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Effect of Surface Roughness in a Narrow Porous Journal Bearing With a Heterogeneous Slip/No-Slip Surface

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Abstract: *In this paper, a numerical simulation of the effect of surface roughness in hydrodynamic lubrication of a porous journal bearing with a heterogeneous slip/no-slip surface is studied. The modified Reynolds equations accounting for the heterogeneous surface on which slip occurs in certain region and is absent in others, by considering surface roughness structure, are mathematically formulated. The well established Christensen stochastic theory is the basis for present work. The problem is formulated and solved by using appropriate boundary conditions for heterogeneous surface. The result shows that with a roughness pattern, a significant increase in load support can be achieved with an appropriate surface pattern.*

Keywords: *Porous, journal bearing, roughness, heterogeneous surface*

1. INTRODUCTION

In earlier days, linguistic study of hydrodynamic lubrication, it has been assumed that the bearing surfaces are smooth. But, it was not true for the bearings with small film thickness. This is because of the fact that all the surfaces are rough to some extent and generally the roughness asperities are of the same order as the mean separation between lubricated contacts. Most of the researchers in this area have confined their work to non-porous bearings, as the surface roughness is inherent to the process used in their manufacture. The surface irregularities strongly affect the bearing performances. The first paper which deals with the surface roughness effect is by Ting [1]. Then the stochastic theory of hydrodynamic lubrication of rough surface was developed by Christensen [2]. The behavior of a fluid film bearing depends on the boundary conditions at the interface between the liquid and the solid bearing surfaces by considering no - slip boundary conditions. However, Prakas and Vij [3] studied the analysis of narrow porous journal bearing using Beavers – Joseph criterion of velocity slip. Gururajan and Prakash [4, 5] have studied the effect of surface roughness in an infinitely long porous journal bearing and in an infinitely short porous journal bearing by considering slip condition. After that many researchers like Naduvinamani, Bujurke, Gururbasavaraj [6, 7, and 8], have examined the effect of surface roughness on porous hydrodynamic bearings with couple stress fluids. Ramesh Kudenatti, Shalini Patil, Dinesh and Vinay have studied the combined effect of surface roughness and magnetictic field between rough and porous rectangular plates [9], and combined effect of couple stresses and MHD on squeeze film lubrication between two parallel plates [10].

In recent, several studies number of researchers have found that slip occurs in engineered heterogeneous bearing surface, on which slip occurs in certain regions and is absent in others. The desired effect is for the fluid that first flow through the slip region and then exit through the no – slip region. Later, Numerical simulation of Mechanical seal, hydrodynamic bearing, slider bearing and journal bearing with heterogeneous slip/no- slip surface have been studied by many authors like Salant and Fortier [see 11,12 13, 14, 15].

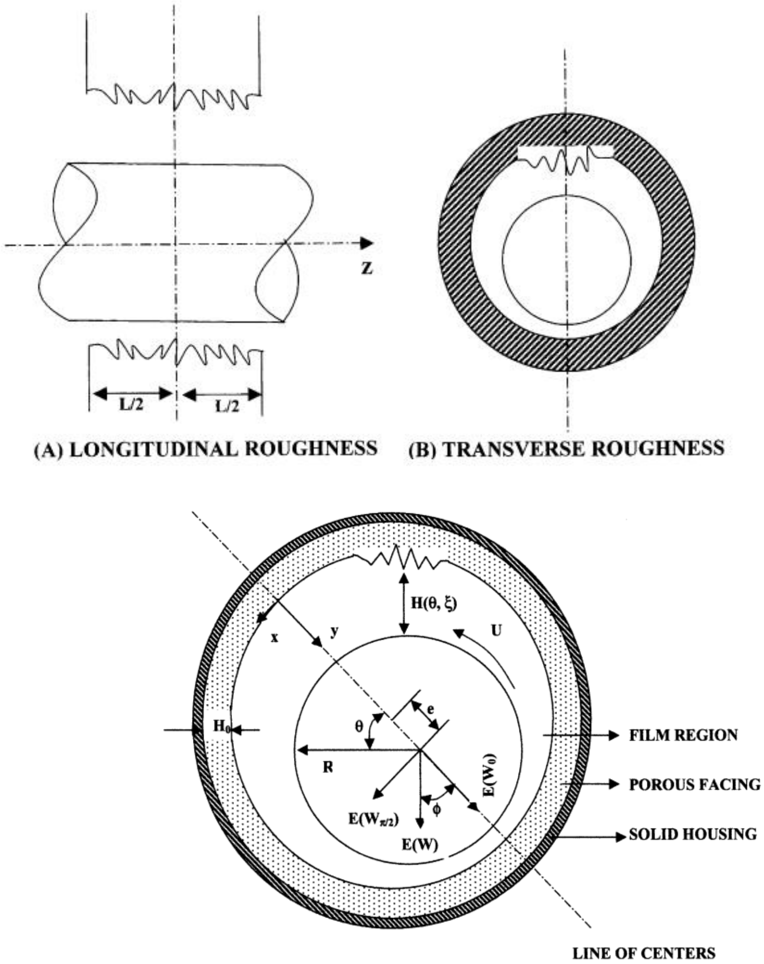


Fig.1. *Bearing Geometry and Journal bearing configuration*

In the present study, the effect of surface roughness of a narrow porous journal bearing with a heterogeneous slip/no – slip surface is analyzed. The stochastic theory of hydrodynamic lubrication of rough surface developed by Christensen [2] forms the basis of present work.

2. ANALYSIS

Consider the journal bearing as shown in fig.1. The shaft and sleeve have a clearance denoted Δr. The operational eccentricity of the shaft and sleeve, as measured along the line of centers, is denoted by e. The film thickness is a function of bearing clearance, bearing eccentricity, and circumferential location. The mathematical expression is given by

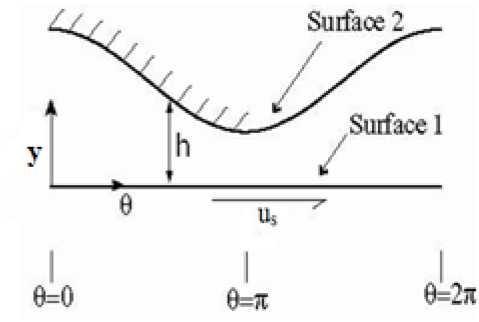
$$h(\theta) = \Delta r + e\cos(\theta)\,. \tag{1}$$

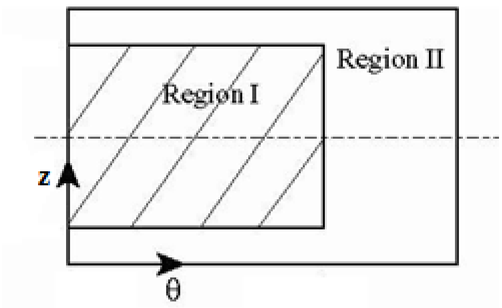
The Fig. 2(a) shows the film thickness distribution. In this figure surface 1 corresponds to shaft moving with speed, u_s and surface 2 is stationary corresponds to bearing sleeve. The slip/no-slip pattern is applied on the surface 2 as shown in Fig. 2(b). Note that the fluid first flows through slip region and exit through the no-slip region.

