & 257 & 259 & 261 & 263 & 265 & 267 & 269 \\
271 & 273 & 275 & 277 & 279 & 281 & 283 & 285 & 287 & 289 & 291 & 293 & 295 & 297 & 299 \\
301 & 303 & 305 & 307 & 309 & 311 & 313 & 315 & 317 & 319 & 321 & 323 & 325 & 327 & 329 \\
331 & 333 & 335 & 337 & 339 & 341 & 343 & 345 & 347 & 349 & 351 & 353 & 355 & 357 & 359 \\
361 & 363 & 365 & 367 & 369 & 371 & 373 & 375 & 377 & 379 & 381 & 383 & 385 & 387 & 389 \\
391 & 393 & 395 & 397 & 399 & 401 & 403 & 405 & 407 & 409 & 411 & 413 & 415 & 417 & 419 \\
421 & 423 & 425 & 427 & 429 & 431 & 433 & 435 & 437 & 439 & 441 & 443 & 445 & 447 & 449
\end{bmatrix}
$$

3. MATERIAL AND METHODS

The authors adopted Brown's algorithm to solve this special case of 15x15 game in which row and columns both dominated. Brown's Algorithm:

Step 1:Player A chooses one of the possible actions(A_{i_1}) from A1-A15 to play, and Player B then plays with the possible action B_{j_1} corresponding to the smallest element in the selected action A_{i_1}.

Step 2:Player A then picks out the possible action (A_{i_2}) from A1 - A15 to play corresponding to the largest element in the possible action (B_{j_1}) selected by Player B in step 1.

Step 3:Player B sums the actions of Player A has played thus far, and plays with the possible action of B_{j_2} corresponding to a smallest sum element.

Step 4:Player A sums the actions of Player B has played thus far, and plays the possible action (A_{i_3}) corresponding to a largest sum element. After the required iterations are computed, then go to step 5; otherwise, come back to step 3.

Step 5: Compute an upper and lower bound $\underline{\gamma}$ and $\overline{\gamma}$ respectively.

$$\overline{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad and \quad \underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6:let Xi be the portion of the time Player A played row i with i=1,2,...,m and let Yi be the proportion of the time Player B played column j with j=1,2,...,n. These strategies approximate the optimal mini max strategies. Upper and Lower bounds on the value of the game where $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ are calculated in step 5. The Process completes.

4. RESULTS

This peculiar game is solved by Brown's algorithm to obtain the best optimum mixed strategies for both the players from 50th iteration to 500th iteration with the help of Java Program. The influencing values on all possible actions of player A from the player B are tabulated in the following tables from Table (1) to Table (20).

Table-1. *Player A Vs Player B at 50th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
50	20630	20660	20690	20720	20750	20780	20810	20840
1550	20730	20760	20790	20820	20850	20880	20910	20940
3050	20830	20860	20890	20920	20950	20980	21010	21040
4550	20930	20960	20990	21020	21050	21080	21110	21140
6050	21030	21060	21090	21120	21150	21180	21210	21240
7550	21130	21160	21190	21220	21250	21280	21310	21340
9050	21230	21260	21290	21320	21350	21380	21410	21440
10550	21330	21360	21390	21420	21450	21480	21510	21540
12050	21430	21460	21490	21520	21550	21580	21610	21640
13550	21530	21560	21590	21620	21650	21680	21710	21740
15050	21630	21660	21690	21720	21750	21780	21810	21840
16550	21730	21760	21790	21820	21850	21880	21910	21940
18050	21830	21860	21890	21920	21950	21980	22010	22040
19550	21930	21960	21990	22020	22050	22080	22110	22140
21050	22030	22060	22090	22120	22150	22180	22210	22240

