Lossy Transmission Lines Terminated by Triode Generator Circuit

Vasil G. Angelov

Faculty of Mine Electromechanics / Department of Mathematics
University of Mining and Geology"St. I. Rilski"
1700 Sofia, Bulgaria
angelov@mgu.bg

Abstract: *In the present paper we consider lossy transmission lines terminated by a circuit consisting of in series connected CR-loads parallel to L-load. Such a configuration of nonlinear elements is a simplified scheme of triode generator circuit. On the base of Kirchhoff's law boundary conditions are derived. Then a mixed problem for lossy transmission line system is formulated. The reducing of the mixed problem leads to an initial value problem for a neutral system on the boundary. We are not able to exclude some transitional current functions and obtain a system of four equations for four unknown functions. We show that natural solutions are oscillatory ones. Further on we prove an existence-uniqueness theorem for oscillatory solution vanishing exponentially at infinity by a fixed point method.*

Keywords: *Lossy Transmission Line, Triode Generator, Initial Value Problem, Mixed Problem, Neutral System, Oscillatory Solution, Fixed Point Theorem.*

1. INTRODUCTION

The main purpose of the present paper is to consider transmission lines terminated by a circuit consisting of in series connected *CR*-loads parallel to *L*-load. Such a nonlinear circuit corresponds to a triode generator of oscillations (cf. [1]). A lot of results and applications of transmission lines terminated by various circuits of nonlinear loads can be found in [2]-[20].

Basically we follow the procedure from [20] and technique from [21]. The paper consists of eight sections. Section 2 contains derivation of boundary conditions using Kirchhoff's law. Then the mixed problem for lossy transmission line system is formulated. It must be emphasized that in this case the *CR* current function cannot be excluded and we need to consider a system of four equations instead of the usual two [20]. In Section 3 using Heaviside condition $R / L = G / C$ the mixed problem to an initial value problem on the boundary is reduced. Finally heuristic reasoning is given which justify why one should look for oscillatory and not for periodic solutions. In Section 4 we consider the arising nonlinearities and introduce suitable function spaces and corresponding families of pseudo-metrics. In Section 5 an operator presentation of the oscillatory problem is given. Section 6 contains some elementary Lipschitz estimates of the right-hand sides of the system necessary in the next section. Section 7 contains the main result − an existence-uniqueness of an oscillatory solution by fixed point method [21]. Section 8 contains a numerical example that demonstrates the advantages of our method introduced in [20].

2. DERIVATION OF BOUNDARY CONDITIONS HEADINGS

Since we consider lossy transmission lines we proceed from the following system (cf. [20])

$$C \frac{\partial u(x,t)}{\partial t} + \frac{\partial i(x,t)}{\partial x} + Gu(x,t) = 0,$$

$$L \frac{\partial i(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} + Ri(x,t) = 0 \qquad (1)$$

$$(x,t) \in \Pi = \left\{ (x,t) \in \Pi^2 : (x,t) \in [0,\Lambda] \times [0,\infty) \right\}$$

where $u(x,t)$ and $i(x,t)$ are the unknown functions (voltage and current), while *L, C, R* and *G* are prescribed specific parameters of the line. For system (1) we formulate the following mixed problem: to find $u(x,t)$ and $i(x,t)$ in Π with initial conditions

$$u(x,0) = u_0(x), \ i(x,0) = i_0(x), \ x \in [0, \Lambda]. \tag{2}$$

To obtain the mixed problem for (1) we have to derive boundary conditions for $x = 0$ and $x = \Lambda$. In accordance to Kirchhoff's law (cf. Fig. 1) we have to sum the voltages of the elements C_0 and R_0 after that to sum the current of $C_0 R_0$ and the current of L_0. We assume that the second end is terminated by the same configuration (cf. Fig. 1). If Λ is the length of the transmission line then the time for propagation of the wave along the line is $T = \Lambda/(1/\sqrt{LC}) = \Lambda\sqrt{LC}$, where $1/\sqrt{LC}$ is the speed of propagation, *L* is per unit-length inductance and *C* – per unit-length capacitance.
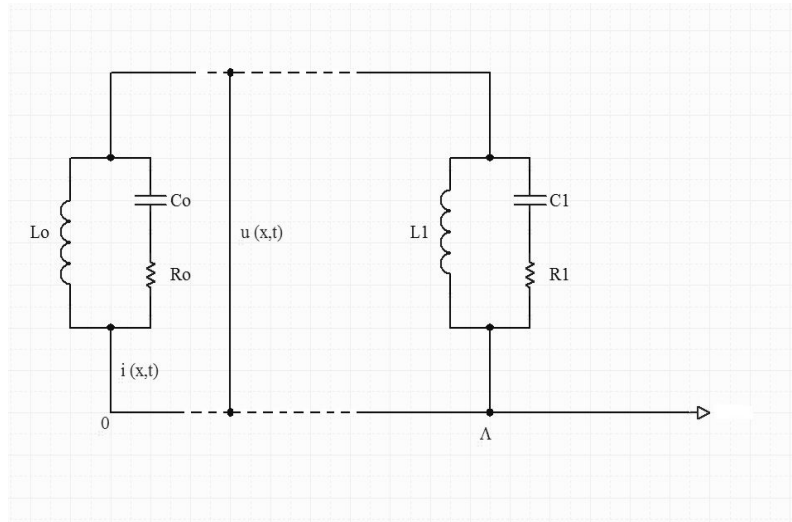

Fig. 1. *A lossy transmission line terminated by nonlinear triode generator circuit*

Assume C_p, R_p and $L_p (p = 0,1)$ are nonlinear elements, that is, $C_p = C_p(u)$, $R_p = R_p(i)$, $L_p = L_p(i)$ are prescribed nonlinear functions (characteristics). Introducing denotations

$\widetilde{C}_p(u_{C_p}) = u_{C_p}.C_p(u_{C_p})$, $\widetilde{L}_p(i_{L_p}) = i_{L_p}.L_p(i_{L_p})$ we have $u_{L_p} = \dfrac{d\Psi_p}{dt} = \dfrac{d\widetilde{L}_p(i_{L_p})}{dt}$. In particular

$$\frac{d\left(i_{L_p}.L_p(i_{L_p})\right)}{dt} = \left(L_p(i_{L_p}) + i_{L_p}\frac{dL_p(i_{L_p})}{di_{L_p}}\right)\frac{di_{L_p}}{dt},$$

$$u_{R_p} = R_p(i_{R_p}),$$

$$i_{C_p} = \frac{d\widetilde{C}_p(u_{C_p})}{dt} \equiv \frac{d\left(u_{C_p}.C_p(u_{C_p})\right)}{dt} = \left(C_p(u_{C_p}) + u_{C_p}\frac{dC_p(u_{C_p})}{du_{C_p}}\right)\frac{du_{C_p}}{dt},$$

and $u_{C_p R_p} = R_p(i_{R_p}) + \dfrac{d\widetilde{L}_p(i_{C_p R_p})}{di_{C_p R_p}}\dfrac{di_{C_p R_p}}{dt}$.

Then Kirchhoff's law yields $-i(0,t) = i_{C_0 R_0} + i_{L0}$. But $i_{L0} = \dfrac{d\widetilde{C}_0(u_{L0})}{dt}$ and $u_{L0} = u(0,t)$. Therefore

$$-i(0,t) = i_{C_0 R_0}(t) + \frac{d\widetilde{C}_0(u_{L0})}{dt} \Leftrightarrow -i(0,t) = i_{C_0 R_0}(t) + \frac{d\widetilde{C}_0(u(0,t))}{dt}.$$

On the other hand $u_{C_0} + u_{R_0} = u_{C_0 R_0} = u(0,t) = u_{L_0}$.