The relevant Navier boundary condition [15] for slip velocity by considering porous media is given by

$$\text{at} \quad y = 0, \qquad u = u_s\,, \qquad v = -V_0, \qquad w = 0\,, \tag{2}$$

at $\quad y = h, \qquad u = -\alpha\mu \frac{\partial u}{\partial y}, \quad v = 0, \quad w = -\alpha\mu \frac{\partial w}{\partial z},$ (3)





(a) (b)

Fig. 2. *(a) Diagram of journal bearing in Cartesian coordinate configuration. (b) Diagram of surface pattern applied to Surface 2*

where α is the slip coefficient, u_s is shaft surface speed, V_0 is the velocity with which lubricant getting into the pores of bearing material, which is governed by Darcy's law. In order to conserve mass, lubricant must be continuously supplied to the bearing to compensate for the lubricant lost through side leakage. This ensures that the bearing is not starved and is able to maintain the prescribed film thickness. In the present model the pressure is atmospheric at $\theta = 0$ to simulate the lubricant inlet location.

In the present study, modified form of the Reynolds equation for a lubricant film in the journal bearing by considering the boundary conditions (2) and (3) [see 14], is developed by using theGururajan and Prakash postulate [4, 5] across the film thickness of a rough journal bearing is given by

$$\frac{\partial}{\partial x}\left[\frac{h^3}{12\mu}\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h^3}{12\mu}\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)\frac{\partial p}{\partial z}\right]=\frac{\partial}{\partial x}\left[\frac{u_s h}{2}\left(1+\frac{\alpha\mu}{h+\alpha\mu}\right)\right]+V_0 \quad (4)$$

or,

$$\frac{\partial}{\partial x}\left[h^3\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[h^3\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)\frac{\partial p}{\partial z}\right]=6\mu u_s\frac{\partial}{\partial x}\left[h\left(1+\frac{\alpha\mu}{h+\alpha\mu}\right)\right]+12\mu V_0. \quad (5)$$

Equation (5) contains two unknowns p and V_0, can be solved for p only when V_0 has been determined. When the bearing is non-porous V_0 vanishes, but in the case of porous bearing it determines the velocity of lubricant across the bearing interface and therefore is nonzero.

Velocity component across the porous boundary is given by [5]

$$V_0 = \frac{k}{\mu}\left[\frac{\partial p}{\partial y}\right]_{y=0} . \quad (6)$$

The bearing wall thickness H_0 assumed to be small. Prakash and Vij [3] showed that

$$\left[\frac{\partial p}{\partial y}\right]_{y=0} \approx -H_0\left[\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}\right] . \quad (7)$$

Substituting (7) in (6), we have

$$V_0 = -\frac{k}{\mu}H_0\left[\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}\right] . \quad (8)$$

Substituting for $V_0$from equation (8) to equation (5), the modified Reynolds equation becomes

$$\frac{\partial}{\partial x}\left[\left\{h^3\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)+12kH_0\right\}\frac{\partial p}{\partial x}\right]+\quad\frac{\partial}{\partial z}\left[\left\{h^3\left(1+\frac{3\alpha\mu}{h+\alpha\mu}\right)+12kH_0\right\}\frac{\partial p}{\partial z}\right]$$

$$= 6\mu u_s \frac{\partial}{\partial x}\left[h\left(1+\frac{\alpha\mu}{h+\alpha\mu}\right)\right] . \tag{9}$$

To represent surface roughness mathematical expression for film thickness is

$$H = h(\theta) + h_s = \Delta r(1+\varepsilon cos\theta) + h_s(\theta, y, \xi) , \tag{10}$$

where $h(\theta) = \Delta r(1+\varepsilon cos\theta)$ denote the nominal smooth part of the film geometry, while h_s denote the roughness and is randomly varying quantity with zero mean, ξ is the random variable. Δr is the radial clearance and $\theta = x/R$.

Non dimensionalsing equation (9)

$$\frac{\partial}{\partial x}\left[\{H^3(1+3\Sigma)+12kH_0\}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[\{H^3(1+3\Sigma)+12kH_0\}\frac{\partial p}{\partial z}\right] = 6\mu u_s \frac{\partial}{\partial x}[H(1+\Sigma)] , \tag{11}$$

where $A = \frac{\alpha\mu}{\Delta r}$, $\overline{H} = \frac{H}{\Delta r}$ and $\Sigma = \frac{A}{\overline{H}+A}$.

In the limiting case as $A \to 0$, equation (11) reduces to the case studied by Gururajan and Prakash [4].

Taking expected value on both side of equation (11), we get

$$\frac{\partial}{\partial x}\left[E\left\langle\{H^3(1+3\Sigma)+12kH_0\}\frac{\partial p}{\partial x}\right\rangle\right] + \frac{\partial}{\partial z}\left[E\left\langle\{H^3(1+3\Sigma)+12kH_0\}\frac{\partial p}{\partial z}\right\rangle\right]$$
$$= 6\mu u_s \frac{\partial}{\partial x} E[H(1+\Sigma)] . \tag{12}$$

where E ( ) is the expectancy operator defined by

$$E(\quad) = \int_{-\infty}^{\infty}(\quad) f(h_s)\, dh_s , \tag{13}$$

where $f(h_s)$ is the probability density distribution function of the stochastic variable h_s.

In order to evaluate the average of fluxes of equation (12), subjected to a specific roughness arrangement, as assumed in accordance with Christensen [2] that pressure gradient in the direction of roughness and the flux perpendicular to it are stochastic variables with zero or negligible variance.

The roughness distribution function used to evaluate the expected value is

$$f(h_s) = \frac{35}{32C^7}\left(C^2 - {h_s}^2\right)^3, \qquad -C < h_s < C,$$
$$= 0, \qquad elsewhere, \tag{14}$$

where $C = \pm 3\sigma$ and σ is the standard deviation.

In the context of stochastic theory, following two types of roughness are studied,

(a) The Longitudinal roughness, having the form of long, narrow ridges and valleys, running in the direction of rotation, and

(b) The Transverse roughness, where the ridges and valleys are running in the transverse direction.

