

A Special Case in Game Theory- A Scientific Study

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Abstract: *A special case of game theory problem is discussed with the help of Brown's Algorithm in this article. The problem consists the dominance nature for both rows and columns. It is established with the strategy of increasing in an action B1 to B15 of player B with the addition of successive natural numbers according to the influences of action A1 to A15 of Player A. few notable conclusions are obtained by computing maximum number of possible iterations in the classical Java program. The errors are calculated for each computation and the Lower bounds and Upper bounds are also computed.*

Keywords: *Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound*

AMS Classification: 91A05, 91A18, 91A43, 91A90

1. INTRODUCTION

The concepts of game theory were initiated in 20th century. The flavor of mathematical treatment in games took place mainly in 1944 by John Von Neumann and Morgenstern. The theory of games was linked with economic behavior. The fundamental idea of minimization of the maximum loss was utilized by minimax principle. Almost all competitive problems can be handled and analyzed successfully with the concepts of game premise. In addition to this, artificially designed games provide some suitable solutions as a model for the existence of real situations.

K.V.L.N.Acharyulu et.al[1-3] obtained some fruitful results in some peculiar cases of game theory in their earlier work. McKinsey [9] discussed the theory of Games in 1952. Raiffa, R. D [8] established the nature of games and possible decisions in 1958. Later Drescher, M [7] developed the strategies and applications of game theory in 1961. Afterwards Rapoport [6], Levin and Desjardins [5] opened new ears in the field of game theory. Billy E. Gillett [4] originated the many concepts and explained how to solve the large size of problems in the games by using Brown's algorithm.

The authors considered a 15x15 game problem which is a special case of game theory and discussed with the help of Brown's Algorithm. The problem consists the dominance nature for both the rows and columns. The principle is adopted for constructing this model by increasing in an action B1 to B15 of player B with the addition of successive natural numbers according to the influences of action A1 to A15 of Player A. some results are found by computing maximum number of possible iterations. The traced results are specified in the conclusions. The errors are also identified in each iteration and tabulated in a table. Lower bounds and Upper bounds are calculated in each iteration for classifying the nature of the game. The maximum possible iterations have been computed to obtain the best optimum mixed strategies for the players. The iterations are computed from 50th iteration to 500th iteration. The authors used Brown's algorithm with the help of programming language of Java for this investigation. The influences among the actions of Player A and the actions of Player B are observed.

2. BASIC FORMATION OF 15x15 GAME

A special game is framed with 15 rows and 15 columns of player A & Player B with all 15 possible opposing actions on one and another. One player chooses only one single action from his/her set possible actions. It comprises of fifteen possible actions of A i.e A1,A2,A3,A4, A5,A6,A7,A8,A9,A10, A11,A12,A13,A14,A15 which will effect on the other fifteen possible actions of player B i.e B1,B2,B3, B4,B5, B6,B7,B8,B9,B10,B11, B12,B13, B14,B15. The pay off matrix of constructed game having the size of 15x15 is given below in matrix representation form.

		Player B														
		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
		32	34	36	38	40	42	44	46	48	50	52	54	56	58	60
		62	64	66	68	70	72	74	76	78	80	82	84	86	88	90
		92	94	96	98	100	102	104	106	108	110	112	114	116	118	120
		122	124	126	128	130	132	134	136	138	140	142	144	146	148	150
		152	154	156	158	160	162	164	166	168	170	172	174	176	178	180
		182	184	186	188	190	192	194	196	198	200	202	204	206	208	210
Player A		212	214	216	218	220	222	224	226	228	230	232	234	236	238	240
		242	244	246	248	250	252	254	256	258	260	262	264	266	268	270
		272	274	276	278	280	282	284	286	288	290	292	294	296	298	300
		302	304	306	308	310	312	314	316	318	320	322	324	326	328	330
		332	334	336	338	340	342	344	346	348	350	352	354	356	358	360
		362	364	366	368	370	372	374	376	378	380	382	384	386	388	390
		392	394	396	398	400	402	404	406	408	410	412	414	416	418	420
		422	424	426	428	430	432	434	436	438	440	442	444	446	448	450

3. MATERIAL AND METHODS

The authors adopted Brown’s algorithm to solve this special case of 15x15 game in which row and columns both dominated Brown’s Algorithm.