But $u_{C_0} = \dfrac{d\widetilde{L}_0(i_{C_0})}{dt}$ and $u_{R_0} = R_0(i_{R_0})$ and consequently

$$u_{C_0 R_0} = \frac{d\widetilde{L}_0(i_{C_0})}{dt} + R_0(i_{R_0}) \Rightarrow \frac{d\widetilde{L}_0(i_{C_0 R_0})}{dt} + R_0(i_{R_0}) = u(0,t).$$

In view of $i_{C_0} = i_{R_0} = i_{C_0 R_0}$ and $-i(0,t) = i_{C_0 R_0}(t) + i_{L_0}$ we obtain boundary conditions for (1)

$$-i(0,t) = i_{C_0 R_0}(t) + \frac{d\widetilde{C}_0(u(0,t))}{dt} \; ; \; \frac{d\widetilde{L}_0(i_{C_0 R_0})}{dt} + R_0(i_{C_0 R_0}) = u(0,t) \tag{3-1}$$

and for the right end

$$i(\Lambda,t) = i_{C_1 R_1}(t) + \frac{d\widetilde{C}_1(u(\Lambda,t))}{dt} \; ; \; \frac{d\widetilde{L}_1(i_{C_1 R_1})}{dt} + R_1(i_{C_1 R_1}) = u(\Lambda,t) \tag{3-2}$$

which jointly with initial conditions (2) form a mixed problem.

3. REDUCING THE MIXED PROBLEM TO AN INITIAL VALUE PROBLEM ON THE BOUNDARY USING THE HEAVISIDE CONDITION $R/L = G/C$

First we present (1) in the form

$$\frac{\partial u(x,t)}{\partial t} + \frac{1}{C}\frac{\partial i(x,t)}{\partial x} + \frac{G}{C}u(x,t) = 0, \quad \frac{\partial i(x,t)}{\partial t} + \frac{1}{L}\frac{\partial u(x,t)}{\partial x} + \frac{R}{L}i(x,t) = 0$$

and then in the matrix form

$$\frac{\partial U(x,t)}{\partial t} + A\frac{\partial U(x,t)}{\partial x} + BU(x,t) = 0 \tag{4}$$

where $U(x,t) = \begin{bmatrix} u(x,t) \\ i(x,t) \end{bmatrix}$, $\dfrac{\partial U(x,t)}{\partial t} = \begin{bmatrix} \dfrac{\partial u(x,t)}{\partial t} \\ \dfrac{\partial i(x,t)}{\partial t} \end{bmatrix}$, $\dfrac{\partial U(x,t)}{\partial x} = \begin{bmatrix} \dfrac{\partial u(x,t)}{\partial x} \\ \dfrac{\partial i(x,t)}{\partial x} \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1/C \\ 1/L & 0 \end{bmatrix}$, $B = \begin{bmatrix} G/C & 0 \\ 0 & R/L \end{bmatrix}$.

We follow the method from [20]. Transform the matrix $A = \begin{bmatrix} 0 & 1/C \\ 1/L & 0 \end{bmatrix}$ in diagonal form by solving the characteristic equation $\begin{vmatrix} -\lambda & 1/C \\ 1/L & -\lambda \end{vmatrix} = 0$ whose roots are $\lambda_1 = \dfrac{1}{\sqrt{LC}}$ and $\lambda_2 = -\dfrac{1}{\sqrt{LC}}$.

Denote by H the matrix formed by eigenvectors $H = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix}$ and by

$H^{-1} = \begin{bmatrix} 1/(2\sqrt{C}) & -1/(2\sqrt{C}) \\ 1/(2\sqrt{L}) & 1/(2\sqrt{L}) \end{bmatrix}$ its inverse one. Denote by $A^{can} = \begin{bmatrix} 1/\sqrt{LC} & 0 \\ 0 & -1/\sqrt{LC} \end{bmatrix} = |HAH^{-1}|$.

Introduce new variables $Z = \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix}$ such that $Z = HU$ ($U = H^{-1}Z$) where $U = \begin{bmatrix} u(x,t) \\ i(x,t) \end{bmatrix}$. Then

$$\begin{vmatrix} V(x,t) = \sqrt{C}\, u(x,t) + \sqrt{L}\, i(x,t) \\ I(x,t) = -\sqrt{C}\, u(x,t) + \sqrt{L}\, i(x,t) \end{vmatrix} \tag{5}$$

and

$$\left| \begin{array}{l} u(x,t) = \dfrac{1}{2\sqrt{C}} V(x,t) - \dfrac{1}{2\sqrt{C}} I(x,t) \\[3mm] i(x,t) = \dfrac{1}{2\sqrt{L}} V(x,t) + \dfrac{1}{2\sqrt{L}} I(x,t). \end{array} \right.$$ (6)

Substituting $U = H^{-1}Z$ in (4) we obtain

$$\frac{\partial}{\partial t} H^{-1}Z(x,t) + A \frac{\partial}{\partial x} H^{-1}Z(x,t) + B\left(H^{-1}Z(x,t)\right) = 0.$$

Multiplying the above matrix equation from the left by H we obtain

$$\frac{\partial Z(x,t)}{\partial t} + A^{can} \frac{\partial Z(x,t)}{\partial x} + H\left(BH^{-1}\right)Z(x,t) = 0.$$ (7)

Taking into account the Heaviside condition $R/L = G/C$ we obtain

$$HBH^{-1} = \begin{bmatrix} \dfrac{1}{2}\left(\dfrac{G}{C} + \dfrac{R}{L}\right) & \dfrac{1}{2}\left(-\dfrac{G}{C} + \dfrac{R}{L}\right) \\[3mm] \dfrac{1}{2}\left(-\dfrac{G}{C} + \dfrac{R}{L}\right) & \dfrac{1}{2}\left(\dfrac{G}{C} + \dfrac{R}{L}\right) \end{bmatrix} = \begin{bmatrix} \dfrac{R}{L} & 0 \\[3mm] 0 & \dfrac{R}{L} \end{bmatrix}$$

and then (7) becomes:

$$\begin{bmatrix} \dfrac{\partial V(x,t)}{\partial t} \\[3mm] \dfrac{\partial I(x,t)}{\partial t} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\sqrt{LC}} & 0 \\[3mm] 0 & -\dfrac{1}{\sqrt{LC}} \end{bmatrix} \begin{bmatrix} \dfrac{\partial V(x,t)}{\partial x} \\[3mm] \dfrac{\partial I(x,t)}{\partial x} \end{bmatrix} + \begin{bmatrix} \dfrac{R}{L} & 0 \\[3mm] 0 & \dfrac{R}{L} \end{bmatrix} \begin{bmatrix} V(x,t) \\[3mm] I(x,t) \end{bmatrix} = \begin{bmatrix} 0 \\[3mm] 0 \end{bmatrix}.$$ (8)

The new initial conditions for $x \in [0, \Lambda]$ become

$$V(x,0) = \sqrt{C}\, u(x,0) + \sqrt{L}\, i(x,0) = \sqrt{C}\, u_0(x) + \sqrt{L}\, i_0(x) \equiv V_0(x),$$ (9)

$$I(x,0) = -\sqrt{C}\, u(x,0) + \sqrt{L}\, i(x,0) = -\sqrt{C}\, u_0(x) + \sqrt{L}\, i_0(x) \equiv I_0(x).$$ (10)

We can simplify (8) by the following substitution:

$$V(x,t) = e^{-\frac{R}{L}t} W(x,t), \; I(x,t) = e^{-\frac{R}{L}t} J(x,t),$$ (11)

or

$$W(x,t) = e^{\frac{R}{L}t} V(x,t), \; J(x,t) = e^{\frac{R}{L}t} I(x,t).$$ (12)

Replacing (11) in (6) we obtain

$$\left| \begin{array}{l} u(x,t) = \dfrac{e^{-\frac{R}{L}t} W(x,t)}{2\sqrt{C}} - \dfrac{e^{-\frac{R}{L}t} J(x,t)}{2\sqrt{C}} \\[5mm] i(x,t) = \dfrac{e^{-\frac{R}{L}t} W(x,t)}{2\sqrt{L}} + \dfrac{e^{-\frac{R}{L}t} J(x,t)}{2\sqrt{L}}. \end{array} \right.$$ (13)

Rewrite (8) in the form:

$$\left|\begin{array}{l}\dfrac{\partial V(x,t)}{\partial t}+\dfrac{1}{\sqrt{LC}}\dfrac{\partial V(x,t)}{\partial x}+\dfrac{R}{L}V(x,t)=0,\\[2mm]\dfrac{\partial I(x,t)}{\partial t}-\dfrac{1}{\sqrt{LC}}\dfrac{\partial I(x,t)}{\partial x}+\dfrac{R}{L}I(x,t)=0\end{array}\right. \tag{14}$$

and substituting $V(x,t)$ and $I(x,t)$ from (11) into (14) we obtain

$$\left|\begin{array}{l}\dfrac{\partial W(x,t)}{\partial t}+\dfrac{1}{\sqrt{LC}}\dfrac{\partial W(x,t)}{\partial x}=0,\\[2mm]\dfrac{\partial J(x,t)}{\partial t}-\dfrac{1}{\sqrt{LC}}\dfrac{\partial J(x,t)}{\partial x}=0.\end{array}\right. \tag{15}$$

The mixed problem for (15) can be reduced to an equivalent initial value problem for a neutral system on the boundary (cf. [20]).

