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Parametric Fitting of Non – Function – Like Curves by Minmaxion and Minaddition

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Abstract: *It has been the desire of every Scientist or Engineer to give precise mathematical formulations to the model he envisages. The model could represent some physical, biological or sociological process.*

Fitting curves to complex shapes has always been a challenging problem and continues to be so. While global fits to such data cannot serve the purpose, one usually thinks of piecewise fitting strategies. Even then, each curve segment may have a lot in detail and hence fitting an explicit equation to it may not produce the desired shape. Other options include (a) a parametric representation to a curve segment and (b) a multi-resolution representation using wavelets.

Given a discrete set of n points $(x_i, y_i), i = 1 : n$*one can guess what the curve that fits the points looks like. In many applications, the data that are captured from physical or biological experiments do not have simple structures in the sense that, if one were to fit a curve to this data, the curve would not appear as a function in the classical sense. Further, in most such situations, the genesis of the data is unknown.*

The modeller will have to address two questions: (a) ordering the points and (b) to give an analytic expression to the approximating curve. The resulting fitted curve must not only conform to statistical standards but also appear pleasing to the eye; it should essentially capture the shape of the data.

Parametric representation to a curve segment by a novel approach is being explored. The technique is purely data-guided and performs a dual role: ordering of points in the data set and parameterization leading to a fit of good quality.

This approach requires the use of two matrix operations namely minmaxion and minaddition. As these are nascent, their definitions and some of their relevant properties are given below.

Definition-I: Minmaxion

C*is min-max product of* A *and* B

$C \underline{\Delta} A \otimes B$ where $c_{ij} = min_x\{max(a_{ix}, b_{xj})\}$

Definition-Ii: Minaddition

$C$is min-ad product of A and B

$C \underline{\Delta} A \oplus B$where $c_{ij} = min_x\{max(a_{ix} + b_{xj})\}$

Both minmaxion and minaddition are similar to the usual matrix multiplication, satisfy the associative law, are non-commutative, satisfy the power law for square matrices and obey the transposition rule analogous to conventional matrix multiplication.

Another property of minmaxion and minaddition is "satiety" which holds in the case of zero diagonal matrices with non-negative entries. By satiety we mean, if A is a zero diagonal matrix of order n such that $A^{k+1} = A^k$ for some positive integer $k<n$, we say A^k is the satiated matrix of A.

The concepts of approachability distance and connective distance between node pairs, crucial to this procedure, is introduced through the satiety property of minmaxion and minaddition.

These new distances are instrumental in imputing an ordering among intermediate points connecting node pairs. Further, we propose the ordering index itself as the parameter for curve fitting.

As a test case this procedure has been applied on a data set. Ordering of the points as well as parametric fitting proved satisfactory, albeit for one class of curves possessing lineal shapes

Keywords and Phrases: *Analytic expression, parameterization, ordering of points, MINMAXION, MINADDITION, approachability distance, connective distance, ordering index.*

1. INTRODUCTION

Recognition of objects has been one of the challenges in several areas of image analysis like biomedical image analysis, biometrics, military target recognition and general computer vision.

There are many applications where image analysis can be reduced to the analysis of shapes. To describe shape through object boundary is a preliminary but crucial step in the overall description of the shape.

We have intuitive ideas about curves because of their striking visual nature. A curve, in general, has no simple mathematical definition. Given a discrete set of n data points (x_i, y_i), i=1:n, one can guess what the curve that fits the points should looks like. We fit the data either by means of interpolants or by approximating curves.

In many applications, the data that are captured from physical or biological experiments do not have simple structures in the sense that, if one were to fit a curve to this data, the curve would not appear as a *function* in the classical sense. Figure 1 shows two typical curves which are not function-like. A global explicit fit to such complicated shapes is impossible. These curves do not qualify to be called functions.

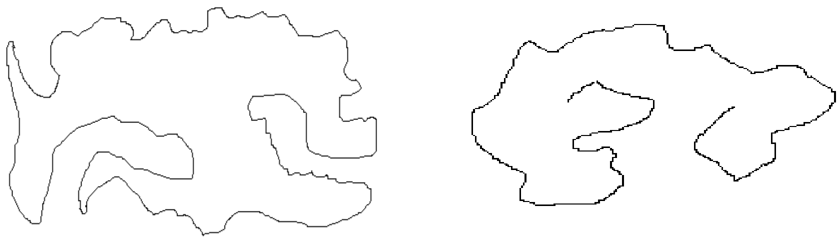


Figure 1. *Curves which are not functions*

One usual approach is to describe the overall shape by a piecewise approximation technique. Several strategies have been suggested which include segmenting the curves at points of high curvature and then going for a piecewise fit of each segment. One then may have to express such curve segments either implicitly, or through parameterization. The implicit form of representing curves does not have this limitation. Conic sections are classic examples where the implicit form has been successfully tried.

Fundamentally, one has to answer the question of *ordering* the points. This is particularly true of situations where we have no idea of how the data were generated.

The parametric form is preferred to fit closed and multiple valued curves. Since a point on a parametric curve is specified by a single value of a parameter, the parametric form is, in a sense, axis independent.

In the parametric form, each coordinate of a point on a curve is represented as a function of a single parameter t. For a two dimensional curve with t as a parameter, the Cartesian coordinates of a point on the curve are given by$x = x\ (t), y = y\ (t);\ a \le t \le b$.