Step 1: Player A chooses one of the possible actions(Ai₁) from A1-A15 to play, and Player B then plays with the possible action Bj₁ corresponding to the smallest element in the selected action Ai₁.

Step 2: Player A then picks out the possible action (Ai₂) from A1 - A15 to play corresponding to the largest element in the possible action (Bj₁) selected by Player B in step 1.

Step 3: Player B sums the actions of Player A has played thus far, and plays with the possible action of Bj₂ corresponding to a smallest sum element.

Step 4: Player A sums the actions of Player B has played thus far, and plays the possible action (Ai₃) corresponding to a largest sum element. After the required iterations are computed, then go to step 5; otherwise, come back to step 3.

Step 5: Compute an upper and lower bound $\underline{\gamma}$ and $\overline{\gamma}$ respectively.

$$\underline{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad \text{and} \quad \overline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

Step 6: Let Xi be the portion of the time Player A played row i with i=1,2,...,m and let Yi be the proportion of the time Player B played column j with j=1,2,...,n. These strategies approximate the optimal mini max strategies. Upper and Lower bounds on the value of the game where $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ are calculated in step 5. The Process completes.

4. RESULTS

The game is determined by Brown's algorithm to obtain the best optimum mixed strategies for both the players from 50th iteration to 500th iteration with the aid of Java Program. The effects on all possible actions of player A from the player B are given in the following tables from Table (1) to Table (20).

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Table-1. Player A vs. Player B at 50th Iteration from Action A1 to A8

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
100	20680	20710	20740	20770	20800	20830	20860	20890
1600	20780	20810	20840	20870	20900	20930	20960	20990
3100	20880	20910	20940	20970	21000	21030	21060	21090
4600	20980	21010	21040	21070	21100	21130	21160	21190
6100	21080	21110	21140	21170	21200	21230	21260	21290
7600	21180	21210	21240	21270	21300	21330	21360	21390
9100	21280	21310	21340	21370	21400	21430	21460	21490
10600	21380	21410	21440	21470	21500	21530	21560	21590
12100	21480	21510	21540	21570	21600	21630	21660	21690
13600	21580	21610	21640	21670	21700	21730	21760	21790
15100	21680	21710	21740	21770	21800	21830	21860	21890
16600	21780	21810	21840	21870	21900	21930	21960	21990
18100	21880	21910	21940	21970	22000	22030	22060	22090
19600	21980	22010	22040	22070	22100	22130	22160	22190
21100	22080	22110	22140	22170	22200	22230	22260	22290

Table-2. Player A vs. Player B at 50th Iteration from Action A9 to A15

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
100	20920	20950	20980	21010	21040	21070	21100
1600	21020	21050	21080	21110	21140	21170	21200
3100	21120	21150	21180	21210	21240	21270	21300
4600	21220	21250	21280	21310	21340	21370	21400
6100	21320	21350	21380	21410	21440	21470	21500
7600	21420	21450	21480	21510	21540	21570	21600
9100	21520	21550	21580	21610	21640	21670	21700
10600	21620	21650	21680	21710	21740	21770	21800
12100	21720	21750	21780	21810	21840	21870	21900
13600	21820	21850	21880	21910	21940	21970	22000
15100	21920	21950	21980	22010	22040	22070	22100
16600	22020	22050	22080	22110	22140	22170	22200
18100	22120	22150	22180	22210	22240	22270	22300
19600	22220	22250	22280	22310	22340	22370	22400
21100	22320	22350	22380	22410	22440	22470	22500