Table-2. *Player A Vs Player B at 50th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
50	20870	20900	20930	20960	20990	21020	21050
1550	20970	21000	21030	21060	21090	21120	21150
3050	21070	21100	21130	21160	21190	21220	21250
4550	21170	21200	21230	21260	21290	21320	21350
6050	21270	21300	21330	21360	21390	21420	21450
7550	21370	21400	21430	21460	21490	21520	21550
9050	21470	21500	21530	21560	21590	21620	21650
10550	21570	21600	21630	21660	21690	21720	21750
12050	21670	21700	21730	21760	21790	21820	21850
13550	21770	21800	21830	21860	21890	21920	21950
15050	21870	21900	21930	21960	21990	22020	22050
16550	21970	22000	22030	22060	22090	22120	22150
18050	22070	22100	22130	22160	22190	22220	22250
19550	22170	22200	22230	22260	22290	22320	22350
21050	22270	22300	22330	22360	22390	22420	22450

Table-3. *Player A Vs Player B at 100th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
100	41680	41710	41740	41770	41800	41830	41860	41890
3100	41880	41910	41940	41970	42000	42030	42060	42090
6100	42080	42110	42140	42170	42200	42230	42260	42290
9100	42280	42310	42340	42370	42400	42430	42460	42490
12100	42480	42510	42540	42570	42600	42630	42660	42690
15100	42680	42710	42740	42770	42800	42830	42860	42890
18100	42880	42910	42940	42970	43000	43030	43060	43090
21100	43080	43110	43140	43170	43200	43230	43260	43290
24100	43280	43310	43340	43370	43400	43430	43460	43490
27100	43480	43510	43540	43570	43600	43630	43660	43690
30100	43680	43710	43740	43770	43800	43830	43860	43890
33100	43880	43910	43940	43970	44000	44030	44060	44090

36100	44080	44110	44140	44170	44200	44230	44260	44290
39100	44280	44310	44340	44370	44400	44430	44460	44490
42100	44480	44510	44540	44570	44600	44630	44660	44690

Table-4. *Player A Vs Player B at 100ᵗʰ Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
100	41920	41950	41980	42010	42040	42070	42100
3100	42120	42150	42180	42210	42240	42270	42300
6100	42320	42350	42380	42410	42440	42470	42500
9100	42520	42550	42580	42610	42640	42670	42700
12100	42720	42750	42780	42810	42840	42870	42900
15100	42920	42950	42980	43010	43040	43070	43100
18100	43120	43150	43180	43210	43240	43270	43300
21100	43320	43350	43380	43410	43440	43470	43500
24100	43520	43550	43580	43610	43640	43670	43700
27100	43720	43750	43780	43810	43840	43870	43900
30100	43920	43950	43980	44010	44040	44070	44100
33100	44120	44150	44180	44210	44240	44270	44300
36100	44320	44350	44380	44410	44440	44470	44500
39100	44520	44550	44580	44610	44640	44670	44700
42100	44720	44750	44780	44810	44840	44870	44900

Table-5. *Player A Vs Player B at 150ᵗʰ Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
150	62730	62760	62790	62820	62850	62880	62910	62940
4650	63030	63060	63090	63120	63150	63180	63210	63240
9150	63330	63360	63390	63420	63450	63480	63510	63540
13650	63630	63660	63690	63720	63750	63780	63810	63840
18150	63930	63960	63990	64020	64050	64080	64110	64140
22650	64230	64260	64290	64320	64350	64380	64410	64440
27150	64530	64560	64590	64620	64650	64680	64710	64740
31650	64830	64860	64890	64920	64950	64980	65010	65040
36150	65130	65160	65190	65220	65250	65280	65310	65340
40650	65430	65460	65490	65520	65550	65580	65610	65640
45150	65730	65760	65790	65820	65850	65880	65910	65940
49650	66030	66060	66090	66120	66150	66180	66210	66240
54150	66330	66360	66390	66420	66450	66480	66510	66540
58650	66630	66660	66690	66720	66750	66780	66810	66840
63150	66930	66960	66990	67020	67050	67080	67110	67140