Let us put (13) into the boundary conditions (3-1):

$$\frac{d\widetilde{C}_0(e^{-\frac{R}{L}t}\left(W(0,t)-J(0,t)\right)/2\sqrt{C})}{du}\,\frac{d}{dt}\left(\frac{e^{-\frac{R}{L}t}\left(W(0,t)-J(0,t)\right)}{2\sqrt{C}}\right)=i_{C_0R_0}(t)-\frac{e^{-\frac{R}{L}t}W(0,t)}{2\sqrt{L}}-\frac{e^{-\frac{R}{L}t}J(0,t)}{2\sqrt{L}};$$

$$\frac{d\widetilde{L}_0(i_{C_0R_0})}{di_{C_0R_0}}\,\frac{di_{C_0R_0}}{dt}=\frac{e^{-\frac{R}{L}t}\left(W(0,t)-J(0,t)\right)}{2\sqrt{C}}-R_0(i_{C_0R_0})\ ;$$

$$\frac{d\widetilde{C}_1(e^{-\frac{R}{L}t}\left(W(\Lambda,t)-J(\Lambda,t)\right)/2\sqrt{C})}{du}\,\frac{d}{dt}\frac{e^{-\frac{R}{L}t}W(\Lambda,t)-e^{-\frac{R}{L}t}J(\Lambda,t)}{2\sqrt{C}}=\frac{e^{-\frac{R}{L}t}W(\Lambda,t)}{2\sqrt{L}}+\frac{e^{-\frac{R}{L}t}J(\Lambda,t)}{2\sqrt{L}}-i_{C_1R_1}(t);$$

$$\frac{d\widetilde{L}_1(i_{C_1R_1})}{di_{C_1R_1}}\,\frac{di_{C_1R_1}}{dt}=\frac{e^{-\frac{R}{L}t}W(\Lambda,t)-e^{-\frac{R}{L}t}J(\Lambda,t)}{2\sqrt{C}}-R_1(i_{C_1R_1}).$$

Choose $W(0,t)=W(t)$, $J(t)=J(\Lambda,t)$ to be the unknown functions. Then in view of $W(0,t)=W(\Lambda,t+T)$, $J(0,t+T)=J(\Lambda,t)$ (cf. [20]) and $(RT)/L=\left(R\Lambda\sqrt{LC}\right)/L=(R\Lambda)/Z_0$ we have:

$$\frac{d}{dt}i_{C_0R_0}(t)=\frac{1}{d\tilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\times\left[\frac{e^{-\frac{R}{L}t}W(t)-e^{-\frac{R}{L}(t-T)}J(t-T)}{2\sqrt{C}}-R_0(i_{C_0R_0}(t))\right];$$

$$\frac{d}{dt}\frac{e^{-\frac{R}{L}t}W(t)-e^{-\frac{R}{L}(t-T)}J(t-T)}{2\sqrt{C}}=\frac{i_{C_0R_0}(t)-\dfrac{e^{-\frac{R}{L}t}W(t)-e^{-\frac{R}{L}(t-T)}J(t-T)}{2\sqrt{L}}}{\dfrac{d}{du}\tilde{C}_0\left(\left(e^{-\frac{R}{L}t}W(t)-e^{-\frac{R}{L}(t-T)}J(t-T)\right)/2\sqrt{C}\right)};$$

$$\frac{d}{dt}i_{C_1R_1}(t)=\frac{1}{d\tilde{L}_1(i_{C_1R_1})/di_{C_1R_1}}\times\left[\frac{e^{-\frac{R}{L}(t-T)}W(t-T)-e^{-\frac{R}{L}t}J(t)}{2\sqrt{C}}-R_1(i_{C_1R_1}(t))\right];$$

$$\frac{d}{dt}\frac{e^{-\frac{R}{L}(t-T)}W(t-T)-e^{-\frac{R}{L}t}J(t)}{2\sqrt{C}}=\frac{\dfrac{e^{-\frac{R}{L}(t-T)}W(t-T)+e^{-\frac{R}{L}t}J(t)}{2\sqrt{L}}-i_{C_1R_1}(t)}{\dfrac{d}{du}C_1\!\left(\!\left(e^{-\frac{R}{L}(t-T)}W(t-T)-e^{-\frac{R}{L}t}J(t)\right)\!/2\sqrt{C}\right)}. \tag{16}$$

A natural problem is to formulate conditions for existence-uniqueness of a periodic solution of the above system. We make arguments, however, which justify why we should look for oscillating and not periodic solutions. First we notice that the right-hand sides of the above system are not periodic functions. Therefore it is a natural to look for solutions of the type $e^{-\frac{R}{L}t}W(t)$, $e^{-\frac{R}{L}t}J(t)$, where $W(t)$, $J(t)$ might be periodic functions. Then the functions $\widehat{W}(t)=e^{-\frac{R}{L}t}W(t)$ and $\widehat{J}(t)=e^{-\frac{R}{L}t}J(t)$ become an oscillatory solution and (16) could be rewritten as

$$\frac{di_{C_0R_0}(t)}{dt}=\frac{1}{d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\left[\frac{\widehat{W}(t)-\widehat{J}(t-T)}{2\sqrt{C}}-R_0(i_{C_0R_0}(t))\right]; \tag{17}$$

$$\frac{d\widehat{W}(t)}{dt}=\frac{d\widehat{J}(t-T)}{dt}+2\sqrt{C}\frac{i_{C_0R_0}(t)-\left(\widehat{W}(t)-\widehat{J}(t-T)\right)/2\sqrt{L}}{d\widetilde{C}_0\!\left(\left(\widehat{W}(t)-\widehat{J}(t-T)\right)/2\sqrt{C}\right)/du};$$

$$\frac{di_{C_1R_1}(t)}{dt}=\frac{1}{\widetilde{L}_1(i_{C_1R_1})/di_{C_1R_1}}\left[\frac{\widehat{W}(t-T)-\widehat{J}(t)}{2\sqrt{C}}-R_1(i_{C_1R_1}(t))\right];$$

$$\frac{d\widehat{J}(t)}{dt}=\frac{d\widehat{W}(t-T)}{dt}-2\sqrt{C}\frac{\left(\widehat{W}(t-T)+\widehat{J}(t)\right)/2\sqrt{L}-i_{C_1R_1}(t)}{d\widetilde{C}_1\!\left(\widehat{W}(t-T)-\widehat{J}(t)/2\sqrt{C}\right)/du}.$$

4. Estimates of the Arising Nonlinearities and Introducing a Family of Pseudo-Metrics

Here we assume the nonlinear capacitive elements possess characteristics $C_p(u)=\dfrac{c_p}{\sqrt[h]{1-(u/\Phi_p)}}$, where $c_p>0$, $\Phi_p>0$, $h\in[2,3]$ ($p=0,1$) are constants and $|u|\le\phi_0<\hat{\Phi}=\min\{\Phi_0,\Phi_1\}$.

If $u\in[-\phi_0,\phi_0]$ then $\widetilde{C}_p(u)=C_p(u).u$ and $d\widetilde{C}_p(u)/du$ have strictly positive lower bounds.