Table-3: Player A Vs Player B at 100th Iteration from Action A1 to A8

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
200	41780	41810	41840	41870	41900	41930	41960	41990
3200	41980	42010	42040	42070	42100	42130	42160	42190
6200	42180	42210	42240	42270	42300	42330	42360	42390
9200	42380	42410	42440	42470	42500	42530	42560	42590
12200	42580	42610	42640	42670	42700	42730	42760	42790
15200	42780	42810	42840	42870	42900	42930	42960	42990
18200	42980	43010	43040	43070	43100	43130	43160	43190
21200	43180	43210	43240	43270	43300	43330	43360	43390
24200	43380	43410	43440	43470	43500	43530	43560	43590
27200	43580	43610	43640	43670	43700	43730	43760	43790

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30200	43780	43810	43840	43870	43900	43930	43960	43990
33200	43980	44010	44040	44070	44100	44130	44160	44190
36200	44180	44210	44240	44270	44300	44330	44360	44390
39200	44380	44410	44440	44470	44500	44530	44560	44590
42200	44580	44610	44640	44670	44700	44730	44760	44790

Table-4. Player A vs. Player B at 100th Iteration from Action A9 to A15

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
200	42020	42050	42080	42110	42140	42170	42200
3200	42220	42250	42280	42310	42340	42370	42400
6200	42420	42450	42480	42510	42540	42570	42600
9200	42620	42650	42680	42710	42740	42770	42800
12200	42820	42850	42880	42910	42940	42970	43000
15200	43020	43050	43080	43110	43140	43170	43200
18200	43220	43250	43280	43310	43340	43370	43400
21200	43420	43450	43480	43510	43540	43570	43600
24200	43620	43650	43680	43710	43740	43770	43800
27200	43820	43850	43880	43910	43940	43970	44000
30200	44020	44050	44080	44110	44140	44170	44200
33200	44220	44250	44280	44310	44340	44370	44400
36200	44420	44450	44480	44510	44540	44570	44600
39200	44620	44650	44680	44710	44740	44770	44800
42200	44820	44850	44880	44910	44940	44970	45000

Table-5. Player A vs. Player B at 150th Iteration from Action A1 to A8

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
300	62880	62910	62940	62970	63000	63030	63060	63090
4800	63180	63210	63240	63270	63300	63330	63360	63390
9300	63480	63510	63540	63570	63600	63630	63660	63690
13800	63780	63810	63840	63870	63900	63930	63960	63990
18300	64080	64110	64140	64170	64200	64230	64260	64290
22800	64380	64410	64440	64470	64500	64530	64560	64590
27300	64680	64710	64740	64770	64800	64830	64860	64890
31800	64980	65010	65040	65070	65100	65130	65160	65190
36300	65280	65310	65340	65370	65400	65430	65460	65490
40800	65580	65610	65640	65670	65700	65730	65760	65790
45300	65880	65910	65940	65970	66000	66030	66060	66090
49800	66180	66210	66240	66270	66300	66330	66360	66390
54300	66480	66510	66540	66570	66600	66630	66660	66690
58800	66780	66810	66840	66870	66900	66930	66960	66990
63300	67080	67110	67140	67170	67200	67230	67260	67290

Table-6. Player A vs. Player B at 150th Iteration from Action A9 to A15

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
300	63120	63150	63180	63210	63240	63270	63300
4800	63420	63450	63480	63510	63540	63570	63600
9300	63720	63750	63780	63810	63840	63870	63900
13800	64020	64050	64080	64110	64140	64170	64200
18300	64320	64350	64380	64410	64440	64470	64500

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22800	64620	64650	64680	64710	64740	64770	64800
27300	64920	64950	64980	65010	65040	65070	65100
31800	65220	65250	65280	65310	65340	65370	65400
36300	65520	65550	65580	65610	65640	65670	65700
40800	65820	65850	65880	65910	65940	65970	66000
45300	66120	66150	66180	66210	66240	66270	66300
49800	66420	66450	66480	66510	66540	66570	66600
54300	66720	66750	66780	66810	66840	66870	66900
58800	67020	67050	67080	67110	67140	67170	67200
63300	67320	67350	67380	67410	67440	67470	67500