2.1 Longitudinal, One – Dimensional Roughness

In this model, roughness pattern has the form of narrow ridges running the direction of rotation, the film thickness assumes the form

$$H = \Delta r(1+\varepsilon cos\theta) + h_s(y, \xi) . \tag{15}$$

The generalized equation (12) is given by

$$\frac{\partial}{\partial x}\left[E\left\langle\{H^3(1+3\Sigma)+12kH_0\}\frac{\partial p}{\partial x}\right\rangle\right]+\frac{\partial}{\partial z}\left[\left\langle\left\{\frac{1}{E\langle 1/H^3(1+3\Sigma)\rangle}+12kH_0\right\}\frac{\partial p}{\partial z}\right\rangle\right]$$
$$=6\mu u_s\frac{\partial}{\partial x}E[H(1+\Sigma)]\ . \tag{16}$$

2.2 Transverse, One-Dimensional Roughness

In this model, roughness pattern has the form of ridges and a valley running in the transverse direction, the film thickness assumes the form

$$H=\Delta r(1+\varepsilon cos\theta)+h_s(\theta,\xi). \tag{17}$$

In this case, the Reynolds equation (12) takes the form

$$\frac{\partial}{\partial x}\left[\left\{\frac{1}{E\{1/H^3(1+3\Sigma)\}}+12kH_0\right\}\frac{\partial}{\partial x}[E(P)]\right]+\frac{\partial}{\partial z}\left[\{E\langle H^3(1+3\Sigma)\rangle+12kH_0\}\frac{\partial}{\partial z}[E(P)]\right]$$
$$=6\mu u_s\frac{\partial}{\partial x}\left[\frac{E\{(1+\Sigma)/H^2(1+3\Sigma)\}}{E\{1/H^3(1+3\Sigma)\}}\right]. \tag{18}$$

3. NARROW JOURNAL BEARING

The modified Reynolds equations (16) and (18), applicable to the two types of directional roughness structure, do not yield an analytical solution for a journal bearing geometry except for the limiting case of infinitely short journal bearing. Using short bearing approximation, the circumferential pressure variations may be neglected in comparison with axial pressure variation.

3.1 Longitudinal Roughness

In comparison with axial pressure variation $\frac{\partial}{\partial x}[E(p)]=0$. Equation (16) becomes

$$\frac{\partial}{\partial z}\left[\left\{\frac{1}{E\langle 1/H^3(1+3\Sigma)\rangle}+12kH_0\right\}\frac{\partial}{\partial z}[E(P)]\right]=6\mu u_s\frac{\partial}{\partial x}E[H(1+\Sigma)]\ . \tag{19}$$

Integrating equation (19) twice with respect to z and using the following boundary conditions for pressure,

$$\text{i.e., } E(p)=0 \qquad at\ z=\pm\frac{L}{2},\quad 0\le\theta\le 2\pi\ , \tag{20}$$

we get,

$$E(P)=\frac{3\mu u_s}{R}L^2\frac{\frac{d}{d\theta}E\langle H(1+\Sigma)\rangle}{\left[\frac{1}{E\{1/H^3(1+3\Sigma)\}}+12kH_0\right]}\left(\frac{z^2}{L^2}-\frac{1}{4}\right), \tag{21}$$

$$E(P)=\frac{3\mu u_s L^2}{(\Delta r)^2R}\frac{\varepsilon sin\theta[1+A^2*G_3(\bar{h}+A,C)]}{[1/G_9(\bar{h},C)+12\ \psi\frac{H_0}{R}]}\left(\frac{1}{4}-\bar{z}^2\right), \tag{22}$$

where $\psi=\frac{kH_0}{(\Delta r)^3}$, $\bar{z}=\frac{z}{L}$, $\bar{h}=\frac{h}{\Delta r}$, $C=\frac{c}{\Delta r}$.

The non-dimensional mean pressure, $\bar{p}$ is given as

$$\bar{p}=\frac{E(P)(\Delta r)^2R}{\mu u_s L^2}=\frac{3\varepsilon\sin\theta[1+A^2*G_3(\bar{h}+A,C)]}{[1/G_9(\bar{h},C)+12\ \psi\frac{H_0}{R}]}\left(\frac{1}{4}-\bar{z}^2\right), \tag{23}$$

where ψ is the permeability parameter. In the limiting case as $A\to 0$, equation (22) reduces to case studied by Gururajan and Prakash [4].

Load carrying capacity of the short journal bearing is obtained by integrating the pressure, taking the direction into account and by using Half Somerfield boundary conditions, followed Cameron et al.[16]. Let the attitude angle ϕ which makes the load line with line of centres, the mean component acting along the line of centre is given by

$$E(W_0)=E(W)cos\phi=-LR\int_0^{\pi}E(p)\ cos\theta\ \ d\theta \tag{24}$$

and that acting along the normal line of centers is given by

$$E\left(W_{\pi/2}\right) = E(W)\, sin\phi = LR \int_0^\pi E(p)\, sin\theta \;\; d\theta \;\; . \tag{25}$$

The non-dimensional forms of (24) and (25) are

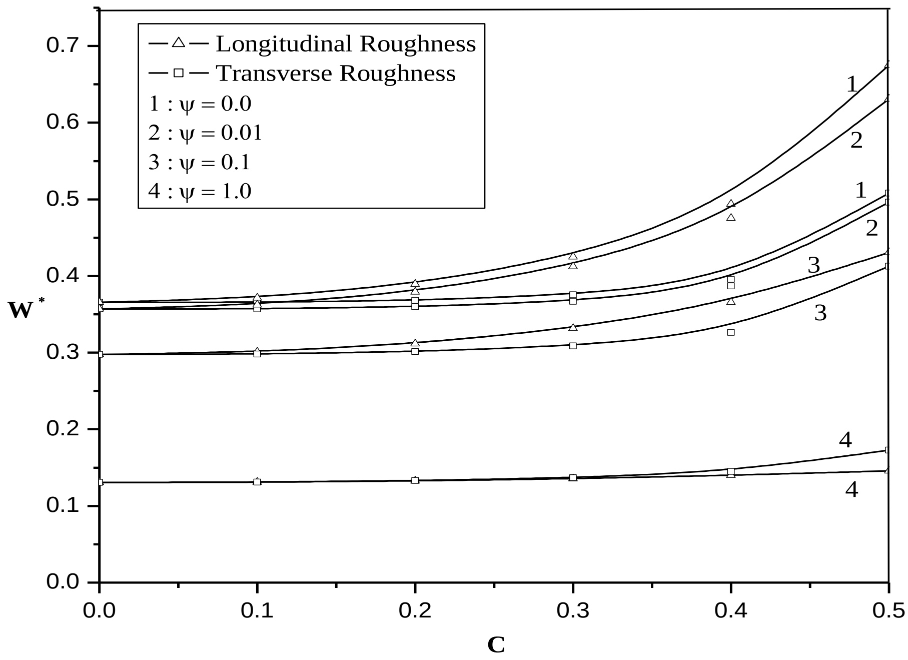


Fig. 3 Non dimensional load carrying capacity verses roughness parameter C for different values of ψ with A=5, ε=0.5