Table-6. *Player A Vs Player B at 150ᵗʰ Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
150	62970	63000	63030	63060	63090	63120	63150
4650	63270	63300	63330	63360	63390	63420	63450
9150	63570	63600	63630	63660	63690	63720	63750
13650	63870	63900	63930	63960	63990	64020	64050
18150	64170	64200	64230	64260	64290	64320	64350
22650	64470	64500	64530	64560	64590	64620	64650
27150	64770	64800	64830	64860	64890	64920	64950
31650	65070	65100	65130	65160	65190	65220	65250
36150	65370	65400	65430	65460	65490	65520	65550
40650	65670	65700	65730	65760	65790	65820	65850
45150	65970	66000	66030	66060	66090	66120	66150

49650	66270	66300	66330	66360	66390	66420	66450
54150	66570	66600	66630	66660	66690	66720	66750
58650	66870	66900	66930	66960	66990	67020	67050
63150	67170	67200	67230	67260	67290	67320	67350

Table-7. *Player A Vs Player B at 200th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
200	83780	83810	83840	83870	83900	83930	83960	83990
6200	84180	84210	84240	84270	84300	84330	84360	84390
12200	84580	84610	84640	84670	84700	84730	84760	84790
18200	84980	85010	85040	85070	85100	85130	85160	85190
24200	85380	85410	85440	85470	85500	85530	85560	85590
30200	85780	85810	85840	85870	85900	85930	85960	85990
36200	86180	86210	86240	86270	86300	86330	86360	86390
42200	86580	86610	86640	86670	86700	86730	86760	86790
48200	86980	87010	87040	87070	87100	87130	87160	87190
54200	87380	87410	87440	87470	87500	87530	87560	87590
60200	87780	87810	87840	87870	87900	87930	87960	87990
66200	88180	88210	88240	88270	88300	88330	88360	88390
72200	88580	88610	88640	88670	88700	88730	88760	88790
78200	88980	89010	89040	89070	89100	89130	89160	89190
84200	89380	89410	89440	89470	89500	89530	89560	89590

Table-8. *Player A Vs Player B at 200th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
200	84020	84050	84080	84110	84140	84170	84200
6200	84420	84450	84480	84510	84540	84570	84600
12200	84820	84850	84880	84910	84940	84970	85000
18200	85220	85250	85280	85310	85340	85370	85400
24200	85620	85650	85680	85710	85740	85770	85800
30200	86020	86050	86080	86110	86140	86170	86200
36200	86420	86450	86480	86510	86540	86570	86600
42200	86820	86850	86880	86910	86940	86970	87000
48200	87220	87250	87280	87310	87340	87370	87400
54200	87620	87650	87680	87710	87740	87770	87800
60200	88020	88050	88080	88110	88140	88170	88200
66200	88420	88450	88480	88510	88540	88570	88600
72200	88820	88850	88880	88910	88940	88970	89000
78200	89220	89250	89280	89310	89340	89370	89400
84200	89620	89650	89680	89710	89740	89770	89800

Table-9. *Player A Vs Player B at 250th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
250	104830	104860	104890	104920	104950	104980	105010	105040
7750	105330	105360	105390	105420	105450	105480	105510	105540
15250	105830	105860	105890	105920	105950	105980	106010	106040
22750	106330	106360	106390	106420	106450	106480	106510	106540
30250	106830	106860	106890	106920	106950	106980	107010	107040
37750	107330	107360	107390	107420	107450	107480	107510	107540
45250	107830	107860	107890	107920	107950	107980	108010	108040
52750	108330	108360	108390	108420	108450	108480	108510	108540

60250	108830	108860	108890	108920	108950	108980	109010	109040
67750	109330	109360	109390	109420	109450	109480	109510	109540
75250	109830	109860	109890	109920	109950	109980	110010	110040
82750	110330	110360	110390	110420	110450	110480	110510	110540
90250	110830	110860	110890	110920	110950	110980	111010	111040
97750	111330	111360	111390	111420	111450	111480	111510	111540
105250	111830	111860	111890	111920	111950	111980	112010	112040