Table-7. *Player A vs. Player B at 200th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
400	83980	84010	84040	84070	84100	84130	84160	84190
6400	84380	84410	84440	84470	84500	84530	84560	84590
12400	84780	84810	84840	84870	84900	84930	84960	84990
18400	85180	85210	85240	85270	85300	85330	85360	85390
24400	85580	85610	85640	85670	85700	85730	85760	85790
30400	85980	86010	86040	86070	86100	86130	86160	86190
36400	86380	86410	86440	86470	86500	86530	86560	86590
42400	86780	86810	86840	86870	86900	86930	86960	86990
48400	87180	87210	87240	87270	87300	87330	87360	87390
54400	87580	87610	87640	87670	87700	87730	87760	87790
60400	87980	88010	88040	88070	88100	88130	88160	88190
66400	88380	88410	88440	88470	88500	88530	88560	88590
72400	88780	88810	88840	88870	88900	88930	88960	88990
78400	89180	89210	89240	89270	89300	89330	89360	89390
84400	89580	89610	89640	89670	89700	89730	89760	89790

Table-8. *Player A vs. Player B at 200th Iteration from Action A9 to A15*

Player A	Player B							
	A9	A10	A11	A12	A13	A14	A15	
400	84220	84250	84280	84310	84340	84370	84400	
6400	84620	84650	84680	84710	84740	84770	84800	
12400	85020	85050	85080	85110	85140	85170	85200	
18400	85420	85450	85480	85510	85540	85570	85600	
24400	85820	85850	85880	85910	85940	85970	86000	
30400	86220	86250	86280	86310	86340	86370	86400	
36400	86620	86650	86680	86710	86740	86770	86800	
42400	87020	87050	87080	87110	87140	87170	87200	
48400	87420	87450	87480	87510	87540	87570	87600	
54400	87820	87850	87880	87910	87940	87970	88000	
60400	88220	88250	88280	88310	88340	88370	88400	
66400	88620	88650	88680	88710	88740	88770	88800	
72400	89020	89050	89080	89110	89140	89170	89200	
78400	89420	89450	89480	89510	89540	89570	89600	
84400	89820	89850	89880	89910	89940	89970	90000	

Table-9. *Player A vs. Player B at 250th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
500	105080	105110	105140	105170	105200	105230	105260	105290
8000	105580	105610	105640	105670	105700	105730	105760	105790
15500	106080	106110	106140	106170	106200	106230	106260	106290
23000	106580	106610	106640	106670	106700	106730	106760	106790
30500	107080	107110	107140	107170	107200	107230	107260	107290
38000	107580	107610	107640	107670	107700	107730	107760	107790
45500	108080	108110	108140	108170	108200	108230	108260	108290
53000	108580	108610	108640	108670	108700	108730	108760	108790
60500	109080	109110	109140	109170	109200	109230	109260	109290
68000	109580	109610	109640	109670	109700	109730	109760	109790
75500	110080	110110	110140	110170	110200	110230	110260	110290
83000	110580	110610	110640	110670	110700	110730	110760	110790
90500	111080	111110	111140	111170	111200	111230	111260	111290
98000	111580	111610	111640	111670	111700	111730	111760	111790
105500	112080	112110	112140	112170	112200	112230	112260	112290

Table-10. *Player A vs. Player B at 250th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
500	105320	105350	105380	105410	105440	105470	105500
8000	105820	105850	105880	105910	105940	105970	106000
15500	106320	106350	106380	106410	106440	106470	106500
23000	106820	106850	106880	106910	106940	106970	107000
30500	107320	107350	107380	107410	107440	107470	107500
38000	107820	107850	107880	107910	107940	107970	108000
45500	108320	108350	108380	108410	108440	108470	108500
53000	108820	108850	108880	108910	108940	108970	109000
60500	109320	109350	109380	109410	109440	109470	109500
68000	109820	109850	109880	109910	109940	109970	110000
75500	110320	110350	110380	110410	110440	110470	110500
83000	110820	110850	110880	110910	110940	110970	111000
90500	111320	111350	111380	111410	111440	111470	111500
98000	111820	111850	111880	111910	111940	111970	112000
105500	112320	112350	112380	112410	112440	112470	112500