Indeed

$$\frac{d\widetilde{C}_p(u)}{du}=C_p(u)+u\frac{dC_p(u)}{du}=c_p\sqrt[h]{\Phi_p}\left(\Phi_p-\frac{h-1}{h}u\right)\!/(\Phi_p-u)^{\frac{1}{h}+1},$$

$$\frac{d^2\widetilde{C}_p(u)}{du^2}=2\frac{dC_p(u)}{du}+u\frac{d^2C_p(u)}{du^2}=\frac{2c_p\sqrt[h]{\Phi_p}}{h}\left(\Phi_p-\frac{2h+1}{h}u\right)\!/(\Phi_p-u)^{\frac{1}{h}+2}.$$

If we choose $\phi_0<\hat{\Phi}\min\left\{\dfrac{h}{1+2h},\dfrac{h}{h-1}\right\}=\dfrac{h}{1+2h}$ it follows $d\widetilde{C}_p(u)/du>0$ and $d^2\widetilde{C}_p(u)/du^2>0$ for $u\in[-\phi_0,\phi_0]$. Therefore

$$\min\left\{\frac{d\widetilde{C}_p(u)}{du}:|u|\le\phi_0\right\}=\frac{d\widetilde{C}_p(-\phi_0)}{du}=c_p\sqrt[h]{\Phi_p}\frac{\Phi_p+((h-1)/h)\phi_0}{\left(\Phi_p+\phi_0\right)^{\frac{1}{h}+1}}=\widehat{C}_p^1>0,(p=0,1).$$

We need also the estimates

$$\left|\frac{d\widetilde{C}_p(u)}{du}\right| \le \frac{2c_p\sqrt[h]{\Phi_p}}{\left(\Phi_p - \phi_0\right)^{\frac{1}{h}+2}} \frac{\Phi_p + ((2h+1)/h)\phi_0}{h} \equiv \breve{C}_p^{\ 1},$$

$$\left|\frac{d^2\widetilde{C}_p(u)}{du^2}\right| \le 2c_p\sqrt[h]{\Phi_p} \frac{h(\Phi_p - \phi_0) + |u|(1+h)}{h^2 \sqrt[h]{\left(\Phi_p - \phi_0\right)^{1+2h}}} \le \frac{2c_p\sqrt[h]{\Phi_p}\left(h\Phi_p + \phi_0\right)}{h^2 \sqrt[h]{\left(\Phi_p - \phi_0\right)^{1+2h}}} = \breve{C}_p^{\ 2} \quad (p = 0,1).$$

Denoting $\widehat{W}(t)$, $\widehat{J}(t)$ again by $W(t)$, $J(t)$ we consider the problem for existence-uniqueness of an oscillatory solution of the following system:

$$\frac{di_{C_0R_0}(t)}{dt} = \frac{1}{d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}} \left[\frac{W(t) - J(t-T)}{2\sqrt{C}} - R_0(i_{C_0R_0}(t))\right] \equiv I_{C_0}(i_{C_0R_0}, W, J);$$

$$\frac{dW(t)}{dt} = \frac{dJ(t-T)}{dt} + 2\sqrt{C} \frac{i_{C_0R_0}(t) - \dfrac{W(t) - J(t-T)}{2\sqrt{L}}}{d\widetilde{C}_0\left(\dfrac{W(t) - J(t-T)}{2\sqrt{L}}\right)/du} \equiv U(i_{C_0R_0}, W, J); \tag{18}$$

$$\frac{di_{C_1R_1}(t)}{dt} = \frac{1}{d\widetilde{L}_1(i_{C_1R_1})/di_{C_1R_1}} \left[\frac{W(t-T) - J(t)}{2\sqrt{C}} - R_1(i_{C_1R_1}(t))\right] \equiv I_{C_1}(W, i_{C_1R_1}, J);$$

$$\frac{dJ(t)}{dt} = \frac{dW(t-T)}{dt} + \frac{\dfrac{W(t-T) - J(t)}{2\sqrt{C}} - i_{C_1R_1}(t)}{d\widetilde{C}_1\left(\dfrac{W(t-T) - J(t)}{2\sqrt{C}}\right)/du} \equiv I(W, i_{C_1R_1}, J)$$

for $t \in [T, \infty)$ with initial conditions

$$i_{C_0R_0}(T) = i_{C_0R_0}^{(0)} = 0, \ W(t) = \widetilde{W}_0(t), \ i_{C_1R_1}(T) = i_{C_1R_1}^{(0)} = 0, \ J(t) = \widetilde{J}_0(t), \ t \in [0,T].$$

We look for a solution of (18) with advanced prescribed zeroes on $[t_0, \infty)$, $T \equiv t_0$, where $\widetilde{W}_0(t)$, $\widetilde{J}_0(t)$, $t \in [0, t_0]$ are prescribed initial oscillatory functions on the interval $[0, t_0 \equiv T]$.

Let $S_T = \{\tau_k\}_{k=0}^n, n \in N$ be the set of zeroes of the initial function, that is, $\widetilde{W}_0(\tau_k) = 0$, $\widetilde{J}_0(\tau_k) = 0$, such that $\tau_0 = 0$, $\tau_n = T \equiv t_0$. Besides $\max\{\tau_{k+1} - \tau_k : k = 0,1,...,n\} \le T_0$.

Let $S = \{t_k\}_{k=0}^\infty$ be a strictly increasing sequence of real numbers satisfying the following conditions **(C)**:

(C1) $\lim_{k\to\infty} t_k = \infty$; **(C2)** for every k there is $s < k$ such that $t_k - T = t_s$ where $t_s \in S_T \cup S$.

It follows
(C3) $0 < \inf\{t_{k+1} - t_k : k = 0,1,2,...\} \le \sup\{t_{k+1} - t_k : k = 0,1,2,...\} = T_0 < \infty$ and $\tau_k + T = t_k$.

Introduce the set $C^1[t_0, \infty)$ consisting of all continuous differentiable bounded functions with bounded derivatives on every interval $[t_k, t_{k+1}]$.

Assumption (IN-1): The initial functions $\widetilde{W}_0(.)$, $\widetilde{J}_0(.) \in C_{T_0}^1[0,T]$ are prescribed oscillatory ones on $[0, t_0]$ with zero points $S_T = \{\tau_k\}_{k=0}^n$.

Remark 4.1. Let us note the left and right derivatives at t_k of $W(.)$, $J(.) \in C^1[t_0, \infty)$ may not coincide. That is why we introduce a suitable topology for continuous functions with piecewise continuous derivatives.

In view of the above calculations we make the following

Assumption (L): $\dfrac{d\widetilde{L}_p(i)}{di} \geq \hat{L}_p^{\,1} > 0$ $(p = 0,1)$.

Assumption (IN-2): The initial functions $\widetilde{W}_0(.)$, $\widetilde{J}_0(.) \in C^1_{T_0}[0, T]$ satisfy the inequalities

$$\left|\widetilde{W}_0(t)\right| \leq W_0 e^{-\beta} e^{-\frac{R}{L}(t+T)} \; ; \quad \left|\widetilde{J}_0(t)\right| \leq J_0 e^{-\beta} e^{-\frac{R}{L}(t+T)}, \; t \in [0, T].$$

It follows $\left|\widetilde{W}_0(t)\right| \leq W_0 e^{\mu(t-\tau_k)}$, $\left|\widetilde{J}_0(t)\right| \leq J_0 e^{\mu(t-\tau_k)}$ for $t \in [\tau_k, \tau_{k+1}]$ $(k = 0,1,2,...,n-1)$.

Let us put $\vec{W}_0(t) = \widetilde{W}_0(t-T)$, $\vec{J}_0(t) = \widetilde{J}_0(t-T)$, $t \in [T, 2T]$. Then

$$\left|\vec{W}_0(t)\right| = \left|\widetilde{W}_0(t-T)\right| \leq W_0 e^{\mu(t-T-\tau_k)} \leq W_0 e^{\mu(t-t_k)}, \quad \left|\vec{J}_0(t)\right| = \left|\widetilde{J}_0(t-T)\right| \leq J_0 e^{\mu(t-t_k)} \quad \text{for} \quad t \in [t_k, t_{k+1}]$$
$(k = 0,1,2,...,n-1)$.

Assumption (C): $e^{\mu T_0}(W_0 + J_0)/2\sqrt{C} \leq \phi_0$.