Table-10. *Player A Vs Player B at 250th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
250	105070	105100	105130	105160	105190	105220	105250
7750	105570	105600	105630	105660	105690	105720	105750
15250	106070	106100	106130	106160	106190	106220	106250
22750	106570	106600	106630	106660	106690	106720	106750
30250	107070	107100	107130	107160	107190	107220	107250
37750	107570	107600	107630	107660	107690	107720	107750
45250	108070	108100	108130	108160	108190	108220	108250
52750	108570	108600	108630	108660	108690	108720	108750
60250	109070	109100	109130	109160	109190	109220	109250
67750	109570	109600	109630	109660	109690	109720	109750
75250	110070	110100	110130	110160	110190	110220	110250
82750	110570	110600	110630	110660	110690	110720	110750
90250	111070	111100	111130	111160	111190	111220	111250
97750	111570	111600	111630	111660	111690	111720	111750
105250	112070	112100	112130	112160	112190	112220	112250

Table-11. *Player A Vs Player B at 300th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
300	125880	125910	125940	125970	126000	126030	126060	126090
9300	126480	126510	126540	126570	126600	126630	126660	126690
18300	127080	127110	127140	127170	127200	127230	127260	127290
27300	127680	127710	127740	127770	127800	127830	127860	127890
36300	128280	128310	128340	128370	128400	128430	128460	128490
45300	128880	128910	128940	128970	129000	129030	129060	129090
54300	129480	129510	129540	129570	129600	129630	129660	129690
63300	130080	130110	130140	130170	130200	130230	130260	130290
72300	130680	130710	130740	130770	130800	130830	130860	130890
81300	131280	131310	131340	131370	131400	131430	131460	131490
90300	131880	131910	131940	131970	132000	132030	132060	132090
99300	132480	132510	132540	132570	132600	132630	132660	132690
108300	133080	133110	133140	133170	133200	133230	133260	133290
117300	133680	133710	133740	133770	133800	133830	133860	133890
126300	134280	134310	134340	134370	134400	134430	134460	134490

Table-12. *Player A Vs Player B at 300th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
300	126120	126150	126180	126210	126240	126270	126300
9300	126720	126750	126780	126810	126840	126870	126900
18300	127320	127350	127380	127410	127440	127470	127500
27300	127920	127950	127980	128010	128040	128070	128100
36300	128520	128550	128580	128610	128640	128670	128700
45300	129120	129150	129180	129210	129240	129270	129300

54300	129720	129750	129780	129810	129840	129870	129900
63300	130320	130350	130380	130410	130440	130470	130500
72300	130920	130950	130980	131010	131040	131070	131100
81300	131520	131550	131580	131610	131640	131670	131700
90300	132120	132150	132180	132210	132240	132270	132300
99300	132720	132750	132780	132810	132840	132870	132900
108300	133320	133350	133380	133410	133440	133470	133500
117300	133920	133950	133980	134010	134040	134070	134100
126300	134520	134550	134580	134610	134640	134670	134700

Table-13. *Player A Vs Player B at 350th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
350	146930	146960	146990	147020	147050	147080	147110	147140
10850	147630	147660	147690	147720	147750	147780	147810	147840
21350	148330	148360	148390	148420	148450	148480	148510	148540
31850	149030	149060	149090	149120	149150	149180	149210	149240
42350	149730	149760	149790	149820	149850	149880	149910	149940
52850	150430	150460	150490	150520	150550	150580	150610	150640
63350	151130	151160	151190	151220	151250	151280	151310	151340
73850	151830	151860	151890	151920	151950	151980	152010	152040
84350	152530	152560	152590	152620	152650	152680	152710	152740
94850	153230	153260	153290	153320	153350	153380	153410	153440
105350	153930	153960	153990	154020	154050	154080	154110	154140
115850	154630	154660	154690	154720	154750	154780	154810	154840
126350	155330	155360	155390	155420	155450	155480	155510	155540
136850	156030	156060	156090	156120	156150	156180	156210	156240
147350	156730	156760	156790	156820	156850	156880	156910	156940