Table-11. *Player A vs. Player B at 300th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
600	126180	126210	126240	126270	126300	126330	126360	126390
9600	126780	126810	126840	126870	126900	126930	126960	126990
18600	127380	127410	127440	127470	127500	127530	127560	127590
27600	127980	128010	128040	128070	128100	128130	128160	128190
36600	128580	128610	128640	128670	128700	128730	128760	128790
45600	129180	129210	129240	129270	129300	129330	129360	129390
54600	129780	129810	129840	129870	129900	129930	129960	129990
63600	130380	130410	130440	130470	130500	130530	130560	130590
72600	130980	131010	131040	131070	131100	131130	131160	131190
81600	131580	131610	131640	131670	131700	131730	131760	131790

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90600	132180	132210	132240	132270	132300	132330	132360	132390
99600	132780	132810	132840	132870	132900	132930	132960	132990
108600	133380	133410	133440	133470	133500	133530	133560	133590
117600	133980	134010	134040	134070	134100	134130	134160	134190
126600	134580	134610	134640	134670	134700	134730	134760	134790

Table-12. *Player A vs. Player B at 300th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
600	126420	126450	126480	126510	126540	126570	126600
9600	127020	127050	127080	127110	127140	127170	127200
18600	127620	127650	127680	127710	127740	127770	127800
27600	128220	128250	128280	128310	128340	128370	128400
36600	128820	128850	128880	128910	128940	128970	129000
45600	129420	129450	129480	129510	129540	129570	129600
54600	130020	130050	130080	130110	130140	130170	130200
63600	130620	130650	130680	130710	130740	130770	130800
72600	131220	131250	131280	131310	131340	131370	131400
81600	131820	131850	131880	131910	131940	131970	132000
90600	132420	132450	132480	132510	132540	132570	132600
99600	133020	133050	133080	133110	133140	133170	133200
108600	133620	133650	133680	133710	133740	133770	133800
117600	134220	134250	134280	134310	134340	134370	134400
126600	134820	134850	134880	134910	134940	134970	135000

Table-13. *Player A vs. Player B at 350th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
700	147280	147310	147340	147370	147400	147430	147460	147490
11200	147980	148010	148040	148070	148100	148130	148160	148190
21700	148680	148710	148740	148770	148800	148830	148860	148890
32200	149380	149410	149440	149470	149500	149530	149560	149590
42700	150080	150110	150140	150170	150200	150230	150260	150290
53200	150780	150810	150840	150870	150900	150930	150960	150990
63700	151480	151510	151540	151570	151600	151630	151660	151690
74200	152180	152210	152240	152270	152300	152330	152360	152390
84700	152880	152910	152940	152970	153000	153030	153060	153090
95200	153580	153610	153640	153670	153700	153730	153760	153790
105700	154280	154310	154340	154370	154400	154430	154460	154490
116200	154980	155010	155040	155070	155100	155130	155160	155190
126700	155680	155710	155740	155770	155800	155830	155860	155890
137200	156380	156410	156440	156470	156500	156530	156560	156590
147700	157080	157110	157140	157170	157200	157230	157260	157290

Table-14. *Player A vs. Player B at 350th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
700	147520	147550	147580	147610	147640	147670	147700
11200	148220	148250	148280	148310	148340	148370	148400
21700	148920	148950	148980	149010	149040	149070	149100
32200	149620	149650	149680	149710	149740	149770	149800

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42700	150320	150350	150380	150410	150440	150470	150500
53200	151020	151050	151080	151110	151140	151170	151200
63700	151720	151750	151780	151810	151840	151870	151900
74200	152420	152450	152480	152510	152540	152570	152600
84700	153120	153150	153180	153210	153240	153270	153300
95200	153820	153850	153880	153910	153940	153970	154000
105700	154520	154550	154580	154610	154640	154670	154700
116200	155220	155250	155280	155310	155340	155370	155400
126700	155920	155950	155980	156010	156040	156070	156100
137200	156620	156650	156680	156710	156740	156770	156800
147700	157320	157350	157380	157410	157440	157470	157500