There is no unique parametric representation of a curve. For example,

$$x = cos\theta\ ,\ 0 \le \theta \le \frac{\pi}{2}\ , \qquad y = sin\theta\ ,\ 0 \le \theta \le \frac{\pi}{2}$$

And

$$x = \frac{1 - t^2}{1 + t^2}\ ,\ 0 \le t \le 1, \qquad y = \frac{2t}{1 + t^2}\ , \qquad 0 \le t \le 1,$$

are two different parameterizations of the unit circular are in the first quadrant.

The curve end points and the length are fixed by the parameter range. Often it is convenient to normalize the parameter range for the curve segment of interest to$0 \le t \le\ 1$.

We introduce a new parameterization procedure. This procedure requires the use of two matrix operations namely MINMAXION and MINADDITION [3, 4, 5, 6, and 8].Since these are nascent, we shall first present their definitions and some of their relevant properties .Thereafter; we shall show how these operations will be useful in the context of curve parameterization.

2. MINMAXION AND MINADDITION

Definition I: (MINMAXION).$C$is the min-max product of A and B

$$C \triangleq A \otimes B \quad where\ c_{ij} = min_x\{max\ \ (a_{ix}, b_{xj})\}.$$

Definition II :(MINADDITION).CIs the min-ad product of A and B

$$C \triangleq A \oplus B \quad where\ c_{ij} = min_x\{max\ \ (a_{ix} +\ b_{xj})\}.$$

Both MINMAXION and MINADDITION are similar to the usual matrix multiplication, satisfy the associative law, are non-commutative, satisfy the power law for square matrices and obey the transposition rule analogous to conventional matrix multiplication.

Another property of minmaxion and minaddition is "satiety" which holds in the case of zero diagonal matrices with non-negative entries. By satiety we mean, if $A$is a zero diagonal matrix of order n such that $A^{k+1} = A^k$ for some positive integer$k < n$, we sayA^k is the satiated matrix of A. In fact, one can define satiated minmaxion and satiated minadditioneven when the zero-diagonal matrix $D = [dij]$ is not symmetric. Further, it is not necessary that the d_{ij} satisfy the usual metric laws, even though in the present context they are Euclidean distances.

2.1. MOTIVATION FOR THE MINMAXION AND MINADDITION OPERATIONS

Consider a network with n points where the direct distance between each point pair (i,j) isd_{ij}. As mentioned earlier, the d_{ij}'s need not be "metric". They need not be even symmetric. They can be any "scalars" allowing comparison (<,=,> between two d_{ij}'s) and addition. For instance, d_{ij} may be the time taken from hilltop i to the base j or the cost of going from $itoj$. Now consider a path ij through these points in r steps i.e. $a = x_1 \rightarrow x_2 \rightarrow \cdots . \rightarrow x_{r+1} = b$.The *approachability distance* for this path is defined as the largest of the distances between the consecutive point pairs along the path. Among all the paths of the same number of steps r, there will be at least one path for which the approachability distance is minimum. This is defined as the *connective distance* for the step length r.

The smallest among these$(r = 1 : .... : n - 1)$ distances is defined as the connective distance. It can be easily seen that the minmax power sequence is term-wise monotonic. The connective distance of step length r is given by $a^{(r)}_{i\,j}$ and the elements of the satiated minmaxion matrix give the (overall) connective lengths between the node pairs. Similarly, in case of minaddition, the satiated matrix gives the length of the total distance along the shortest paths between the ordered node pairs. One may refer to, Reddy [8].

2.2. Ordering the Points and Parameterization by MINMAXION and MINADDITION

Consider a test curve on which we take a discrete set of points. Once we have the co-ordinates of the point-pairs, we can compute inter-node distances d_{ij} (say, Euclidean distances) and store these distances in the distance matrix D.

$$D = [d_{ij}] = \begin{cases} 0, & i = j \\ > 0, & i \neq j \end{cases}$$

Let $S = D^*$be the minmax satiated matrix of D, i.e. $D^* = D^r = D^{r+1}$ for some $r < n$. The element d_{ij}^* of D^*gives the (r^{th}order) connective distance from i to j. Each of these paths will have a link of largest length. Then d_{ij}^* is the smallest among these largest links in the different paths. Let p_{ij}^* be the number of steps from i to j along this optimal path. The number and the actual path itself can be obtained by the use of minaddition.

One can now define the Direct Link Matrix P from the matrix S as follows.

$$P = [p_{ij}] = \begin{cases} 0 & i = j, \\ 1 & d_{ij} = s_{ij},\ i \neq j \\ \infty & otherwise \end{cases}$$

The minad satiated matrix of P, denoted by P^*called the step length matrix, gives the number of steps between point-pairs along these paths. Choosing a point-pair with largest step length, say αtoβ, one gets the path fromα toβ on which a relatively large number of points lie in an ordered fashion; the number of steps between any point-pair along this path will be less than this number and one can take this path as an arterial path along which many points lie in a well-defined sequence. If it so happens that, $P_{\alpha\beta}^* = (n-1) or P_{\alpha\beta}^* \sim (n-1)$ one may infer that nodes $\alpha to \beta$ are the end points of a long connective path. Since the sequence of points between $\alpha to \beta$ is now available, one can accept this sequence of points along this path as the appropriate *ordering* among the n points.

Ordering of points along a curve, in general a difficult problem by itself has now been addressed, particularly in the case of open curves, what remains to be tackled is curve parameterization.

We propose the *ordering index* of the connective path itself as the parameter t. The coordinates $x\ (t)\ and y\ (t)$ can now be fitted as functions of$'t'$. Of course, $'t'$ is in the ordinal scale; but as a first approximation, can be used as values in an interval scale.

The present study investigates curves which are not closed and are in a sense, convex.