It follows

$$\left|u(0,t)\right| \leq \frac{|W(t)| + |J(t-T)|}{2\sqrt{C}} = \frac{|W(t)| + |\vec{J}_0(t)|}{2\sqrt{C}} \leq e^{\mu(t-t_k)} \frac{W_0 + J_0}{2\sqrt{C}} \leq e^{\mu T_0} \frac{W_0 + J_0}{2\sqrt{C}} \leq \phi_0;$$

$$\left|u(\Lambda,t)\right| \leq \frac{|W(t-T)| + |J(t)|}{2\sqrt{C}} = \frac{|\vec{W}_0(t)| + |J(t)|}{2\sqrt{C}} \leq e^{\mu(t-t_k)} \frac{W_0 + J_0}{2\sqrt{C}} \leq e^{\mu T_0} \frac{W_0 + J_0}{2\sqrt{C}} \leq \phi_0.$$

For the resistors we use *V-I* characteristics with polynomial type nonlinearities:

Assumption (R): $R_p(i_{C_p R_p}) = \sum\limits_{n=1}^{m} r_n^{(p)}(i_{C_p R_p})^n$, $(p = 0,1)$.

Introduce the sets

$$M_0 = \left\{ i_{C_0 R_0}(.) \in C^1[t_0, \infty) : i_{C_0 R_0}(t_k) = 0 \wedge \left| i_{C_0 R_0}(t) \right| \leq I_{C_0} e^{-\frac{R}{L}t}, \; t \in [t_k, t_{k+1}] \right\},$$

$$M_W = \left\{ W(.) \in C^1[t_0, \infty) : W(t_k) = 0 \wedge |W(t)| \leq W_0 e^{-\frac{R}{L}t}, t \in [t_k, t_{k+1}] \right\},$$

$$M_1 = \left\{ i_{C_1 R_1}(.) \in C^1[t_0, \infty) : i_{C_1 R_1}(t_k) = 0 \wedge \left| i_{C_1 R_1}(t) \right| \leq I_{C_1} e^{-\frac{R}{L}t}, t \in [t_k, t_{k+1}] \right\},$$

$$M_J = \left\{ J(.) \in C^1[t_0, \infty) : J(t_k) = 0 \wedge |J(t)| \leq J_0 e^{-\frac{R}{L}t}, t \in [t_k, t_{k+1}] \right\}.$$

Remark 4.2. It follows that the functions from M_0, M_W, M_1 and M_J satisfy the inequalities

$$\left| i_{C_0 R_0}(t) \right| \leq I_{C_0} e^{\mu(t-t_k)}, \; |W(t)| \leq W_0 e^{\mu(t-t_k)}, \; \left| i_{C_1 R_1}(t) \right| \leq I_{C_1} e^{\mu(t-t_k)}, \; |J(t)| \leq J_0 e^{\mu(t-t_k)}, t \in [t_k, t_{k+1}]$$

$(k = 0,1,2,...$ $)$ where $I_{C_0}, W_0, I_{C_1}, J_0, \mu, \mu T_0 = \mu_0$ are positive constants.

Introduce the following family of pseudo-metrics

$$\rho^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) = \max\left\{\left|i_{C_pR_p}(t) - \bar{i}_{C_pR_p}(t)\right| : t \in [t_k, t_{k+1}]\right\}, (p = 0,1);$$

$$\rho^{(k)}(W, \overline{W}) = \max\left\{\left|W(t) - \overline{W}(t)\right| : t \in [t_k, t_{k+1}]\right\}, \quad \rho^{(k)}(J, \bar{J}) = \max\left\{\left|J(t) - \bar{J}(t)\right| : t \in [t_k, t_{k+1}]\right\};$$

$$\hat{\rho}^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) = \max\left\{\left|i_{C_pR_p}(t) - \bar{i}_{C_pR_p}(t)\right| : t \in [t_0, t_{k+1}]\right\}, (p = 0,1);$$

$$\hat{\rho}^{(k)}(W, \overline{W}) = \max\left\{\left|W(t) - \overline{W}(t)\right| : t \in [t_0, t_{k+1}]\right\};$$

$$\hat{\rho}^{(k)}(J, \bar{J}) = \max\left\{\left|J(t) - \bar{J}(t)\right| : t \in [t_0, t_{k+1}]\right\};$$

$$\rho_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) = \max\left\{\left|i_{C_pR_p}(t) - \bar{i}_{C_pR_p}(t)\right| : t \in [t_0, t_{k+1}]\right\}, (p = 0,1);$$

$$\rho_\mu^{(k)}(W, \overline{W}) = \max\left\{e^{-\mu(t-t_k)}\left|W(t) - \overline{W}(t)\right| : t \in [t_k, t_{k+1}]\right\};$$

$$\rho_\mu^{(k)}(J, \bar{J}) = \max\left\{e^{-\mu(t-t_k)}\left|J(t) - \bar{J}(t)\right| : t \in [t_k, t_{k+1}]\right\};$$

$$\hat{\rho}_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) = \max\left\{\rho_\mu^{(0)}(i_{C_pR_p}, \bar{i}_{C_pR_p}), \rho_\mu^{(1)}(i_{C_pR_p}, \bar{i}_{C_pR_p}), ..., \rho_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p})\right\}, (p = 0,1);$$

$$\hat{\rho}_\mu^{(k)}(W, \overline{W}) = \max\left\{\rho_\mu^{(0)}(W, \overline{W}), ..., \rho_\mu^{(k)}(W, \overline{W})\right\};$$

$$\hat{\rho}_\mu^{(k)}(J, \bar{J}) = \max\left\{\rho_\mu^{(0)}(J, \bar{J}), ..., \rho_\mu^{(k)}(J, \bar{J})\right\};$$

$$\rho_\mu^{(k)}(\dot{i}_{C_pR_p}, \dot{\bar{i}}_{C_pR_p}) = \max\left\{e^{-\mu(t-t_k)}\left|\dot{i}_{C_pR_p}(t) - \dot{\bar{i}}_{C_pR_p}(t)\right| : t \in [t_k, t_{k+1}]\right\}, (p = 0,1);$$

$$\rho_\mu^{(k)}(\dot{W}, \dot{\overline{W}}) = \max\left\{e^{-\mu(t-t_k)}\left|\dot{W}(t) - \dot{\overline{W}}(t)\right| : t \in [t_k, t_{k+1}]\right\};$$

$$\rho_\mu^{(k)}(\dot{J}, \dot{\bar{J}}) = \max\left\{e^{-\mu(t-t_k)}\left|\dot{J}(t) - \dot{\bar{J}}(t)\right| : t \in [t_k, t_{k+1}]\right\};$$

$$\hat{\rho}_\mu^{(k)}(\dot{i}_{R_0L_0}, \dot{\bar{i}}_{R_0L_0}) = \max\left\{\rho_\mu^{(0)}(\dot{i}_{C_pR_p}, \dot{\bar{i}}_{C_pR_p}), ..., \rho_\mu^{(k)}(\dot{i}_{C_pR_p}, \dot{\bar{i}}_{C_pR_p})\right\};$$

$$\hat{\rho}_\mu^{(k)}(\dot{W}, \dot{\overline{W}}) = \max\left\{\rho_\mu^{(0)}(\dot{W}, \dot{\overline{W}}), ..., \rho_\mu^{(k)}(\dot{W}, \dot{\overline{W}})\right\}; \quad \hat{\rho}_\mu^{(k)}(\dot{J}, \dot{\bar{J}}) = \max\left\{\rho_\mu^{(0)}(\dot{J}, \dot{\bar{J}}), ..., \rho_\mu^{(k)}(\dot{J}, \dot{\bar{J}})\right\}.$$

The following inequalities imply the equivalence of the families of pseudo-metrics

$$\rho_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) \le \rho^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) \le e^{\mu T_0} \rho_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}),$$

$$\rho_\mu^{(k)}(W, \overline{W}) \le \rho^{(k)}(W, \overline{W}) \le e^{\mu T_0} \rho_\mu^{(k)}(W, \overline{W}), \rho_\mu^{(k)}(J, \bar{J}) \le \rho^{(k)}(J, \bar{J}) \le e^{\mu T_0} \rho_\mu^{(k)}(J, \bar{J}) \ (k = 0,1,2,...).$$