Table-15. Player A vs. Player B at 400th Iteration from Action A1 to A8

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
800	168380	168410	168440	168470	168500	168530	168560	168590
12800	169180	169210	169240	169270	169300	169330	169360	169390
24800	169980	170010	170040	170070	170100	170130	170160	170190
36800	170780	170810	170840	170870	170900	170930	170960	170990
48800	171580	171610	171640	171670	171700	171730	171760	171790
60800	172380	172410	172440	172470	172500	172530	172560	172590
72800	173180	173210	173240	173270	173300	173330	173360	173390
84800	173980	174010	174040	174070	174100	174130	174160	174190
96800	174780	174810	174840	174870	174900	174930	174960	174990
108800	175580	175610	175640	175670	175700	175730	175760	175790
120800	176380	176410	176440	176470	176500	176530	176560	176590
132800	177180	177210	177240	177270	177300	177330	177360	177390
144800	177980	178010	178040	178070	178100	178130	178160	178190
156800	178780	178810	178840	178870	178900	178930	178960	178990
168800	179580	179610	179640	179670	179700	179730	179760	179790

Table-16. Player A vs Player B at 400th Iteration from Action A9 to A15

Player A	Player B							
	A9	A10	A11	A12	A13	A14	A15	
800	168620	168650	168680	168710	168740	168770	168800	
12800	169420	169450	169480	169510	169540	169570	169600	
24800	170220	170250	170280	170310	170340	170370	170400	
36800	171020	171050	171080	171110	171140	171170	171200	
48800	171820	171850	171880	171910	171940	171970	172000	
60800	172620	172650	172680	172710	172740	172770	172800	
72800	173420	173450	173480	173510	173540	173570	173600	
84800	174220	174250	174280	174310	174340	174370	174400	
96800	175020	175050	175080	175110	175140	175170	175200	
108800	175820	175850	175880	175910	175940	175970	176000	
120800	176620	176650	176680	176710	176740	176770	176800	
132800	177420	177450	177480	177510	177540	177570	177600	
144800	178220	178250	178280	178310	178340	178370	178400	
156800	179020	179050	179080	179110	179140	179170	179200	
168800	179820	179850	179880	179910	179940	179970	180000	

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Table-17. *Player A vs. Player B at 450th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
900	189480	189510	189540	189570	189600	189630	189660	189690
14400	190380	190410	190440	190470	190500	190530	190560	190590
27900	191280	191310	191340	191370	191400	191430	191460	191490
41400	192180	192210	192240	192270	192300	192330	192360	192390
54900	193080	193110	193140	193170	193200	193230	193260	193290
68400	193980	194010	194040	194070	194100	194130	194160	194190
81900	194880	194910	194940	194970	195000	195030	195060	195090
95400	195780	195810	195840	195870	195900	195930	195960	195990
108900	196680	196710	196740	196770	196800	196830	196860	196890
122400	197580	197610	197640	197670	197700	197730	197760	197790
135900	198480	198510	198540	198570	198600	198630	198660	198690
149400	199380	199410	199440	199470	199500	199530	199560	199590
162900	200280	200310	200340	200370	200400	200430	200460	200490
176400	201180	201210	201240	201270	201300	201330	201360	201390
189900	202080	202110	202140	202170	202200	202230	202260	202290