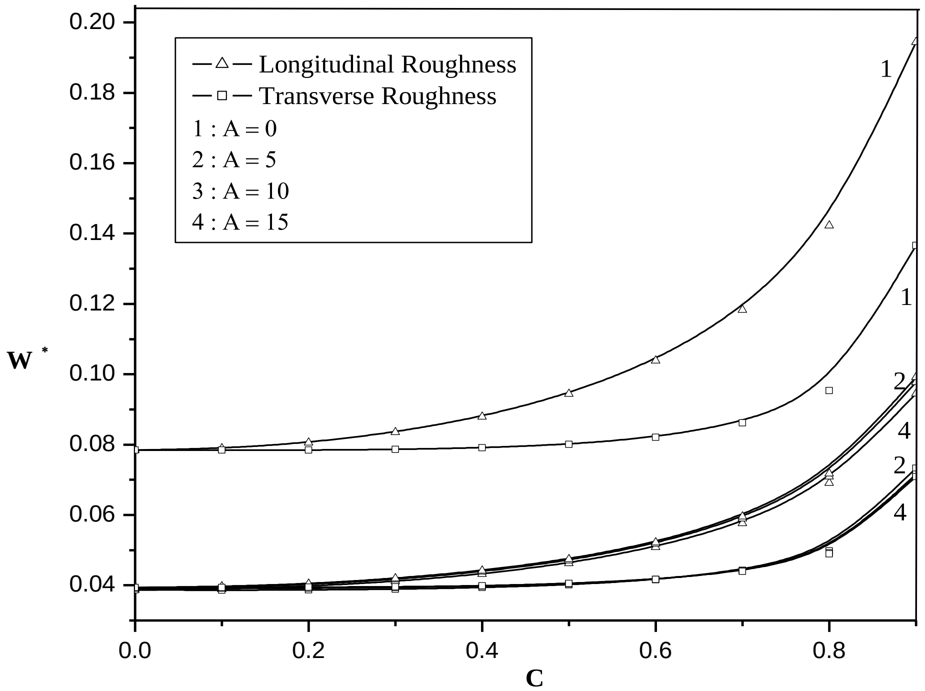


Fig. 4 Non dimensional load carrying capacity verses roughness parameter C for different values of A with ψ=0.01, ε=0.1

$$\overline{W_0} = \frac{E(W_0)(\Delta r)^2}{\mu u_s L R^2} = -\frac{\varepsilon}{2}\int_0^{\pi} \frac{\sin\theta \cos\theta \, [1+A^2 * G_3(\bar{h}+A,C)]}{[\{1/G_9(\bar{h},C)\}+12\,\psi\frac{H_0}{R}]}\, d\theta \tag{26}$$

and

$$\overline{W_{\pi/2}} = \frac{E(W_{\pi/2})(\Delta r)^2}{\mu u_s L R^2} = \frac{\varepsilon}{2}\int_0^{\pi} \frac{\sin\theta \sin\theta \, [1+A^2 * G_3(\bar{h}+A,C)]}{[\{1/G_9(\bar{h},C)\}+12\,\psi\frac{H_0}{R}]}\, d\theta \,. \tag{27}$$

The total non-dimensional load carrying capacity is given by

$$W^* = \frac{E(W)(\Delta r)^2}{\mu u_s L R^2} = \sqrt{(\overline{W_0})^2 + \left(\overline{W_{\pi/2}}\right)^2} \,. \tag{28}$$

The mean circumferential frictional force acting on the journal surface at $y = H$ is given by

$$E(F) = \int_{-L/2}^{L/2}\int_0^{2\pi} E(\tau_H)\, R\, d\theta\, dz\,, \tag{29}$$

where

$$\tau_H = \mu\left(\frac{\partial u}{\partial y}\right)_{y=H} - \eta\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=H} .$$

Shear stress acting on the journal bearing [17] is given by

$$\tau_H = \frac{1}{2}\frac{\partial P}{\partial x}\left(\frac{H^2}{H+A}\right) + \mu\, u_s \frac{1}{H+A} \,. \tag{30}$$

Taking expectation on both sides of (30), we get

$$E(\tau_H) = \frac{\partial E(P)}{\partial x}\; E\left(\frac{H^2}{H+A}\right) + \mu\, u_s\; E\left(\frac{1}{H+A}\right). \tag{31}$$

Substituting the expression (31) in (29) and then integrating, the non-dimensional frictional force is given by

$$F^* = \frac{E(F)(\Delta r)}{\mu u_s L R} = \frac{\varepsilon^2}{4}\int_0^{2\pi} \frac{\sin^2\theta\, \left[1 - A^4 G_3{}^2(\bar{h}+A,C)\right]}{[\{1/G_9(\bar{h},C)\}+12\,\psi\frac{H_0}{R}]} d\theta + \int_0^{2\pi} G_2(\bar{h}+A,C)\; d\theta \,. \tag{32}$$

The mean coefficient of friction $E(f)$ is the ratio of mean frictional force $E(F)$ by mean load carrying capacity $E(W)$.

3.2 Transverse Roughness

In comparison with axial pressure variation $\frac{\partial}{\partial x}[E(p)] = 0$ equation (18) becomes

$$\frac{\partial}{\partial z}\left[\{E\langle H^3(1+3\Sigma)\rangle + 12kH_0\}\frac{\partial}{\partial z}[E(P)]\right] = 6\mu u_s\; \frac{\partial}{\partial x}\left[\frac{E\{(1+\Sigma)/H^2(1+3\Sigma)\}}{E\{1/H^3(1+3\Sigma)\}}\right]. \tag{33}$$

Integrating (33) and using the boundary conditions (20), yields the non-dimensional mean pressure in the form

$$\bar{p} = \frac{E(P)(\Delta r)^2 R}{\mu u_s L^2} = \frac{3\frac{d}{d\theta}\left[\frac{E\{(1+\Sigma)/H^2(1+3\Sigma)\}}{E\{1/H^3(1+3\Sigma)\}}\right]}{\left[E\{H^3(1+3\Sigma)\}+12\,\psi\frac{H_0}{R}\right]}\left(\bar{z}^2 - \frac{1}{4}\right) \tag{34}$$

or

$$\bar{p} = \frac{3G_{14}(\bar{h},C)}{[G_{15}(\bar{h},C)+12\,\psi]}\left(\bar{z}^2 - \frac{1}{4}\right). \tag{35}$$

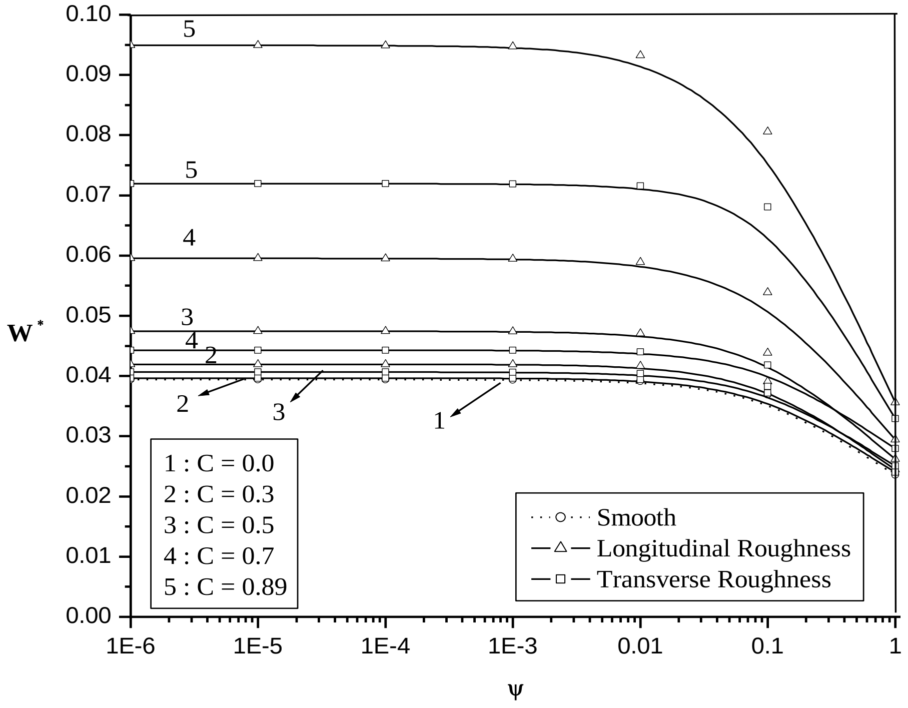


Fig. 5 Non-dimensional load carrying capacity verses permeability parameter ψ for different values of C with A=10, ε=0.1