Illustration. This is generated as 31 points on the curve $y = sign\left(\frac{x}{3}\right).sin(2.x)$in the range 0.1 : 0.2 : 0.61 and subjecting the curve to rotation through 36°, getting the *new* as $[u(i), v(j)], i = 0 : 30$. These values are presented as Table I(a) and (b) and are graphically represented in Figure II(a) and (b), respectively.

Consider the data set given in *Table I(a).* This set is plotted as a scatter plot in *Figure II(a)*. This set is actually obtained by evaluating y as the function $y = sign\left(\frac{x}{3}\right).sin(2.x)$ in the range 0.1:0.2: 0.61. *Figure II (b)* (and the corresponding Table I (b) are the plotted values of (u, v) got by a rotation of the data (x, y) by 36°. It is seen that *x* is not a function of *y*: neither u is a function of *v*.

Matrix I (a) gives *D*, the (squared) Euclidean distance matrix between the points in *Table I(a).*

Matrix I (b) gives $S = D^*$,*Minmax* satiated matrix for *D*.

Matrix I(c) is the *Minad* satiated matrix $P^*$for $P. P^*$ is the *direct link matrix*

From *Matrix I(c)*, it can be observed that the largest path length is 30 from $i = 1\ to j = 9$.The optimal (Minmax) path from 1 to 9 is

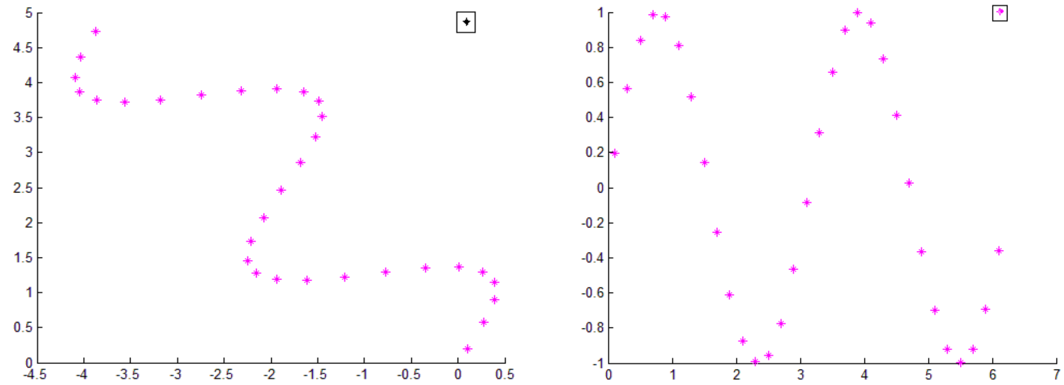
[1→11→3→4→29→23→12→30→2→15→14→27→28→7→17→16→20→24→26→10→22→21→6 → 5→18→31→13→19→8→ 25→9]

This path covers all the points 0, 1, . . ., 30 and hence is the arterial path. The sequence given by this path is x (t), y (t), t = 0 to 30, the same as the coordinates of the points in the sequence above. The ordering indices shown above can now be used to determine new parameters t_1 and t_2 as described below.

Table 1. (x,y): Set of 31 points on the curve$y = sign\left(\frac{x}{3}\right).sin(2.x)$in the range [0.1, 6.1] **(u,v):**Setofcorresponding 31 pointsafter the dataisrotatedby3ss

x	y
0.1000	0.1987
0.3000	0.5646
0.5000	0.8415
0.7000	0.9854
0.9000	0.9738
1.1000	0.8085
1.3000	0.5155
1.5000	0.1411
1.7000	-0.2555
1.9000	-0.6119
2.1000	-0.8716
2.3000	-0.9937
2.5000	-0.9589
2.7000	-0.7728
2.9000	-0.4646
3.1000	-0.0831
3.3000	0.3115
3.5000	0.6570
3.7000	0.8987
3.9000	0.9985
4.1000	0.9407
4.3000	0.7344
4.5000	0.4121
4.7000	0.0248
4.9000	-0.3665
5.1000	-0.6999
5.3000	-0.9228
5.5000	-1.0000
5.7000	-0.9193
5.9000	-0.6935
6.1000	-0.3582

u	v
0.5746	0.2805
3.7454	-3.8618
4.7244	-3.8753
3.8699	-1.6489
3.9104	-1.9333
1.1775	-1.6118
3.8828	-2.3116
3.8169	-2.7425
0.1977	0.1019
3.5216	-1.4477
2.0731	-2.0804
1.7301	-2.2122
1.1455	0.3858
3.2177	-1.5257
1.2251	-1.2060
1.1866	-1.9395
1.3547	-0.3471
1.2965	-0.7675
1.3005	0.2589
2.4591	-1.8894
3.7146	-3.5639
3.7488	-3.1766
0.8991	0.3869
2.8529	-1.6876
1.3651	0.0075
4.0710	-4.0941
4.3656	-4.0290
1.2767	-2.1558
1.4589	-2.2452
3.8618	-4.0418
3.7421	-1.4845



(a) (b)

Figure 2.

The curve fitting of u and v as functions of t is in two stages. Initially, t is the ordinal index and we take the coordinates of the beginning an end points in the ordered curve as (u_b, u_e) and (v_b, v_e) respectively. We then convert this index t which has the range $0:30\, tot_1 = u_b + (0:30) \times (u_e - u_b)/3$and $t_2 = v_b + (0:30) \times (v_e - v_b)/30$. Using these t_1 and t_2 as independent variables and getting the co-ordinates of u(t_1) and v(t_2) as the curves to be fitted to, we go for successive polynomial fits. A scatter plot of the vectors *x* and *y* gives the theoretical fitted approximating curves.