Table-18. *Player A vs. Player B at 450th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
900	189720	189750	189780	189810	189840	189870	189900
14400	190620	190650	190680	190710	190740	190770	190800
27900	191520	191550	191580	191610	191640	191670	191700
41400	192420	192450	192480	192510	192540	192570	192600
54900	193320	193350	193380	193410	193440	193470	193500
68400	194220	194250	194280	194310	194340	194370	194400
81900	195120	195150	195180	195210	195240	195270	195300
95400	196020	196050	196080	196110	196140	196170	196200
108900	196920	196950	196980	197010	197040	197070	197100
122400	197820	197850	197880	197910	197940	197970	198000
135900	198720	198750	198780	198810	198840	198870	198900
149400	199620	199650	199680	199710	199740	199770	199800
162900	200520	200550	200580	200610	200640	200670	200700
176400	201420	201450	201480	201510	201540	201570	201600
189900	202320	202350	202380	202410	202440	202470	202500

Table-19. *Player A vs. Player B at 500th Iteration from Action A1 to A8*

Player A	Player B							
	A1	A2	A3	A4	A5	A6	A7	A8
1000	210580	210610	210640	210670	210700	210730	210760	210790
16000	211580	211610	211640	211670	211700	211730	211760	211790
31000	212580	212610	212640	212670	212700	212730	212760	212790
46000	213580	213610	213640	213670	213700	213730	213760	213790
61000	214580	214610	214640	214670	214700	214730	214760	214790
76000	215580	215610	215640	215670	215700	215730	215760	215790
91000	216580	216610	216640	216670	216700	216730	216760	216790

106000	217580	217610	217640	217670	217700	217730	217760	217790
121000	218580	218610	218640	218670	218700	218730	218760	218790
136000	219580	219610	219640	219670	219700	219730	219760	219790
151000	220580	220610	220640	220670	220700	220730	220760	220790
166000	221580	221610	221640	221670	221700	221730	221760	221790
181000	222580	222610	222640	222670	222700	222730	222760	222790
196000	223580	223610	223640	223670	223700	223730	223760	223790
211000	224580	224610	224640	224670	224700	224730	224760	224790

Table-20. *Player A vs. Player B at 500th Iteration from Action A9 to A15*

Player A	Player B						
	A9	A10	A11	A12	A13	A14	A15
1000	210820	210850	210880	210910	210940	210970	211000
16000	211820	211850	211880	211910	211940	211970	212000
31000	212820	212850	212880	212910	212940	212970	213000
46000	213820	213850	213880	213910	213940	213970	214000
61000	214820	214850	214880	214910	214940	214970	215000
76000	215820	215850	215880	215910	215940	215970	216000
91000	216820	216850	216880	216910	216940	216970	217000
106000	217820	217850	217880	217910	217940	217970	218000
121000	218820	218850	218880	218910	218940	218970	219000
136000	219820	219850	219880	219910	219940	219970	220000
151000	220820	220850	220880	220910	220940	220970	221000
166000	221820	221850	221880	221910	221940	221970	222000
181000	222820	222850	222880	222910	222940	222970	223000
196000	223820	223850	223880	223910	223940	223970	224000
211000	224820	224850	224880	224910	224940	224970	225000

Conclusions

1. The competitor of player A i.e player B influences on all available actions of player A in each computation.
2. It has utmost the level of correlation among scientific computations.
3. There is a step by step accuracy obtained from each iteration.
4. Better improvements have been discovered.
5. Steady fluctuations are obtained.
6. Each available action of player A at any two successive computations has consistent disputes.
7. Systematic developments have been identified in both the players.

5. OPTIMUM MIXIED STRATEGIES OF PLAYER A AND PLAYER B

The optimum mixed strategies of the playerA from the iteration 50 - 500 are obtained as

$$\begin{bmatrix} A1 & A2 & A3 & A4 & A5 & A6 & A7 & A8 & A9 & A10 & A11 & A12 & A13 & A14 & A15 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly the optimum mixed strategies of the player B from the iteration 50 - 500

are also obtained as

$$\begin{bmatrix} B1 & B2 & B3 & B4 & B5 & B6 & B7 & B8 & B9 & B10 & B11 & B12 & B13 & B14 & B15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

6. UPPER BOUNDS, LOWER BOUNDS AND ERRORS AT ALL COMPUTATIONS

The Values of U.Bs and L.Bs in 15x15 game and error estimations are shown in the tables from Table (21) to Table (22).