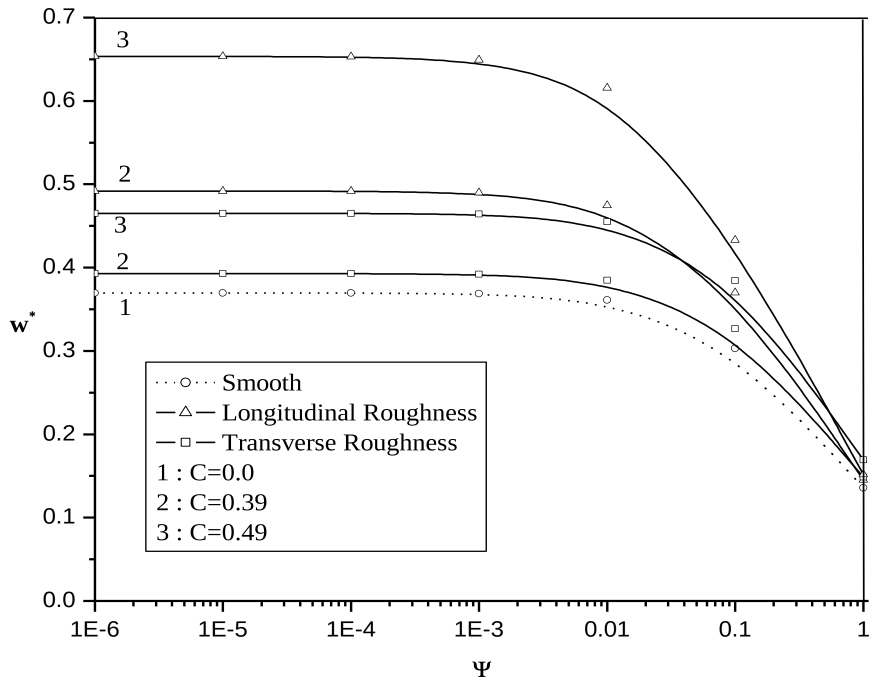


Fig. 6 Non dimensional load carrying capacity verses permeability parameter Ψ for different values of C with A=10, ε=0.5

The mean load components $E(W_0)$ and $E(W_{\pi/2})$ acting along the line of centre and acting normal to the line of centres are given in the non-dimesional forms,

$$\overline{W_0} = \frac{E(W_0)(\Delta r)^2}{\mu u_s L R^2} = \frac{\varepsilon}{2}\int_0^{\pi} sin\theta\; cos\theta \frac{G_{14}(\bar{h},C)}{\left[G_{15}(\bar{h},C)+12\psi\frac{H_0}{R}\right]} d\theta \tag{36}$$

and

$$\overline{W_{\pi/2}} = \frac{E(W_0)(\Delta r)^2}{\mu u_s L R^2} = -\frac{\varepsilon}{2}\int_0^{\pi} sin\theta\; sin\theta\; \frac{G_{14}(\bar{h},C)}{\left[G_{15}(\bar{h},C)+12\,\psi\frac{H_0}{R}\right]} d\theta\;. \tag{37}$$

The total dimensionless load carrying capacity W^* is similar to equation (28).

The mean circumferential frictional force acting on the journal surface $at\ y = H$ is given by

$$F^* = \frac{E(F)(\Delta r)}{\mu u_s L R} = -\frac{1}{6A}\int_0^{\pi} \bar{\mathrm{p}}\; \frac{G_{16}(\bar{h},C)-G_2(\bar{h},C)}{G_9(\bar{h},C)}\, d\theta + \frac{1}{A}\int_0^{2\pi} G_{13}\big(\bar{h},C\big)\; \frac{G_{16}(\bar{h},C)-G_2(\bar{h},C)}{G_9(\bar{h},C)} d\theta$$

$$-\frac{2}{A}\int_0^{2\pi}\big[G_{17}\big(\bar{h},C\big) - G_{18}\big(\bar{h},C\big)\big]d\theta + \int_0^{2\pi} G_2\big(\bar{h}+A,C\big)d\theta\;. \tag{38}$$

The mean coefficient of friction is obtained by dividing the mean frictional force by mean load carrying capacity.

Assuming the roughness asperities like heights are small as compared with the film thickness, i.e, $C/\bar{h}$ is small, the numerical computations are performed after obitaining the Taylor series expansions of the film thickness (Gururajan and Prakash, [4]).

G - functions are given as follows

$$G_2\big(\bar{h},C\big) = E\left(\frac{1}{H}\right) = \frac{1}{\bar{h}}\left\{1 + 105\sum_{\mathrm{n}=1}^{\infty}\frac{\mathrm{X}^{2\mathrm{n}}}{(2\mathrm{n}+1)(2\mathrm{n}+3)(2\mathrm{n}+5)(2\mathrm{n}+7)}\right\},$$

$$G_3\big(\bar{h},C\big) = E\left(\frac{1}{H^2}\right) = \frac{1}{\bar{h}^2}\left\{1 + 105\sum_{\mathrm{n}=1}^{\infty}\frac{\mathrm{X}^{2\mathrm{n}}}{(2\mathrm{n}+3)(2\mathrm{n}+5)(2\mathrm{n}+7)}\right\},$$

$$G_4\big(\bar{h},C\big) = E\left(\frac{1}{H^3}\right) = \frac{1}{\bar{h}^3}\left\{1 + 105\sum_{\mathrm{n}=1}^{\infty}\frac{(\mathrm{n}+1)\mathrm{X}^{2\mathrm{n}}}{(2\mathrm{n}+3)(2\mathrm{n}+5)(2\mathrm{n}+7)}\right\},$$

$$G_5\big(\bar{h},C\big) = E\left(\frac{1}{H^4}\right) = \frac{1}{\bar{h}^4}\left\{1 + 105\sum_{\mathrm{n}=1}^{\infty}\frac{(\mathrm{n}+1)\mathrm{X}^{2\mathrm{n}}}{3(2\mathrm{n}+5)(2\mathrm{n}+7)}\right\},$$