The fitted polynomials $u(t_1)$, $v(t_2)$ are given as parametric equations below. These are plotted, along with the residuals in FigureIII (a) and (b).

In the range $0.1977 \leq t_1 \leq 4.7244$

$$u(t_1) = \begin{cases} 0.1731\, t_1 + 0.9490 \\ 0.1546 t_1^2 + 0.9705\, t_1 - 0.0044 \\ 0.7766 t_1^3 - 0.3826 t_1^2 + 0.6663 t_1 + 0.0908 \\ 0.6396 t_1^4 + 0.0759 t_1^3 + 0.2700 t_1^2 + 0.0323 t_1 - 0.0125 \\ -1.5276 t_1^5 + 10.0261 t_1^4 - 12.6001 t_1^3 + 6.7478 t_1^2 - 1.5278 t_1 + 0.1231 \end{cases}$$

Similarly, in the range $-3.8753 \leq t_2 \leq 0.1019$

$$v(t_2) = \begin{cases} 0.2162 t_2 + 1.0799 \\ 0.1994 t_2^2 + 1.0506\, t_2 - 0.0078 \\ 0.5584 t_2^3 + 2.5530 t_2^2 + 1.0355 t_2 + 0.1843 \\ 0.5192 t_2^4 + 2.1697 t_2^3 + 0.5225 t_2^2 - 0.0337 t_2 - 0.0289 \\ 0.3549 t_2^5 - 3.5900 t_2^4 - 12.6836 t_2^3 - 10.0490 t_2^2 - 3.0827 t_2 - 0.3237 \end{cases}$$