Table-14. *Player A Vs Player B at 350th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
350	147170	147200	147230	147260	147290	147320	147350
10850	147870	147900	147930	147960	147990	148020	148050
21350	148570	148600	148630	148660	148690	148720	148750
31850	149270	149300	149330	149360	149390	149420	149450
42350	149970	150000	150030	150060	150090	150120	150150
52850	150670	150700	150730	150760	150790	150820	150850
63350	151370	151400	151430	151460	151490	151520	151550
73850	152070	152100	152130	152160	152190	152220	152250
84350	152770	152800	152830	152860	152890	152920	152950
94850	153470	153500	153530	153560	153590	153620	153650
105350	154170	154200	154230	154260	154290	154320	154350
115850	154870	154900	154930	154960	154990	155020	155050
126350	155570	155600	155630	155660	155690	155720	155750
136850	156270	156300	156330	156360	156390	156420	156450
147350	156970	157000	157030	157060	157090	157120	157150

Table-15. *Player A Vs Player B at 400th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
400	167980	168010	168040	168070	168100	168130	168160	168190
12400	168780	168810	168840	168870	168900	168930	168960	168990
24400	169580	169610	169640	169670	169700	169730	169760	169790
36400	170380	170410	170440	170470	170500	170530	170560	170590

48400	171180	171210	171240	171270	171300	171330	171360	171390
60400	171980	172010	172040	172070	172100	172130	172160	172190
72400	172780	172810	172840	172870	172900	172930	172960	172990
84400	173580	173610	173640	173670	173700	173730	173760	173790
96400	174380	174410	174440	174470	174500	174530	174560	174590
108400	175180	175210	175240	175270	175300	175330	175360	175390
120400	175980	176010	176040	176070	176100	176130	176160	176190
132400	176780	176810	176840	176870	176900	176930	176960	176990
144400	177580	177610	177640	177670	177700	177730	177760	177790
156400	178380	178410	178440	178470	178500	178530	178560	178590
168400	179180	179210	179240	179270	179300	179330	179360	179390

Table-16. *Player A Vs Player B at 400th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
400	168220	168250	168280	168310	168340	168370	168400
12400	169020	169050	169080	169110	169140	169170	169200
24400	169820	169850	169880	169910	169940	169970	170000
36400	170620	170650	170680	170710	170740	170770	170800
48400	171420	171450	171480	171510	171540	171570	171600
60400	172220	172250	172280	172310	172340	172370	172400
72400	173020	173050	173080	173110	173140	173170	173200
84400	173820	173850	173880	173910	173940	173970	174000
96400	174620	174650	174680	174710	174740	174770	174800
108400	175420	175450	175480	175510	175540	175570	175600
120400	176220	176250	176280	176310	176340	176370	176400
132400	177020	177050	177080	177110	177140	177170	177200
144400	177820	177850	177880	177910	177940	177970	178000
156400	178620	178650	178680	178710	178740	178770	178800
168400	179420	179450	179480	179510	179540	179570	179600

Table-17. *Player A Vs Player B at 450th Iteration from Action A1 to A8*

Player A	Player B						
	A1	A2	A3	A4	A5	A6	A7	A8
450	189030	189060	189090	189120	189150	189180	189210	189240
13950	189930	189960	189990	190020	190050	190080	190110	190140
27450	190830	190860	190890	190920	190950	190980	191010	191040
40950	191730	191760	191790	191820	191850	191880	191910	191940
54450	192630	192660	192690	192720	192750	192780	192810	192840
67950	193530	193560	193590	193620	193650	193680	193710	193740
81450	194430	194460	194490	194520	194550	194580	194610	194640
94950	195330	195360	195390	195420	195450	195480	195510	195540
108450	196230	196260	196290	196320	196350	196380	196410	196440
121950	197130	197160	197190	197220	197250	197280	197310	197340
135450	198030	198060	198090	198120	198150	198180	198210	198240
148950	198930	198960	198990	199020	199050	199080	199110	199140
162450	199830	199860	199890	199920	199950	199980	200010	200040
175950	200730	200760	200790	200820	200850	200880	200910	200940
189450	201630	201660	201690	201720	201750	201780	201810	201840