Where $X = \frac{C}{\bar{h}}$

$$G_3\big(\bar{h}+A,C\big) = E\left\{\frac{1}{(H+A)^2}\right\};\; G_2\big(\bar{h}+A,C\big) = E\left(\frac{1}{H+A}\right);\; G_2\big(\bar{h}+4A,C\big) = E\left(\frac{1}{H+4A}\right),$$

$$G_9\big(\bar{h},C\big) = E\{1/H^3(1+3\Sigma)\} \;= \frac{3}{64A^2}\big\{G_2\big(\bar{h}+4A,C\big) - G_2\big(\bar{h},C\big)\big\} + \frac{3}{16A}G_3\big(\bar{h},C\big) + \frac{1}{4}G_4\big(\bar{h},C\big),$$

$$G_{11}\big(\bar{h},C\big) = E\{(1+\Sigma)/H^2(1+3\Sigma)\} = \frac{1}{8A}\big\{G_2\big(\bar{h},C\big) - G_2\big(\bar{h}+4A,C\big)\big\} + \frac{1}{2}G_3\big(\bar{h},C\big),$$

$$G_{12}\big(\bar{h},C\big) = \frac{d}{d\theta}E\{1/H^3(1+3\Sigma)\}$$

$$= (-\varepsilon sin\theta)\left[\frac{3}{64A^2}\big\{-G_3\big(\bar{h}+4A,C\big) + G_3\big(\bar{h},C\big)\big\} - \frac{3}{8A}G_4\big(\bar{h},C\big) - \frac{3}{4}G_5\big(\bar{h},C\big)\right],$$

$$G_{13}\big(\bar{h},C\big) = \frac{d}{d\theta}E\{(1+\Sigma)/H^2(1+3\Sigma)\}$$
$$= (-\varepsilon sin\theta)\left[\frac{1}{8A}\big\{G_3\big(\bar{h}+4A,C\big) - G_3\big(\bar{h},C\big)\big\} - G_4\big(\bar{h},C\big)\right],$$

$$G_{14}\big(\bar{h},C\big) = \frac{d}{d\theta}\left[\frac{E\{(1+\Sigma)/\,H^2(1+3\Sigma)\}}{E\{1/\,H^3(1+3\Sigma)\}}\right] = \frac{G_9(\bar{h},C).G_{13}(\bar{h},C)-G_{11}(\bar{h},C).G_{12}(\bar{h},C)}{G_9(\bar{h},C)^2},$$

$$G_{15}(\bar{h},C) = E\{H^3(1+3\Sigma)\}$$

$$= E(H^3) + 3AE(H^2) - 3A^2E(H) + 3A^3 - 3A^4G_2(\bar{h}+A,C),$$

$$G_{16}(\bar{h},C) = E\{1/H^2(1+3\Sigma)\} = \frac{3}{16A}\{G_2(\bar{h},C) - G_2(\bar{h}+4A,C)\} + \frac{1}{4}G_3(\bar{h},C),$$

$$G_{17}(\bar{h},C) = \left(\frac{(1+\Sigma)}{(1+3\Sigma)}\right) = 1 - 2AG_2(\bar{h}+4A,C); \qquad G_{18}(\bar{h},C) = 1 + AG_2(\bar{h}+A,C).$$

4. Results and Discussion

On the basis of Christensen stochastic model, the effect of surface roughness is analyzed by assuming the porous journal bearing with heterogeneous slip/no-slip surface. The bearing characteristics depends on ε, H_o/R , ψ, A, and C. The roughness effect is analyzed by the parameter C by modifying the dimensionless slip coefficient A and permeability parameter ψ. The $H_o/R = 0.2$ is independent parameter. The limiting case of non-dimensional slip parameter $A \to 0$ corresponds to the case studied by Gururajan and Prakash [4]. It may be noted that all the results involves definite integrals can be evaluated numerically by using Gaussian quadrature 16-point formula as it gives the minimum accuracy of at least four significant digits.

The effect of surface roughness is observed on the bearing surface with increase of roughness parameter C. The parameters such as C and ε is chosen so as to obtain the maximum effect of surface roughness. The effect of surface roughness on the bearing surface is evaluated by relative difference coefficient R_W, R_F and R_{cf}, where

$$R_W = \left(\frac{W^*_{rough} - W^*_{smooth}}{W^*_{smooth}}\right) X\ 100, \quad R_F = \left(\frac{F^*_{rough} - F^*_{smooth}}{F^*_{smooth}}\right) X\ 100, \quad R_{cf} = \left(\frac{f^*_{rough} - f^*_{smooth}}{f^*_{smooth}}\right) X\ 100$$

4.1 Load Carrying Capacity

The variation of the non-dimensional load carrying capacity with the roughness parameter C, for various values of permeability parameter ψ is shown in Fig.3 for both longitudinal and transverse patterns. A significant increase is observed in W^* for longitudinal roughness when compared to transverse roughness. Also, it is observed that surface roughness effect increases W^* for the smaller values of ψ. The similar type of effect is seen in the journal of Naduvinamani [7]

Fig. 4 shows the variation of W^* with roughness C for different values of A. It is observed that W^* increases with C for both types of roughness and also, it decreases with increase of non-dimensional slip parameter A. It is more pronounced for no-slip surface ($A = 0$) when compared to slip surface. There will no significant effect on the bearing performance once A is larger than 5.

Fig. 5 and Fig. 6 shows the variation of W^* with ψ for different values of roughness C for the fixed value of parameter $A = 10$ for both types of roughness patterns. It is observed that the effect of C increases the load carrying capacity W^* as compared to smooth surface for all values of ψ. Further, it is observed that ψ dependence of W^* is not significant for values of $\psi < 0.001$. However, W^* decreases rapidly with increasing values of $\psi > 0.001$. This is because that as ψ increases, the porous facing has more voids, which permits the quick escape of the lubricants. Then from the porous facing lubricant starts discharging gradually and, therefore the modification of film thickness caused by the surface roughness will have least effect.

We observe that relative significance of load carrying capacity R_W, frictional force R_F and coefficient of friction R_{cf} of surface roughness when compared to smooth bearing are tabulated in Table 1 for different values of ψ. It seen that relative difference R_W decreases with increase of ψ for longitudinal roughness whereas increases with increase of ψ for transverse roughness for all the combinations of (ε, C). Also, it is seen that R_W decreases with decrease of C and increase of ε in both types of roughness patterns.