It is easy to verify that

$$\hat{\rho}^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}) = \max\left\{\rho^{(0)}(i_{C_pR_p}, \bar{i}_{C_pR_p}), ..., \rho^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p})\right\} \le$$

$$\le e^{\mu T_0} \max\left\{\rho_\mu^{(0)}(i_{C_pR_p}, \bar{i}_{C_pR_p}), ..., \rho_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p})\right\} = e^{\mu_0} \hat{\rho}_\mu^{(k)}(i_{C_pR_p}, \bar{i}_{C_pR_p}), (p = 0,1);$$

$$\hat{\rho}^{(k)}(W, \overline{W}) = \max\left\{\rho^{(0)}(W, \overline{W}), \rho^{(1)}(W, \overline{W}), ..., \rho^{(k)}(W, \overline{W})\right\} \le$$

$$\le e^{\mu T_0} \max\left\{\rho_\mu^{(0)}(W, \overline{W}), \rho_\mu^{(1)}(W, \overline{W}), ..., \rho_\mu^{(k)}(W, \overline{W})\right\} = e^{\mu_0} \hat{\rho}_\mu^{(k)}(W, \overline{W});$$

$$\hat{\rho}^{(k)}(J, \bar{J}) = \max\left\{\rho^{(0)}(J, \bar{J}), \rho^{(1)}(J, \bar{J}), ..., \rho^{(k)}(J, \bar{J})\right\} \le$$

$$\le e^{\mu T_0} \max\left\{\rho_\mu^{(0)}(J, \bar{J}), \rho_\mu^{(1)}(J, \bar{J}), ..., \rho_\mu^{(k)}(J, \bar{J})\right\} == e^{\mu_0} \hat{\rho}_\mu^{(k)}(J, \bar{J}).$$

The set $M_0 \times M_W \times M_1 \times M_J$ turns out into a complete uniform space with respect to the family of pseudo-metrics (cf. [21]):

$$\hat{\rho}_{\mu}^{(k)}\Big(\big(i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}\big),\big(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}}\big)\Big)=$$

$$=\max\Big\{\hat{\rho}^{(k)}(i_{C_0R_0},\bar{i}_{C_0R_0}),\hat{\rho}^{(k)}(W,\overline{W}),\hat{\rho}^{(k)}(i_{C_1R_1},\bar{i}_{C_1R_1}),\hat{\rho}^{(k)}(J,\overline{J}),$$

$$\hat{\rho}_{\mu}^{(k)}(\dot{i}_{C_0R_0},\dot{\bar{i}}_{C_0R_0}),\hat{\rho}_{\mu}^{(k)}(\dot{W},\dot{\overline{W}}),\hat{\rho}_{\mu}^{(k)}(\dot{i}_{C_1R_1},\dot{\bar{i}}_{C_1R_1}),\hat{\rho}_{\mu}^{(k)}(\dot{J},\dot{\overline{J}})\Big\}\ (k=0,1,2,...).$$

5. OPERATOR PRESENTATION OF THE OSCILLATORY PROBLEM

Replacing $t=t_0$ in (18) we obtain conformity condition:

$$\frac{dW(t_0)}{dt}=\frac{dJ(0)}{dt}-\frac{W(t_0)-J(0)+2\sqrt{L}\,i_{C_0R_0}(t_0)}{Z_0d\widetilde{C}_0(u(0,t_0))/du};$$

$$\frac{dJ(t_0)}{dt}=\frac{dW(0)}{dt}+\frac{W(0)+J(t_0)+2\sqrt{L}\,i_{C_1R_1}(t_0)}{Z_0d\widetilde{C}_1(u(\Lambda,t_0))/du}.$$

(CC)

We choose $t=t_0$ and $t=0$ to be zero points of the initial functions and then

$$\frac{dW(0)}{dt}=\frac{dJ(0)}{dt}=\frac{dW(t_0)}{dt}=\frac{dJ(t_0)}{dt}=0,\ W(0)=J(0)=W(t_0)=J(t_0)=0,\ i_{C_0R_0}(t_0)=i_{C_1R_1}(t_0)=0$$

imply **(CC).**

Define the operator

$$B=\Big(B_0(i_{C_0R_0},W,i_{C_1R_1},J),B_W(i_{C_0R_0},W,i_{C_1R_1},J),B_1(i_{C_0R_0},W,i_{C_1R_1},J),B_J(i_{C_0R_0},W,i_{C_1R_1},J)\Big)\quad(19)$$

by the formulas

$$B_0(i_{C_0R_0},W,i_{C_1R_1},J)=B_0^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J):=\int_{t_k}^{t}I_{C_0}(i_{C_0R_0},W,J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}}I_{C_0}(i_{C_0R_0},W,J)(s)ds,$$

$$B_W(i_{C_0R_0},W,i_{C_1R_1},J)(t)=B_W^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J)(t):=\int_{t_k}^{t}U(i_{C_0R_0},W,J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}}U(i_{C_0R_0},W,J)(s)ds,$$

$$B_1(i_{C_0R_0},W,i_{C_1R_1},J)=B_1^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J):=\int_{t_k}^{t}I_{C_1}(W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}}I_{C_1}(W,i_{C_1R_1},J)(s)ds,$$

$$B_J(i_{C_0R_0},W,i_{C_1R_1},J)(t)=B_J^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J)(t):=\int_{t_k}^{t}I(W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}}I(W,i_{C_1R_1},J)(s)ds$$

for $t\in[t_k,t_{k+1}]$, $(k=0,1,2,...)$, where

$$I_{C_0}(i_{C_0R_0},W,J)=\frac{1}{d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\left[\frac{W(t)-\bar{J}(t)}{2\sqrt{C}}-R_0(i_{C_0R_0}(t))\right];$$

$$U(i_{C_0R_0},W,J)=\frac{d\bar{J}(t)}{dt}+2\sqrt{C}\,\frac{i_{C_0R_0}(t)-\big(W(t)-\bar{J}(t)\big)/2\sqrt{C}}{d\widetilde{C}_0\!\left(\dfrac{W(t)-\bar{J}(t)}{2\sqrt{C}}\right)/du};$$

$$I_{C_1}(W,i_{C_1R_1},J)=\frac{1}{d\widetilde{L}_1(i_{C_1R_1})/di_{C_1R_1}}\left[\frac{\overline{W}(t)-J(t)}{2\sqrt{C}}-R_1(i_{C_1R_1}(t))\right];$$

$$I(W, i_{C_1R_1}, J) = \frac{d\bar{W}(t)}{dt} + \frac{\left(\bar{W}(t) - J(t)\right)/2\sqrt{C} - i_{C_1R_1}(t)}{d\tilde{C}_1\left(\frac{\bar{W}(t) - J(t)}{2\sqrt{C}}\right)/du} \,.$$

Lemma 5.1. Problem (18) has a solution $(i_{C_0R_0}, W, i_{C_1R_1}, J) \in M_0 \times M_W \times M_1 \times M_J$ iff the operator B has a fixed point in $M_0 \times M_W \times M_1 \times M_J$, that is,

$$(i_{C_0R_0}, W, i_{C_1R_1}, J) = \left(B_0(i_{C_0R_0}, W, i_{C_1R_1}, J), B_W(i_{C_0R_0}, W, i_{C_1R_1}, J), B_1(i_{C_0R_0}, W, i_{C_1R_1}, J), B_J(i_{C_0R_0}, W, i_{C_1R_1}, J)\right).$$

Proof: Let $(W, J) \in M_W \times M_J$ be a solution of (18). Then integrating every equation of (18) on every interval $[t_k, t] \subset [t_k, t_{k+1}]$ $(k = 0,1,2 \dots)$ we obtain

$$i_{C_0R_0}(t) - i_{C_0R_0}(t_k) = \int_{t_k}^{t} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Leftrightarrow i_{R_0L_0}(t) = \int_{t_k}^{t} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$W(t) - W(t_k) = \int_{t_k}^{t} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Leftrightarrow W(t) = \int_{t_k}^{t} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$i_{C_1R_1}(t) - i_{C_1R_1}(t_k) = \int_{t_k}^{t} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Leftrightarrow i_{C_1R_1}(t) = \int_{t_k}^{t} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$J(t) - J(t_k) = \int_{t_k}^{t} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Leftrightarrow J(t) = \int_{t_k}^{t} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds$$

and then

$$i_{C_0R_0}(t) = \int_{t_k}^{t} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Rightarrow 0 = i_{C_0R_0}(t_{k+1}) = \int_{t_k}^{t_{k+1}} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$W(t) = \int_{t_k}^{t} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Rightarrow 0 = W(t_{k+1}) = \int_{t_k}^{t_{k+1}} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$i_{C_1R_1}(t) = \int_{t_k}^{t} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Rightarrow 0 = i_{C_1R_1}(t_{k+1}) = \int_{t_k}^{t_{k+1}} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds;$$

$$J(t) = \int_{t_k}^{t} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \Rightarrow 0 = J(t_{k+1}) = \int_{t_k}^{t_{k+1}} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds.$$