Table-18. *Player A Vs Player B at 450th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
450	189270	189300	189330	189360	189390	189420	189450

13950	190170	190200	190230	190260	190290	190320	190350
27450	191070	191100	191130	191160	191190	191220	191250
40950	191970	192000	192030	192060	192090	192120	192150
54450	192870	192900	192930	192960	192990	193020	193050
67950	193770	193800	193830	193860	193890	193920	193950
81450	194670	194700	194730	194760	194790	194820	194850
94950	195570	195600	195630	195660	195690	195720	195750
108450	196470	196500	196530	196560	196590	196620	196650
121950	197370	197400	197430	197460	197490	197520	197550
135450	198270	198300	198330	198360	198390	198420	198450
148950	199170	199200	199230	199260	199290	199320	199350
162450	200070	200100	200130	200160	200190	200220	200250
175950	200970	201000	201030	201060	201090	201120	201150
189450	201870	201900	201930	201960	201990	202020	202050

Table-19. *Player A Vs Player B at 500th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
500	210080	210110	210140	210170	210200	210230	210260	210290
15500	211080	211110	211140	211170	211200	211230	211260	211290
30500	212080	212110	212140	212170	212200	212230	212260	212290
45500	213080	213110	213140	213170	213200	213230	213260	213290
60500	214080	214110	214140	214170	214200	214230	214260	214290
75500	215080	215110	215140	215170	215200	215230	215260	215290
90500	216080	216110	216140	216170	216200	216230	216260	216290
105500	217080	217110	217140	217170	217200	217230	217260	217290
120500	218080	218110	218140	218170	218200	218230	218260	218290
135500	219080	219110	219140	219170	219200	219230	219260	219290
150500	220080	220110	220140	220170	220200	220230	220260	220290
165500	221080	221110	221140	221170	221200	221230	221260	221290
180500	222080	222110	222140	222170	222200	222230	222260	222290
195500	223080	223110	223140	223170	223200	223230	223260	223290
210500	224080	224110	224140	224170	224200	224230	224260	224290

Table-20. *Player A Vs Player B at 500th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
500	210320	210350	210380	210410	210440	210470	210500
15500	211320	211350	211380	211410	211440	211470	211500
30500	212320	212350	212380	212410	212440	212470	212500
45500	213320	213350	213380	213410	213440	213470	213500
60500	214320	214350	214380	214410	214440	214470	214500
75500	215320	215350	215380	215410	215440	215470	215500
90500	216320	216350	216380	216410	216440	216470	216500
105500	217320	217350	217380	217410	217440	217470	217500
120500	218320	218350	218380	218410	218440	218470	218500
135500	219320	219350	219380	219410	219440	219470	219500
150500	220320	220350	220380	220410	220440	220470	220500
165500	221320	221350	221380	221410	221440	221470	221500
180500	222320	222350	222380	222410	222440	222470	222500
195500	223320	223350	223380	223410	223440	223470	223500
210500	224320	224350	224380	224410	224440	224470	224500

4.1 Conclusions

(i) The player B shows his/her influences on all available actions of player A in each computation.

(ii) The game has maximum possible correlation among scientific computations.

(iii)There is a gradual accuracy obtained from each computation.

(iv). Required improvements have been obtained.

(v). The fluctuations are stabilized.

(vi). Systematic improvements have been traced in both the players.