0	4.17	13.11	10.25	6.593	2.813	7.809	2.657	16.01	14.53	3.897	0.107	1.72	0.306	7.251	9.06	17.7	7.572	3.991	0.666	8.245	3.944	0.48	1.631	3.514	4.072	5.522	11.66	1.022	0.167	0.727
4.17	0	6.746	4.401	0.277	0.159	1.06	5.783	7.584	7.325	11.77	4.297	0.708	4.165	0.441	3.007	7.791	0.646	7.948	2.684	1.84	10.25	3.611	4.86	6.86	9.034	0.098	5.736	1.618	4.073	4.266
13.11	6.746	0	0.25	6.554	6.56	2.994	22.63	0.254	0.047	30.6	11.58	6.601	10.24	5.726	0.761	0.772	4.448	26.98	7.892	1.69	29.49	9.002	19.94	25.06	28.39	6.845	0.046	7.047	14.99	17.3
10.25	4.401	0.25	0	4.325	4.263	1.597	18.28	0.64	0.408	25.81	9.029	4.38	7.926	3.692	0.151	1.117	2.697	22.2	5.765	0.685	24.64	6.827	15.92	20.44	23.54	4.515	0.09	4.895	11.76	13.67
6.593	0.277	6.554	4.325	0	0.832	0.704	7.876	6.863	6.918	15.08	6.737	1.81	6.529	0.043	2.863	6.681	0.23	10.24	4.578	1.588	13.15	5.792	6.993	9.001	11.58	0.05	5.651	3.141	6.413	6.495
2.813	0.159	6.56	4.263	0.832	0	1.45	5.087	7.77	7.275	10.25	2.84	0.196	2.704	1.036	3.02	8.289	1.179	7.214	1.536	2.042	9.064	2.254	4.042	6.199	8.121	0.505	5.524	0.762	2.882	3.269
7.809	1.06	2.994	1.597	0.704	1.45	0	11.72	3.182	3.213	19.41	7.408	2.205	6.816	0.439	0.766	3.153	0.144	14.77	4.644	0.19	17.66	5.88	10.25	13.28	16.18	0.877	2.422	3.329	8.286	9.073
2.657	5.783	22.63	18.28	7.876	5.087	11.72	0	25.3	24.15	1.372	3.823	4.795	4.688	9.017	15.82	26.36	10.25	0.191	5.06	13.57	0.699	5.084	0.126	0.067	0.361	6.768	20.64	4.861	1.492	0.605
16.01	7.584	0.254	0.64	6.863	7.77	3.182	25.3	0	0.091	34.44	14.47	8.213	13.05	5.911	1.107	0.152	4.599	29.86	10.25	1.956	32.94	11.63	22.62	27.78	31.52	7.375	0.413	9.052	17.83	20.06
14.53	7.325	0.047	0.408	6.918	7.275	3.213	24.15	0.091	0	32.61	12.96	7.459	11.57	6.02	0.946	0.475	4.695	28.63	9.021	1.891	31.36	10.24	21.42	26.62	30.14	7.305	0.16	8.047	16.44	18.76
3.897	11.77	30.6	25.81	15.08	10.25	19.41	1.372	34.44	32.61	0	5.145	9.079	6.262	16.55	23.36	36.31	17.93	0.979	7.704	21.19	0.174	7.1	1.54	1.241	0.573	13.45	28.3	8.28	2.766	1.963
0.107	4.297	11.58	9.029	6.737	2.84	7.408	3.823	14.47	12.96	5.145	0	1.622	0.055	7.284	8.095	16.26	7.419	5.409	0.37	7.563	5.303	0.168	2.564	4.846	5.495	5.694	10.24	0.806	0.539	1.386
1.72	0.708	6.601	4.38	1.81	0.196	2.205	4.795	8.213	7.459	9.079	1.622	0	1.469	2.048	3.32	9.076	2.108	6.902	0.636	2.572	8.26	1.127	3.598	5.957	7.615	1.324	5.545	0.186	1.99	2.61
0.306	4.165	10.24	7.926	6.529	2.704	6.816	4.688	13.05	11.57	6.262	0.055	1.469	0	6.982	7.153	14.84	6.986	6.477	0.209	6.797	6.428	0.041	3.278	5.831	6.608	5.541	9.005	0.64	0.905	1.928
7.251	0.441	5.726	3.692	0.043	1.036	0.439	9.017	5.911	6.02	16.55	7.284	2.048	6.982	0	2.349	5.687	0.083	11.56	4.896	1.199	14.58	6.169	8.021	10.24	12.97	0.162	4.913	3.415	7.191	7.399
9.06	3.007	0.761	0.151	2.863	3.02	0.766	15.82	1.107	0.946	23.36	8.095	3.32	7.153	2.349	0	1.44	1.572	19.47	5.005	0.193	22.03	6.111	13.74	17.8	20.82	3.041	0.469	4.01	10.25	11.82
17.7	7.791	0.772	1.117	6.681	8.289	3.153	26.36	0.152	0.475	36.31	16.26	9.076	14.84	5.687	1.44	0	4.434	30.97	11.73	2.107	34.49	13.32	23.8	28.81	32.8	7.34	0.959	10.25	19.37	21.41
7.572	0.646	4.448	2.697	0.23	1.179	0.144	10.25	4.599	4.695	17.93	7.419	2.108	6.986	0.083	1.572	4.434	0	13.02	4.832	0.664	16.03	6.107	9.048	11.62	14.45	0.387	3.746	3.398	7.74	8.192
3.991	7.948	26.98	22.2	10.24	7.214	14.77	0.191	29.86	28.63	0.979	5.409	6.902	6.477	11.56	19.47	30.97	13.02	0	7.091	16.92	0.337	7.021	0.581	0.04	0.061	9.008	24.8	6.943	2.54	1.353
0.666	2.684	7.892	5.765	4.578	1.536	4.644	5.06	10.25	9.021	7.704	0.37	0.636	0.209	4.896	5.005	11.73	4.832	7.091	0	4.636	7.549	0.075	3.62	6.291	7.446	3.794	6.783	0.135	1.268	2.275
8.245	1.84	1.69	0.685	1.588	2.042	0.19	13.57	1.956	1.891	21.19	7.563	2.572	6.797	1.199	0.193	2.107	0.664	16.92	4.636	0	19.65	5.808	11.8	15.34	18.31	1.764	1.258	3.479	9.079	10.25
3.944	10.25	29.49	24.64	13.15	9.064	17.66	0.699	32.94	31.36	0.174	5.303	8.26	6.428	14.58	22.03	34.49	16.03	0.337	7.549	19.65	0	7.161	1.002	0.527	0.117	11.67	27.21	7.819	2.633	1.619
0.48	3.611	9.002	6.827	5.792	2.254	5.88	5.084	11.63	10.24	7.1	0.168	1.127	0.041	6.169	6.111	13.32	6.107	7.021	0.075	5.808	7.161	0	3.614	6.296	7.241	4.891	7.841	0.404	1.135	2.21
1.631	4.86	19.94	15.92	6.993	4.042	10.25	0.126	22.62	21.42	1.54	2.564	3.598	3.278	8.021	13.74	23.8	9.048	0.581	3.62	11.8	1.002	3.614	0	0.37	0.746	5.907	18.07	3.521	0.755	0.18
3.514	6.86	25.06	20.44	9.001	6.199	13.28	0.067	27.78	26.62	1.241	4.846	5.957	5.831	10.24	17.8	28.81	11.62	0.04	6.291	15.34	0.527	6.296	0.37	0	0.178	7.846	22.96	6.069	2.151	1.053
4.072	9.034	28.39	23.54	11.58	8.121	16.18	0.361	31.52	30.14	0.573	5.495	7.615	6.608	12.97	20.82	32.8	14.45	0.061	7.446	18.31	0.117	7.241	0.746	0.178	0	10.24	26.15	7.466	2.643	1.49
5.522	0.098	6.845	4.515	0.05	0.505	0.877	6.768	7.375	7.305	13.45	5.694	1.324	5.541	0.162	3.041	7.34	0.387	9.008	3.794	1.764	11.67	4.891	5.907	7.846	10.24	0	5.878	2.499	5.332	5.414
11.66	5.736	0.046	0.09	5.651	5.524	2.422	20.64	0.413	0.16	28.3	10.24	5.545	9.005	4.913	0.469	0.959	3.746	24.8	6.783	1.258	27.21	7.841	18.07	22.96	26.15	5.878	0	5.97	13.41	15.57
1.022	1.618	7.047	4.895	3.141	0.762	3.329	4.861	9.052	8.047	8.28	0.806	0.186	0.64	3.415	4.01	10.25	3.398	6.943	0.135	3.479	7.819	0.404	3.521	6.069	7.466	2.499	5.97	0	1.484	2.327
0.167	4.073	14.99	11.76	6.413	2.882	8.286	1.492	17.83	16.44	2.766	0.539	1.99	0.905	7.191	10.25	19.37	7.74	2.54	1.268	9.079	2.633	1.135	0.755	2.151	2.643	5.332	13.41	1.484	0	0.197
0.727	4.266	17.3	13.67	6.495	3.269	9.073	0.605	20.06	18.76	1.963	1.386	2.61	1.928	7.399	11.82	21.41	8.192	1.353	2.275	10.25	1.619	2.21	0.18	1.053	1.49	5.414	15.57	2.327	0.197	0