Table 1. *Relative differences* R_W, R_F *and* R_{cf} *for* $A = 5$

	ε	C	$\psi = 0.0$		$\psi = 0.01$		$\psi = 0.1$		$\psi = 1.0$	
			Trans. roughness	Long. roughness	Trans. roughness	Long. roughness	Trans. roughness	Long. roughness	Trans. roughness	Long. roughness
R_w	0.1	0.89	70.86	136.08	71.06	133.42	72.73	113.75	82.92	48.07
	0.3	0.69	37.79	89.93	37.90	86.85	38.76	67.39	41.147	24.51
	0.5	0.49	28.06	75.12	28.23	68.83	28.92	41.49	26.20	11.21
	0.7	0.29	21.18	65.46	21.80	47.68	21.46	16.77	14.50	3.52
	0.9	0.09	10.72	46.71	11.91	7.16	7.180	1.43	3.40	0.31
R_F	0.1	0.89	44.36	0.47	44.36	0.46	44.36	0.41	44.36	0.28
	0.3	0.69	27.06	2.22	27.06	2.10	27.06	1.39	27.06	0.32
	0.5	0.49	18.08	7.46	18.08	6.50	18.08	2.78	18.08	0.28
	0.7	0.29	11.39	19.86	11.39	13.02	11.39	2.27	11.39	0.12
	0.9	0.09	4.18	24.75	4.18	3.55	4.18	0.21	4.18	0.01
R_{cf}	0.1	0.89	15.51	57.44	15.60	56.96	16.42	53.03	-21.08	32.27
	0.3	0.69	7.78	46.18	7.86	45.36	8.43	39.43	-9.98	19.43
	0.5	0.49	7.80	38.64	7.92	36.92	8.41	27.36	-6.44	9.83
	0.7	0.29	8.08	27.56	8.54	23.47	8.29	12.42	-2.71	3.28
	0.9	0.09	5.90	14.97	6.90	3.37	2.79	1.20	0.76	0.29

Further, it is seen from that Relative difference of friction, R_F decreases with increase of ψ for longitudinal roughness but the effect is not seen much change in transverse roughness. The roughness effects caused for the relative difference of coefficient of friction, R_{cf} for several such (ε, C) combinations is as shown in Table 1. It is seen that roughness effect generally decreases with increase in ε for a fixed ψ .

4.2 Frictional Force

The variation of non-dimensional frictional force F^* with roughness parameter C for different values of ψ is shown in Fig. 7, for fixed values $A = 5$ and $\varepsilon = 0.5$. The roughness increases the frictional force compared to smooth surface (i.e, $C = 0$). This effect accented as C approaches the hydrodynamic limit ($(C + \varepsilon) \rightarrow 1$). The frictional force is less sensitive to C for longitudinal roughness, whereas transverse roughness shows significant increase in F^*. The decrease of frictional force is observed with increase of ψ.

Fig.8 depicts the variation of the non-dimensional frictional force F^* with roughness C for different values of non-dimensional slip parameter A for $\psi = 0.01$ and $\varepsilon = 0.1$. It is seen that frictional force increases for both the roughness. The increase is more significant for transverse type when compared to longitudinal. It is seen that F^* decreases with increase of non-dimensional slip parameter A. It is more pronounced when there is no slip. Once A is larger than 5, the variations in A have an insignificant effect on the bearing performance.

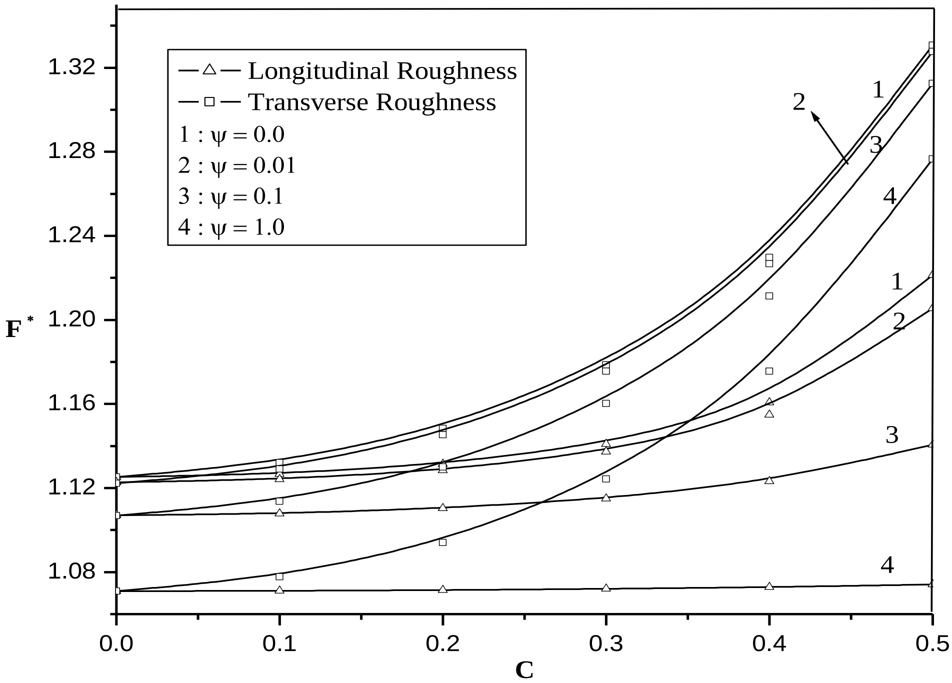


Fig.7 Non dimensional mean frictional force verses roughness parameter for different values of ψ with A=5, ε=0.5

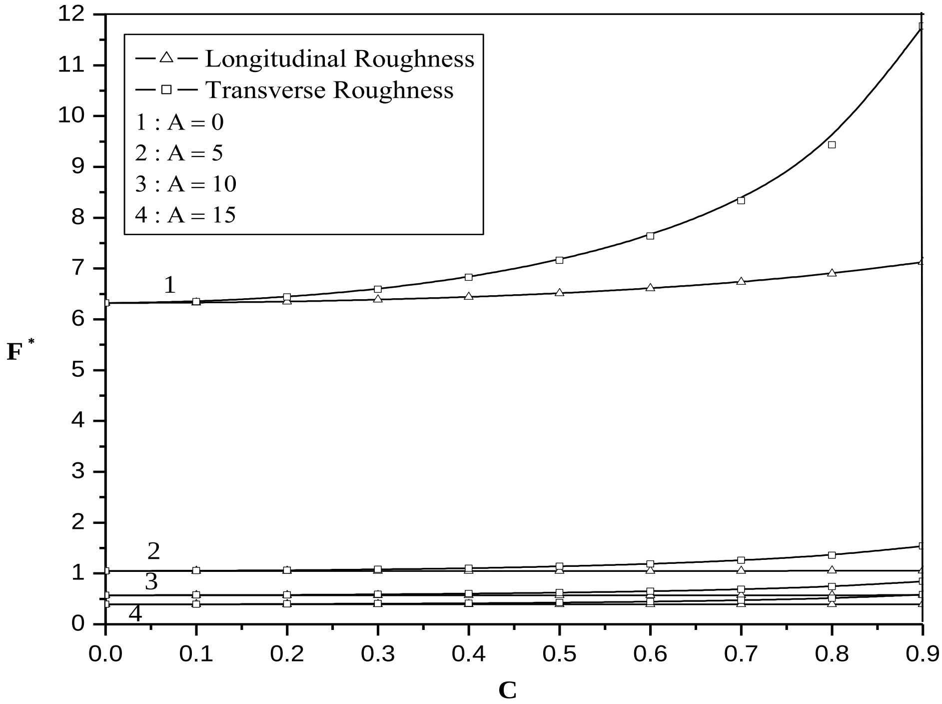


Fig.8 Non dimensional mean frictional force verses roughness parameter for different values of A with ψ=0.01, ε=0.1