5. OPTIMUM MIXIED STRATEGIES OF PLAYER A AND PLAYER B

The optimum mixed strategies of the playerA from the iteration 50 - 500 are obtained as

$$\begin{bmatrix} A1 & A2 & A3 & A4 & A5 & A6 & A7 & A8 & A9 & A10 & A11 & A12 & A13 & A14 & A15 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly the optimum mixed strategies of the player B from the iteration 50 - 500 are also obtained as

$$\begin{bmatrix} B1 & B2 & B3 & B4 & B5 & B6 & B7 & B8 & B9 & B10 & B11 & B12 & B13 & B14 & B15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

6. UPPER BOUNDS ,LOWER BOUNDS AND ERRORS AT ALL COMPUTATIONS

The obtained values of U.Bs and L.Bs in 15x15 game and the exiting errors are shown in the tables from Table (21) to Table (22).

Table 21.

U.B	Lower Bound									
Iterations: 50-500	50	100	150	200	250	300	350	400	450	500
421	412.6	416.8	418.2	418.9	419.32	419.6	419.8	419.95	420.06666	420.16
421	413.2	417.1	418.4	419.05	419.44	419.7	419.8857	420.025	420.13332	420.22
421	413.8	417.4	418.6	419.2	419.56	419.8	419.9714	420.1	420.19998	420.28
421	414.4	417.7	418.8	419.35	419.68	419.9	420.0571	420.175	420.26664	420.34
421	415	418	419	419.5	419.8	420	420.1429	420.25	420.3333	420.4
421	415.6	418.3	419.2	419.65	419.92	420.1	420.2286	420.325	420.39996	420.46
421	416.2	418.6	419.4	419.8	420.04	420.2	420.3143	420.4	420.46662	420.52
421	416.8	418.9	419.6	419.95	420.16	420.3	420.4	420.475	420.53328	420.58
421	417.4	419.2	419.8	420.1	420.28	420.4	420.4857	420.55	420.59994	420.64
421	418	419.5	420	420.25	420.4	420.5	420.5714	420.625	420.6666	420.7
421	418.6	419.8	420.2	420.4	420.52	420.6	420.6571	420.7	420.73326	420.76
421	419.2	420.1	420.4	420.55	420.64	420.7	420.7429	420.775	420.79992	420.82
421	419.8	420.4	420.6	420.7	420.76	420.8	420.8286	420.85	420.86658	420.88
421	420.4	420.7	420.8	420.85	420.88	420.9	420.9143	420.925	420.93324	420.94
421	421	421	421	421	421	421	421	421	421	421

Table 22.

Iterations and Errors									
50	100	150	200	250	300	350	400	450	500
8.4	4.2	2.8	2.1	1.68	1.4	1.2	1.05	0.93334	0.84
7.8	3.9	2.6	1.95	1.56	1.3	1.1143	0.975	0.86668	0.78
7.2	3.6	2.4	1.8	1.44	1.2	1.0286	0.9	0.80002	0.72
6.6	3.3	2.2	1.65	1.32	1.1	0.9429	0.825	0.73336	0.66
6	3	2	1.5	1.2	1	0.8571	0.75	0.6667	0.6
5.4	2.7	1.8	1.35	1.08	0.9	0.7714	0.675	0.60004	0.54
4.8	2.4	1.6	1.2	0.96	0.8	0.6857	0.6	0.53338	0.48
4.2	2.1	1.4	1.05	0.84	0.7	0.6	0.525	0.46672	0.42
3.6	1.8	1.2	0.9	0.72	0.6	0.5143	0.45	0.40006	0.36
3	1.5	1	0.75	0.6	0.5	0.4286	0.375	0.3334	0.3
2.4	1.2	0.8	0.6	0.48	0.4	0.3429	0.3	0.26674	0.24
1.8	0.9	0.6	0.45	0.36	0.3	0.2571	0.225	0.20008	0.18
1.2	0.6	0.4	0.3	0.24	0.2	0.1714	0.15	0.13342	0.12

0.6	0.3	0.2	0.15	0.12	0.1	0.0857	0.075	0.06676	0.06
0	0	0	0	0	0	0	0	0	0

6.1 Conclusions

(i). The optimum mixed pure strategies are same for player A and player B in each computation.