Matrix I (a)

Matrix of (squared) distances between the 31 points in pairs

0	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.107	0.186	0.107	0.196	0.196	0.196	0.196	0.197	0.107	0.196	0.197	0.107	0.197	0.197	0.197	0.196	0.196	0.135	0.167	0.197
0.196	0	0.193	0.193	0.098	0.159	0.144	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.098	0.193	0.193	0.098	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.098	0.193	0.196	0.196	0.197
0.196	0.193	0	0.09	0.193	0.193	0.193	0.197	0.091	0.047	0.197	0.196	0.196	0.196	0.193	0.151	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.046	0.196	0.196	0.197
0.196	0.193	0.09	0	0.193	0.193	0.193	0.197	0.091	0.09	0.197	0.196	0.196	0.196	0.193	0.151	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.09	0.196	0.196	0.197
0.196	0.098	0.193	0.193	0	0.159	0.144	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.043	0.193	0.193	0.083	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.05	0.193	0.196	0.196	0.197
0.196	0.159	0.193	0.193	0.159	0	0.159	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.159	0.193	0.193	0.159	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.159	0.193	0.196	0.196	0.197
0.196	0.144	0.193	0.193	0.144	0.159	0	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.144	0.193	0.193	0.144	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.144	0.193	0.196	0.196	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.067	0.197	0.197	0.117	0.197	0.126	0.067	0.067	0.197	0.197	0.197	0.197	0.18
0.196	0.193	0.091	0.091	0.193	0.193	0.193	0.197	0	0.091	0.197	0.196	0.196	0.196	0.193	0.151	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.091	0.196	0.196	0.197
0.196	0.193	0.047	0.09	0.193	0.193	0.193	0.197	0.091	0	0.197	0.196	0.196	0.196	0.193	0.151	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.047	0.196	0.196	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.174	0.197	0.197	0	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.174	0.197	0.197	0.174	0.197	0.174	0.174	0.174	0.197	0.197	0.197	0.197	0.18
0.107	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0	0.186	0.055	0.196	0.196	0.196	0.196	0.197	0.075	0.196	0.197	0.055	0.197	0.197	0.197	0.196	0.196	0.135	0.167	0.197
0.186	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.186	0	0.186	0.196	0.196	0.196	0.196	0.197	0.186	0.196	0.197	0.186	0.197	0.197	0.197	0.196	0.196	0.186	0.186	0.197
0.107	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.055	0.186	0	0.196	0.196	0.196	0.196	0.197	0.075	0.196	0.197	0.041	0.197	0.197	0.197	0.196	0.196	0.135	0.167	0.197
0.196	0.098	0.193	0.193	0.043	0.159	0.144	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0	0.193	0.193	0.083	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.05	0.193	0.196	0.196	0.197
0.196	0.193	0.151	0.151	0.193	0.193	0.193	0.197	0.151	0.151	0.197	0.196	0.196	0.196	0.193	0	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.151	0.196	0.196	0.197
0.196	0.193	0.152	0.152	0.193	0.193	0.193	0.197	0.152	0.152	0.197	0.196	0.196	0.196	0.193	0.152	0	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0.152	0.196	0.196	0.197
0.196	0.098	0.193	0.193	0.083	0.159	0.144	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.083	0.193	0.193	0	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0.083	0.193	0.196	0.196	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.067	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0	0.197	0.197	0.117	0.197	0.126	0.04	0.061	0.197	0.197	0.197	0.197	0.18
0.107	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.075	0.186	0.075	0.196	0.196	0.196	0.196	0.197	0	0.196	0.197	0.075	0.197	0.197	0.197	0.196	0.196	0.135	0.167	0.197
0.196	0.19	0.193	0.193	0.19	0.19	0.19	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.19	0.193	0.193	0.19	0.197	0.196	0	0.197	0.196	0.197	0.197	0.197	0.19	0.193	0.196	0.196	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.117	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.117	0.197	0.197	0	0.197	0.126	0.117	0.117	0.197	0.197	0.197	0.197	0.18
0.107	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.055	0.186	0.041	0.196	0.196	0.196	0.196	0.197	0.075	0.196	0.197	0	0.197	0.197	0.197	0.196	0.196	0.135	0.167	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.126	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.126	0.197	0.197	0.126	0.197	0	0.126	0.126	0.197	0.197	0.197	0.197	0.18
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.067	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.04	0.197	0.197	0.117	0.197	0.126	0	0.061	0.197	0.197	0.197	0.197	0.18
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.067	0.197	0.197	0.174	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.061	0.197	0.197	0.117	0.197	0.126	0.061	0	0.197	0.197	0.197	0.197	0.18
0.196	0.098	0.193	0.193	0.05	0.159	0.144	0.197	0.193	0.193	0.197	0.196	0.196	0.196	0.05	0.193	0.193	0.083	0.197	0.196	0.19	0.197	0.196	0.197	0.197	0.197	0	0.193	0.196	0.196	0.197
0.196	0.193	0.046	0.09	0.193	0.193	0.193	0.197	0.091	0.047	0.197	0.196	0.196	0.196	0.193	0.151	0.152	0.193	0.197	0.196	0.193	0.197	0.196	0.197	0.197	0.197	0.193	0	0.196	0.196	0.197
0.135	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.135	0.186	0.135	0.196	0.196	0.196	0.196	0.197	0.135	0.196	0.197	0.135	0.197	0.197	0.197	0.196	0.196	0	0.167	0.197
0.167	0.196	0.196	0.196	0.196	0.196	0.196	0.197	0.196	0.196	0.197	0.167	0.186	0.167	0.196	0.196	0.196	0.196	0.197	0.167	0.196	0.197	0.167	0.197	0.197	0.197	0.196	0.196	0.167	0	0.197
0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.18	0.197	0.197	0.18	0.197	0.197	0.197	0.197	0.197	0.197	0.197	0.18	0.197	0.197	0.18	0.197	0.18	0.18	0.18	0.197	0.197	0.197	0.197	0