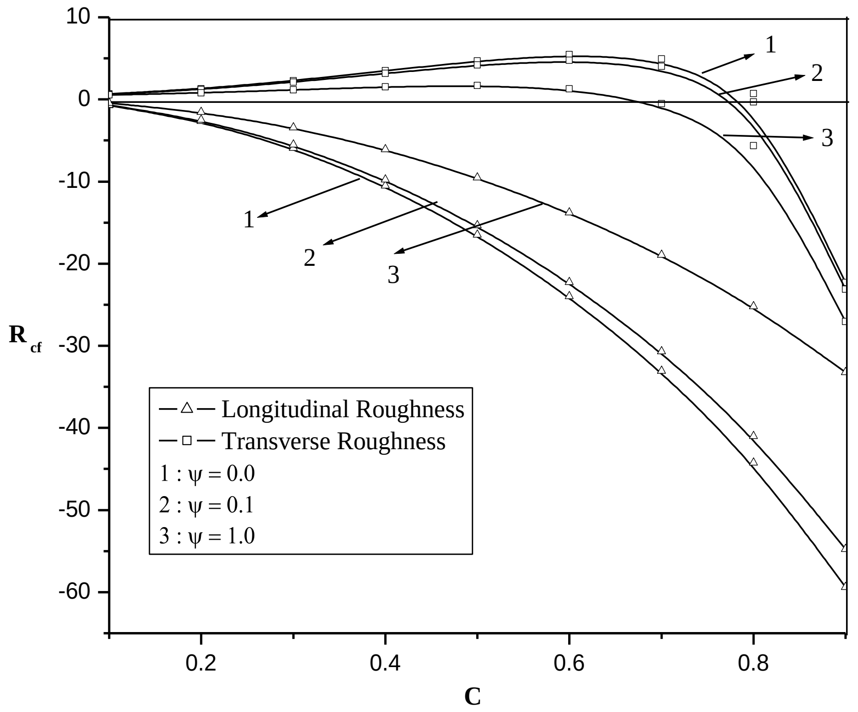


Fig.9 Relative difference of coefficient of friction R_{cf} verses roughness parameter for different values of ψ with A=5, ε=0.1

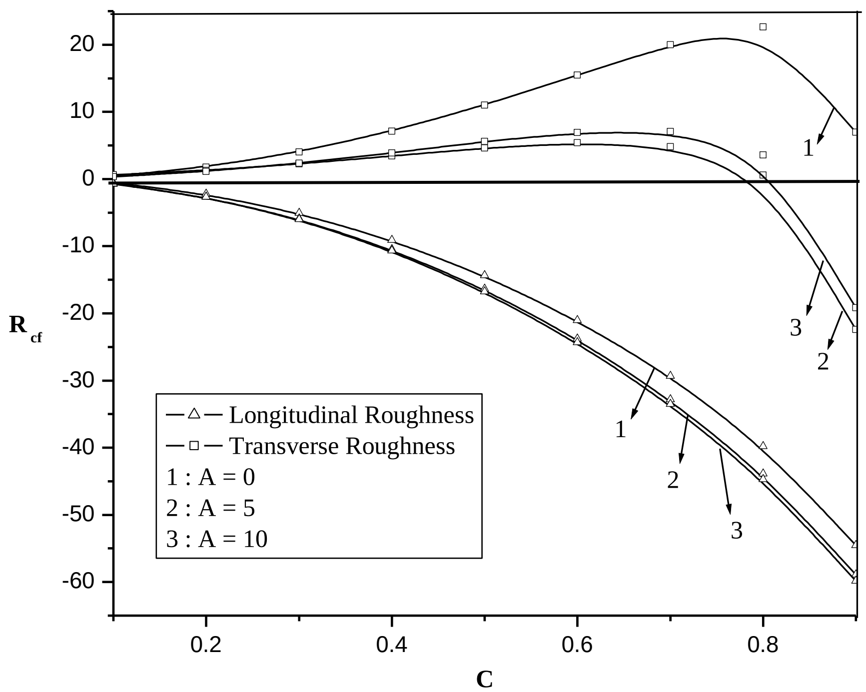


Fig.10 Relative difference of coefficient of friction R_{cf} verses roughness parameter C for different values of A with ψ=0.01, ε=0.1

4.3 Coefficient of Friction

Fig. 9 exhibits the variations of the relative difference of coefficient of friction, R_{cf} with the roughness C for different values of ψ for the slip parameter $A = 5$ and $\varepsilon = 0.1$. For longitudinal roughness, R_{cf} significantly decreases with increase of roughness when compared to transverse roughness.

Fig. 10 shows the variations of R_{cf} with the roughness C for different values of non-dimensional slip parameter A for the fixed value of ψ and ε. It is observed that the effect of roughness decreases R_{cf} for the longitudinal and transverse roughness pattern.

5. Conclusion

On the basis of Christensen stochastic theory and by Navier slip boundary condition , a theory is developed to study the influence of surface roughness in a hydrodynamically lubricated porous journal bearing with heterogeneous slip/no-slip surfaces. As an illustration, the case of an infinitely short journal bearing operating under steady condition is analyzed. It is observed that load carrying capacity, frictional force increases and coefficient of friction decreases with roughness for both the roughness pattern. As a result performance of bearing system is improved.

Nomenclature

A = dimensionless slip coefficient, $\alpha\mu/\Delta r$

Δr = radial clearance

c = maximal asperity deviation from the nominal film height

C = non – dimensional roughness parameter $(= c/\Delta r)$

e = bearing eccentricity

$E(\,)$ = expected value of

f^* = non-dimensional coefficient of friction $\left(=\left(\frac{R}{\Delta r}\,E(F)\right)\right)$

F^* = non-dimensional mean frictional force $\left(=\left(\frac{E(F)\Delta r}{\mu URL}\right)\right)$

h = nominal film height $(= (\Delta r)(1+\varepsilon\, cos\theta))$

h_s = deviation of film height from the nominal level

H = film thickness $(= h + h_s)$

$\bar{H}$ = non-dimensional nominal film height $(= H/\Delta r)$

H_0 = thickness of porous bearing

k = permeability

L = bearing length

R = journal radius

$R_{(\,)}$ = relative difference $(e.g., R_w = ((W_r^* - W_s^*)/W_s^*)100)$

p = pressure in the film thickness

$\bar{p}$ = non-dimensional mean pressure$\left(=\left(\frac{E(p)(\Delta r)^2}{\mu u_s R}\right)\right)$

u_s = Shaft surface speed

V_o = normal velocity of the lubricant at the bearing interface a with which it is getting into Pores

u, v, w = velocity components in the film thickness

W = load capacity

$W_o, W_{\pi/2}$ = load components

W^* = non- dimensional mean load capacity $\left(=\left(\frac{E(w)(\Delta r)^2}{\mu u_s R^2 L}\right)\right)$

x, y, z = Cartesian co-ordinates

α = slip coefficient

ε = eccentricity ratio $(= e/\Delta r)$

μ = viscosity of the lubricant

θ = circumferential co-ordinate $(x = R\theta)$

ξ = random variable

ψ = permeability parameter $\left(=\left(\frac{k\,H_o}{(\Delta r)^3}\right)\right)$

Σ = defined by equation (11)

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