Therefore the ordered four $(i_{C_0R_0}, W, i_{C_1R_1}, J)$ satisfies

$$i_{C_0R_0}(t) = \int_{t_k}^{t} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds - \frac{t - t_k}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds, \, t \in [t_k, t_{k+1}];$$

$$W(t) = \int_{t_k}^{t} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds - \frac{t - t_k}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} U(i_{C_0R_0}, W, i_{C_1R_1} J)(s)ds, \, t \in [t_k, t_{k+1}]; \qquad (20)$$

$$i_{C_1R_1}(t) = \int_{t_k}^{t} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds - \frac{t - t_k}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds, \, t \in [t_k, t_{k+1}];$$

$$J(t) = \int_{t_k}^{t} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds - \frac{t - t_k}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds, \, t \in [t_k, t_{k+1}]$$

that is, $(i_{C_0R_0}, W, i_{C_1R_1}, J)$ is a fixed point of B.

Conversely, let $(i_{C_0R_0}, W, i_{C_1R_1}, J)$ be a fixed point of B, that is, $(i_{C_0R_0}, W, i_{C_1R_1}, J)$ satisfies (20). Then in view of $\mu_0 = \mu T_0 = \text{const.}$ we obtain

$$\left| \int_{t_k}^{t_{k+1}} I_{C_0}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \right| \leq \frac{1}{2\sqrt{C}} \left| \int_{t_k}^{t_{k+1}} \frac{W(s) - \bar{J}(s) - 2\sqrt{C}R_0(i_{C_0R_0}(s))}{d\widetilde{L}_0(i_{C_0R_0}(s))/di_{C_0R_0}} ds \right| \leq$$

$$\leq \frac{1}{2\widehat{L}_0^1\sqrt{C}} \int_{t_k}^{t_{k+1}} \left(|W(s)| + |\bar{J}(s)| + 2\sqrt{C}|R_0(i_{C_0R_0}(s))| \right) ds \leq$$

$$\leq \frac{1}{2\widehat{L}_0^1\sqrt{C}} \int_{t_k}^{t_{k+1}} \left(W_0 e^{\mu(s-t_k)} + J_0 e^{\mu(s-t_k)} + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(0)}| \left| i_{C_0R_0}(s) \right|^n \right) ds \leq$$

$$\leq \frac{1}{2\widehat{L}_0^1\sqrt{C}} \left(W_0 \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} ds + J_0 \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} ds + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(0)}| I_{C_0}^n \int_{t_k}^{t_{k+1}} e^{n\mu(s-t_k)} ds \right) \leq$$

$$\leq \frac{1}{2\widehat{L}_0^1\sqrt{C}} \left(W_0 \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} + J_0 \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(0)}| I_{C_0}^n \frac{e^{n\mu(t_{k+1}-t_k)}-1}{n\mu} \right) \leq$$

$$\leq \frac{e^{\mu_0}-1}{\mu\widehat{L}_0^1} \frac{1}{2\sqrt{C}} \left[W_0 + J_0 + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(0)}| I_{C_0}^n e^{(n-1)\mu_0} \right] \equiv M_0(\mu);$$

$$\left| \int_{t_k}^{t_{k+1}} U(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \right| \leq \left| \int_{t_k}^{t_{k+1}} \frac{d\bar{J}(s)}{dt} ds \right| + \int_{t_k}^{t_{k+1}} \left| \frac{W(t) - \bar{J}(t) + 2\sqrt{L} i_{C_0R_0}(t)}{Z_0 d\widetilde{C}_0(u(0,t))/du} \right| dt \leq$$

$$\leq \frac{1}{Z_0\widehat{C}_0^1} \int_{t_k}^{t_{k+1}} \left(|W(s)| + |\bar{J}(s)| + 2\sqrt{L}|i_{C_0R_0}(s)| \right) ds \leq$$

$$\leq \frac{1}{Z_0\widehat{C}_0^1} \left(W_0 + J_0 + 2\sqrt{L}I_{C_0} + 2\sqrt{L} I_{C_0} \right) \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} ds \leq \frac{e^{\mu_0}-1}{\mu Z_0\widehat{C}_0^1} \left(W_0 + J_0 + 4\sqrt{L}I_{C_0} \right) \equiv M_W(\mu);$$

$$\left| \int_{t_k}^{t_{k+1}} I_{C_1}(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \right| \leq \frac{1}{2\sqrt{C}} \int_{t_k}^{t_{k+1}} \left| \frac{\bar{W}(s) - J(s) - 2\sqrt{C}R_1(i_{C_1R_1}(s))}{d\widetilde{L}_1(i_{C_1R_1}(s))/di_{C_1R_1}} \right| ds \leq$$

$$\leq \frac{1}{2\widehat{L}_1^1\sqrt{C}} \int_{t_k}^{t_{k+1}} \left(|\bar{W}(s)| + |J(s)| + 2\sqrt{C}|R_1(i_{C_1R_1}(s))| \right) ds \leq$$

$$\leq \frac{1}{2\widehat{L}_1^1\sqrt{C}} \int_{t_k}^{t_{k+1}} \left(W_0 e^{\mu(s-t_k)} + J_0 e^{\mu(s-t_k)} + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| \left| i_{C_1R_1}(s) \right|^n ds \right) \leq$$

$$\leq \frac{1}{2\widehat{L}_1^1\sqrt{C}} \left(W_0 \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} + J_0 \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| I_{C_1}^n \frac{e^{n\mu(t_{k+1}-t_k)}-1}{n\mu} \right) \leq$$

$$\leq \frac{e^{\mu_0}-1}{\mu\widehat{L}_1^1} \frac{1}{2\sqrt{C}} \left(W_0 + J_0 + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| I_{C_1}^n e^{(n-1)\mu_0} \right) \equiv M_1(\mu);$$

$$\left| \int_{t_k}^{t_{k+1}} I(i_{C_0R_0}, W, i_{C_1R_1}, J)(s)ds \right| \leq \left| \int_{t_k}^{t_{k+1}} \frac{d\bar{W}(s)}{ds} ds \right| + \frac{1}{Z_0\widehat{C}_1^1} \int_{t_k}^{t_{k+1}} \left(|\bar{W}(s)| + |J(s)| + 2\sqrt{L}|i_{C_1R_1}(s)| \right) ds \leq$$

$$\leq \frac{e^{\mu_0}-1}{\mu Z_0\widehat{C}_1^1} \left(W_0 + J_0 + 4\sqrt{L} I_{C_1} \right) \equiv M_J(\mu).$$

It follows

$$\int\limits_{t_k}^{t_{k+1}} I_{C_0}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds=0,\quad \int\limits_{t_k}^{t_{k+1}} U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds=0,$$

$$\int\limits_{t_k}^{t_{k+1}} I_{C_1}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds=0,\quad \int\limits_{t_k}^{t_{k+1}} I(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds=0.$$

Therefore

$$i_{R_0L_0}(t)=\int\limits_{t_k}^{t} I_{C_0}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int\limits_{t_k}^{t_{k+1}} I_{C_0}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\Leftrightarrow$$

$$i_{R_0L_0}(t)=\int\limits_{t_k}^{t} I_{C_0}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds;$$

$$W(t)=\int\limits_{t_k}^{t} U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int\limits_{t_k}^{t_{k+1}} U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\Leftrightarrow$$

$$W(t)=\int\limits_{t_k}^{t} U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds;$$

$$i_{R_1L_1}(t)=\int\limits_{t_k}^{t} I_{C_1}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int\limits_{t_k}^{t_{k+1}} I_{C_1}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\Leftrightarrow$$

$$i_{R_1L_1}(t)=\int\limits_{t_k}^{t} I_{C_1}(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds;$$

$$J(t)=\int\limits_{t_k}^{t} I(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds-\frac{t-t_k}{t_{k+1}-t_k}\int\limits_{t_k}^{t_{k+1}} I(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\Leftrightarrow$$

$$J(t)=\int\limits_{t_k}^{t} I(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds.$$

Differentiating the last integral equations we obtain (18).