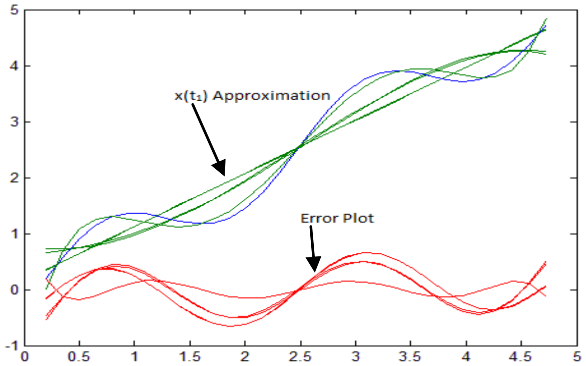
Matrix I (b)

S=D ; The Satiated Minmax Matrix*

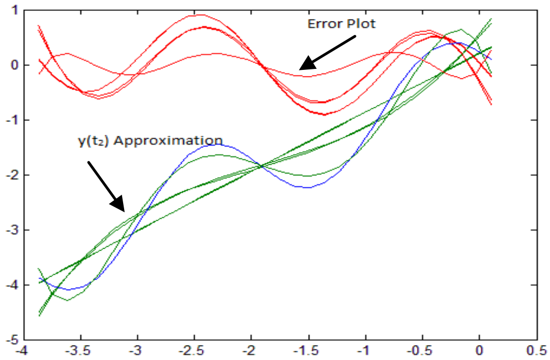
0	25	29	19	20	8	21	22	1	17	13	12	2	16	7	9	5	6	3	14	24	23	1	15	4	27	28	10	11	26	18
25	0	4	6	5	17	4	3	26	8	12	13	23	9	18	16	20	19	22	11	1	2	24	10	21	2	3	15	14	1	7
29	4	0	10	9	21	8	7	30	12	16	17	27	13	22	20	24	23	26	15	5	6	28	14	25	2	1	19	18	3	11
19	6	10	0	1	11	2	3	20	2	6	7	17	3	12	10	14	13	16	5	5	4	18	4	15	8	9	9	8	7	1
20	5	9	1	0	12	1	2	21	3	7	8	18	4	13	11	15	14	17	6	4	3	19	5	16	7	8	10	9	6	2
8	17	21	11	12	0	13	14	9	9	5	4	6	8	1	1	3	2	5	6	16	15	7	7	4	19	20	2	3	18	10
21	4	8	2	1	13	0	1	22	4	8	9	19	5	14	12	16	15	18	7	3	2	20	6	17	6	7	11	10	5	3
22	3	7	3	2	14	1	0	23	5	9	10	20	6	15	13	17	16	19	8	2	1	21	7	18	5	6	12	11	4	4
1	26	30	20	21	9	22	23	0	18	14	13	3	17	8	10	6	7	4	15	25	24	2	16	5	28	29	11	12	27	19
17	8	12	2	3	9	4	5	18	0	4	5	15	1	10	8	12	11	14	3	7	6	16	2	13	10	11	7	6	9	1
13	12	16	6	7	5	8	9	14	4	0	1	11	3	6	4	8	7	10	1	11	10	12	2	9	14	15	3	2	13	5
12	13	17	7	8	4	9	10	13	5	1	0	10	4	5	3	7	6	9	2	12	11	11	3	8	15	16	2	1	14	6
2	23	27	17	18	6	19	20	3	15	11	10	0	14	5	7	3	4	1	12	22	21	1	13	2	25	26	8	9	24	16
16	9	13	3	4	8	5	6	17	1	3	4	14	0	9	7	11	10	13	2	8	7	15	1	12	11	12	6	5	10	2
7	18	22	12	13	1	14	15	8	10	6	5	5	9	0	2	2	1	4	7	17	16	6	8	3	20	21	3	4	19	11
9	16	20	10	11	1	12	13	10	8	4	3	7	7	2	0	4	3	6	5	15	14	8	6	5	18	19	1	2	17	9
5	20	24	14	15	3	16	17	6	12	8	7	3	11	2	4	0	1	2	9	19	18	4	10	1	22	23	5	6	21	13
6	19	23	13	14	2	15	16	7	11	7	6	4	10	1	3	1	0	3	8	18	17	5	9	2	21	22	4	5	20	12
3	22	26	16	17	5	18	19	4	14	10	9	1	13	4	6	2	3	0	11	21	20	2	12	1	24	25	7	8	23	15
14	11	15	5	6	6	7	8	15	3	1	2	12	2	7	5	9	8	11	0	10	9	13	1	10	13	14	4	3	12	4
24	1	5	5	4	16	3	2	25	7	11	12	22	8	17	15	19	18	21	10	0	1	23	9	20	3	4	14	13	2	6
23	2	6	4	3	15	2	1	24	6	10	11	21	7	16	14	18	17	20	9	1	0	22	8	19	4	5	13	12	3	5
1	24	28	18	19	7	20	21	2	16	12	11	1	15	6	8	4	5	2	13	23	22	0	14	3	26	27	9	10	25	17
15	10	14	4	5	7	6	7	16	2	2	3	13	1	8	6	10	9	12	1	9	8	14	0	11	12	13	5	4	11	3
4	21	25	15	16	4	17	18	5	13	9	8	2	12	3	5	1	2	1	10	20	19	3	11	0	23	24	6	7	22	14
27	2	2	8	7	19	6	5	28	10	14	15	25	11	20	18	22	21	24	13	3	4	26	12	23	0	1	17	16	1	9
28	3	1	9	8	20	7	6	29	11	15	16	26	12	21	19	23	22	25	14	4	5	27	13	24	1	0	18	17	2	10
10	15	19	9	10	2	11	12	11	7	3	2	8	6	3	1	5	4	7	4	14	13	9	5	6	17	18	0	1	16	8
11	14	18	8	9	3	10	11	12	6	2	1	9	5	4	2	6	5	8	3	13	12	10	4	7	16	17	1	0	15	7
26	1	3	7	6	18	5	4	27	9	13	14	24	10	19	17	21	20	23	12	2	3	25	11	22	1	2	16	15	0	8
18	7	11	1	2	10	3	4	19	1	5	6	16	2	11	9	13	12	15	4	6	5	17	3	14	9	10	8	7	8	0