(ii). The value of the game is 421.In addition to this, the optimal mini max strategies coincide with the value at saddle position.

(iii).The value of upper bound is fixed in each scientific computation as 421 form initial computation to final computation.

(iv). In the case of lower bound, it is not equal to the value of the game. But it approaches to become the value of the game at last stage in the computation.

(v).The error is obtained initially with 8.4 and it tends step by step to zero at the end.

(iii).The game is concluded as a strictly determinable game. Since lower bound and upper bound are equal to the value of the game.

REFERENCES

[1] K.V.L.N.Acharyulu and Maddi.N.Murali Krishna,(2013). Some Remarkable Results In Row And Column Both Dominance Game With Rown's Algorithm, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol.3,Issue.1,Mar 2013,pp.139-150 .

[2] K.V.L.N.Acharyulu ,Maddi.N.Murali Krishna and K.Praveen kumar (2013), A Computational Study of a Large Network with Arithmetic Progression on Pessimistic Time Estimate, International Journal of Scientific and Innovative Mathematical Research (IJSIMR),Volume1,Issue2,October,PP121-133.

[3] K.V.L.N.Acharyulu and I.Pothu Raju (2014), Geometric Progression in Operations Research (PERT) – A Special Case Study, International Journal of Scientific and Innovative Mathematical Research (IJSIMR) , Volume 2, Issue 1, January, PP 83-93.

[4] K.V.L.N.Acharyulu and P.Prasanna Anjaneyulu (2014), A Special Case in Game Theory- A Scientific Study, International Journal of Scientific and Innovative Mathematical Research (IJSIMR) , Volume 2, Issue 2, March, PP 186-198.

[5] Billy E. Gillett (1979). Introduction to operations Research,Tata McGraw-Hill Edition.

[6] Levin, R. L., and R.B. Desjardins (1970).Theory of Games and Strategies, International Textbook Company, Scranton, Pa.

[7] Rapoport, A.(1966).Two Person Game Theory,The Essential Ideas, University of Michigan Press, Ann Arbor, Mich.

[8] Dresher, M., Games of Strategy (1961). Theory and Applications,Prentice-Hall, Inc., Englewood Cliffs,N.J.

[9] Raiffa, R. D.(1958). Games and Decisions,John Wiley & Sons, Inc., NewYork.

[10] McKinsey,J.C.C.(1952).Introduction of the Theory of Games,McGraw-Hill Book Company, NewYork.

AUTHORS' BIOGRAPHY

Venkata Lakshmi Narasimhacharyulu Kanduri: Who is known as Dr.K.V.L.N.Acharyulu is working as Associate Professor in the Department of Mathematics, Bapatla Engineering College, Bapatla which is a prestigious institution of Andhra Pradesh. He took his M.Phil. Degree in Mathematics from the University of Madras and stood in first Rank,R.K.M. Vivekananda College,Chennai. Nearly for the last twelve years he is rendering his services to the students and he is applauded by one and all for his best way of teaching. He has participated in some seminars and presented his papers on various topics. More than 70 articles were published in various International high impact factor Journals. He obtained his Ph.D from ANU under the able guidance of Prof. N.Ch.Pattabhi Ramacharyulu,NIT,Warangal. He is a Member of Various Professional Bodies and created three world records in research field. He received so many awards and rewards for his research excellency in the field of Mathematics.

I.Pothuraju is working as Assistant professor in Department of Mathematics, Bapatla Engineering College. He has two years of teaching experience. He is doing his M.phil under the guidance of Dr.K.V.L.N.Acharyulu.He did his M.Sc(Mathematics) in Bapatla Engineering College. He obtained MBA from Pydah College,Andhra University. He completed his B.Sc(M.P.C) in Bapatla College of Arts & Science. He has a zeal to invent new findings in Mathematics.