Table-21

U.B	Lower Bound									
Iterations : 50-500	50	100	150	200	250	300	350	400	450	500
422	413.6	417.8	419.2	419.9	420.32	420.6	420.8	420.95	421.0667	421.16
422	414.2	418.1	419.4	420.05	420.44	420.7	420.8857	421.025	421.1333	421.22
422	414.8	418.4	419.6	420.2	420.56	420.8	420.9714	421.1	421.2	421.28
422	415.4	418.7	419.8	420.35	420.68	420.9	421.0571	421.175	421.2666	421.34
422	416	419	420	420.5	420.8	421	421.1429	421.25	421.3333	421.4
422	416.6	419.3	420.2	420.65	420.92	421.1	421.2286	421.325	421.4	421.46
422	417.2	419.6	420.4	420.8	421.04	421.2	421.3143	421.4	421.4666	421.52
422	417.8	419.9	420.6	420.95	421.16	421.3	421.4	421.475	421.5333	421.58
422	418.4	420.2	420.8	421.1	421.28	421.4	421.4857	421.55	421.5999	421.64
422	419	420.5	421	421.25	421.4	421.5	421.5714	421.625	421.6666	421.7
422	419.6	420.8	421.2	421.4	421.52	421.6	421.6571	421.7	421.7333	421.76
422	420.2	421.1	421.4	421.55	421.64	421.7	421.7429	421.775	421.7999	421.82
422	420.8	421.4	421.6	421.7	421.76	421.8	421.8286	421.85	421.8666	421.88
422	421.4	421.7	421.8	421.85	421.88	421.9	421.9143	421.925	421.9333	421.94
422	422	422	422	422	422	422	422	422	422	422

Table-22

Iterations and Errors									
50	100	150	200	250	300	350	400	450	500
8.4	4.2	2.8	2.1	1.68	1.4	1.2	1.05	0.9333	0.84
7.8	3.9	2.6	1.95	1.56	1.3	1.1143	0.975	0.8667	0.78
7.2	3.6	2.4	1.8	1.44	1.2	1.0286	0.9	0.8	0.72
6.6	3.3	2.2	1.65	1.32	1.1	0.9429	0.825	0.7334	0.66
6	3	2	1.5	1.2	1	0.8571	0.75	0.6667	0.6
5.4	2.7	1.8	1.35	1.08	0.9	0.7714	0.675	0.6	0.54
4.8	2.4	1.6	1.2	0.96	0.8	0.6857	0.6	0.5334	0.48
4.2	2.1	1.4	1.05	0.84	0.7	0.6	0.525	0.4667	0.42
3.6	1.8	1.2	0.9	0.72	0.6	0.5143	0.45	0.4001	0.36
3	1.5	1	0.75	0.6	0.5	0.4286	0.375	0.3334	0.3

2.4	1.2	0.8	0.6	0.48	0.4	0.3429	0.3	0.2667	0.24
1.8	0.9	0.6	0.45	0.36	0.3	0.2571	0.225	0.2001	0.18
1.2	0.6	0.4	0.3	0.24	0.2	0.1714	0.15	0.1334	0.12
0.6	0.3	0.2	0.15	0.12	0.1	0.0857	0.075	0.0668	0.06
0	0	0	0	0	0	0	0	0	0

7. CONCLUSIONS

1. The same optimum mixed pure strategies are persisted for player A and player B in each computation.
2. The value of the game is 422. Because the optimal mini max strategies coincide with the value at saddle position.
3. The value of upper bound is unique and same as the value of the game at any stage of computation.
4. At the beginning, the value of lower bound of this game is not equal to the value of the game. But it tends to turn to the value of the game at last stage in the computation.
5. The error has found initially with 8.4 and it converges step by step to zero at the end.
6. The game is identified as a strictly determinable game. Since lower bound and upper bound are equal to the value of the game.
7. The occurred errors are slumped step by step from iteration to iteration.

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