Matrix I (c)

P The satiated Minaddition matrix for P*



(a)



(b)

Figure 3. *(a) and (b): Polynomial approximations to $u(t_1)$ and $v(t_2)$ up to the fifth order (Errors shown in red)*

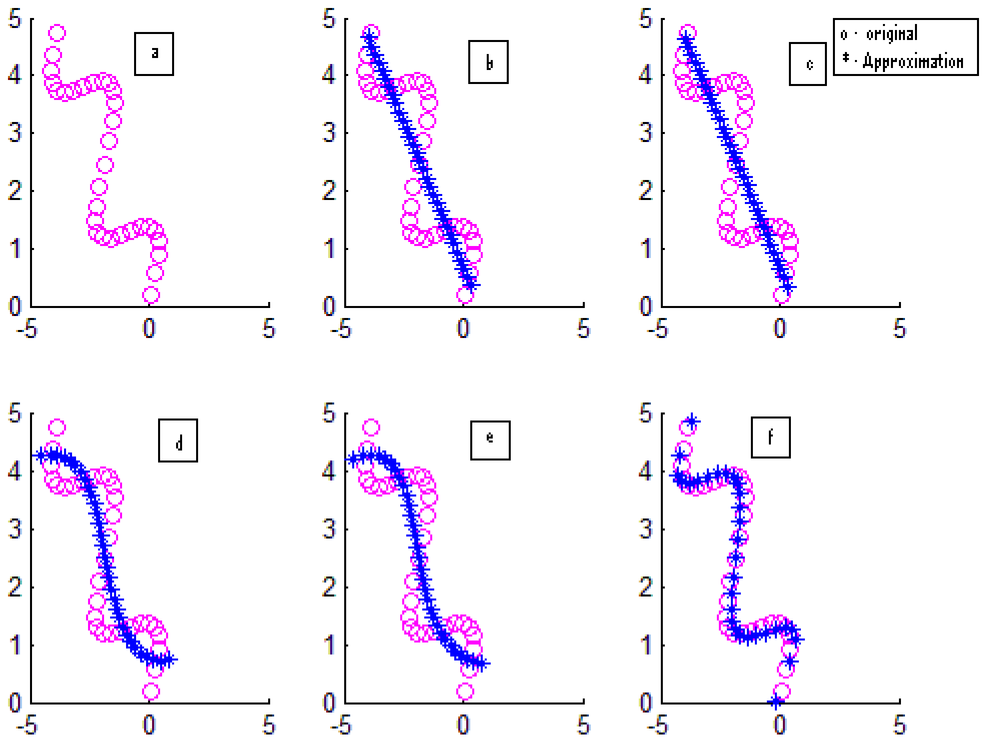


Figure 4.

Progress of the approximation scheme from top left (Original Test Curve (Shown in bubbles) followed by its linear, quadratic, cubic, quartic and quintic approximations- from top left

3. Scope and Future Study

In the case when curves do not appear function-like, we could make an orthogonal transformation on the *(x ,y)* co-ordinates to choose new co-ordinates for the points such that along one co-ordinate axis, there is maximum spread and along the other, a minimum spread. This is easily achieved by a suitable principal component analysis [PCA].Using the first PC score (ξ) as the 'independent' variable, one can fit the second PC score 'η' as a function of 'ξ', hopefully which can be a function. This latter approach can be very rewarding. This, of course can fail if the curve is a closed curve or a non-convex curve.

As already noted, in dealing with complex curves, for instance, self-intersecting curves, non-convex curves or curves having cusps, the ordering of the points would be requiring more local information , particularly as one approaches across over point or a cusp. If one were to summarize curves as complex as in *figure1*, a global fit would be impossible; a piecewise approximation using parametric curves appears the only way out. Fitting parametric curves for each segment separately and then joining up the segments at the knots by adjusting derivatives of appropriate orders can be thought of, as is done in the case of splines.

It is also to be noted that this approach is applicable even to points in three or higher dimensional space where a meaningful distance metric can be defined so that Minmaxion becomes applicable. A tangentially relevant reference is [7] where the matrix operations are used to recognize patterns in a set of points.

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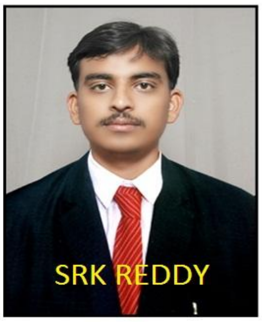

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