Lemma 5.1 is thus proved.

6. PRELIMINARY ESTIMATES OF THE RIGHT-HAND-SIDES

Here we present some preliminary estimates that we use in the next Section 7.

Indeed in view of

$$\left|\frac{d\widetilde{L}_p(i_{C_0R_0})(t)}{di_{C_0R_0}}\right|\le \breve{L}_p^{\;1}<\infty;\quad \left|\frac{d^2\widetilde{L}_p(i_{C_0R_0})(t)}{di_{C_0R_0}^{\;2}}\right|\le \breve{L}_p^{\;2}<\infty$$

we have

$$I_{C_0}(i_{C_0R_0},W,J)=\frac{1}{d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\left[\frac{W(t)-\bar{J}(t)}{2\sqrt{C}}-R_0(i_{C_0R_0}(t))\right];$$

$$\frac{\partial I_{C_0}(i_{C_0R_0},W,J)}{\partial i_{C_0R_0}}=\frac{\partial}{\partial i_{C_0R_0}}\left(\frac{W(t)-\bar{J}(t)-2\sqrt{C}R_0(i_{C_0R_0}(t))}{2\sqrt{C}\;d\widetilde{L}_0(i_{C_0R_0})/i_{C_0R_0}}\right)=$$

$$= \frac{1}{2\sqrt{C}\left(\dfrac{d\widetilde{L}_0(i_{C_0R_0})}{di_{C_0R_0}}\right)^2}\left[-2\sqrt{C}\,\frac{\widetilde{L}_0(i_{C_0R_0})}{di_{C_0R_0}}\frac{dR_0(i_{C_0R_0}(t))}{di_{C_0R_0}}-\left(W(t)-\bar{J}(t)-2\sqrt{C}R_0(i_{C_0R_0})\right)\frac{d^2\widetilde{L}_0(i_{C_0R_0})}{di_{C_0R_0}{}^2}\right];$$

$$\left|\frac{\partial I_{C_0R_0}(i_{C_0R_0},W,J)}{\partial i_{C_0R_0}}\right|\le\frac{\left|dR_0(i_{C_0R_0}(t))/di_{C_0R_0}\right|}{\widehat{L}_0^{\,1}}+\frac{|W(t)|+|\bar{J}(t)|+2\sqrt{C}\left|R_0(i_{C_0R_0}(t))\right|}{2\sqrt{C}\left(\widehat{L}_0^1\right)^2}\breve{L}_0^{\,2}\le$$

$$\le\frac{1}{\widehat{L}_0^{\,1}}\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{C_0}^{n-1}+\frac{\breve{L}_0^{\,2}\left(W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{C_0}^{\,n}\right)}{2\sqrt{C}\left(\widehat{L}_0^1\right)^2};$$

$$\frac{\partial I_{C_0}(i_{C_0R_0},W,J)}{\partial W}=\frac{\partial}{\partial W}\left(\frac{W(t)-\bar{J}(t)-2\sqrt{C}R_0(i_{C_0R_0}(t))}{2\sqrt{C}d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\right)=\frac{1}{2\sqrt{C}\,d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}};$$

$$\frac{\partial I_{C_0}(i_{C_0R_0},W,J)}{\partial J}=\frac{\partial}{\partial J}\left(\frac{W(t)-\bar{J}(t)-2\sqrt{C}R_0(i_{C_0R_0}(t))}{2\sqrt{C}\,d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}}\right)-\frac{1}{2\sqrt{C}\,d\widetilde{L}_0(i_{C_0R_0})/di_{C_0R_0}};$$

$$\left|\frac{\partial I_{C_0}(i_{C_0R_0},W,J)}{\partial W}\right|\le\frac{1}{2\sqrt{C}\,\widehat{L}_0^{\,1}};\quad\left|\frac{\partial I_{C_0}(i_{C_0R_0},W,J)}{\partial J}\right|\le\frac{1}{2\sqrt{C}\,\widehat{L}_0^{\,1}};$$

$$\frac{\partial U(i_{C_0R_0},W,J)}{\partial i_{C_0R_0}}=\frac{\partial}{\partial i_{C_0R_0}}\left(\frac{d\bar{J}(t)}{dt}-\frac{W(t)-\bar{J}(t)+2\sqrt{L}\,i_{C_0R_0}(t)}{Z_0d\widetilde{C}_0\left(\dfrac{W(t)-J(t-T)}{2\sqrt{C}}\right)/du}\right)=$$

$$=\frac{2\sqrt{L}}{Z_0d\widetilde{C}_0(u(0,t))/du}\Rightarrow\left|\frac{\partial U(i_{C_0R_0},W,J)}{\partial i_{C_0R_0}}\right|\le\frac{2\sqrt{L}}{Z_0\widehat{C}_0^{\,1}};$$

$$\frac{\partial U(i_{C_0R_0},W,J)}{\partial W}=\frac{\partial}{\partial W}\left(\frac{dJ(t-T)}{dt}-\frac{W(t)-\bar{J}(t)+2\sqrt{L}i_{C_0R_0}(t)}{Z_0d\widetilde{C}_0\left(\dfrac{W(t)-\bar{J}(t)}{2\sqrt{C}}\right)/du}\right)=$$

$$=\frac{1}{Z_0\left(d\widetilde{C}_0(u(0,t))/du\right)^2}\left[\frac{d\widetilde{C}_0(u(0,t))}{du}-\frac{W(t)-\bar{J}(t)+2\sqrt{L}i_{C_0R_0}(t)}{2\sqrt{C}}\frac{d^2\widetilde{C}_0(u(0,t))}{du^2}\right]\Rightarrow$$

$$\left|\frac{\partial U(i_{C_0R_0},W,J)}{\partial W}\right|\le\frac{1}{Z_0\widehat{C}_0^{\,1}}+\frac{\breve{C}_0^{\,2}\left(W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}\right)}{2Z_0\sqrt{C}\left(\widehat{C}_0^1\right)^2};$$

$$\frac{\partial U(i_{C_0R_0},W,J)}{\partial J}=\frac{\partial}{\partial J}\left(\frac{d\bar{J}(t)}{dt}-\frac{W(t)-\bar{J}(t)+2\sqrt{L}i_{C_0R_0}(t)}{Z_0d\widetilde{C}_0\left(\dfrac{W(t)-\bar{J}(t)}{2\sqrt{C}}\right)/du}\right)=$$

$$=\frac{1}{Z_0\left(d\widetilde{C}_0(u(0,t))/du\right)^2}\left[-\frac{d\widetilde{C}_0(u(0,t))}{du}-\frac{W(t)-\bar{J}(t)+2\sqrt{L}i_{C_0R_0}(t)}{2\sqrt{C}}\frac{d^2\widetilde{C}_0(u(0,t))}{du^2}\right]\Rightarrow$$

$$\left|\frac{\partial U(i_{C_0 R_0}, W, J)}{\partial J}\right| \leq \frac{1}{Z_0 \breve{C}_0^{\ 1}} + \frac{\breve{C}_0^{\ 2}\left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0}\right)}{2Z_0 \sqrt{C}\left(\hat{C}_0^{\ 1}\right)^2}.$$

For the third component we have

$$\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial W} = \frac{\partial}{\partial W}\left(\frac{\bar{W}(t) - J(t) - 2\sqrt{C} R_1(i_{C_1 R_1}(t))}{2\sqrt{C}\ \widetilde{L}_1(i_{C_1 R_1})/di_{C_1 R_1}}\right) = \frac{1}{2\sqrt{C}\ \widetilde{L}_1(i_{C_1 R_1})/di_{C_1 R_1}};$$

$$\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial i_{C_1 R_1}} = \frac{1}{2\sqrt{C}}\frac{\partial}{\partial i_{C_1 R_1}}\left(\frac{\bar{W}(t) - J(t) - 2\sqrt{C} R_1(i_{C_1 R_1})}{\widetilde{L}_1(i_{C_1 R_1})/di_{C_1 R_1}}\right) =$$

$$= \frac{1}{2\sqrt{C}\left(\dfrac{d\widetilde{L}_1(i_{C_1 R_1})}{di_{C_1 R_1}}\right)^2}\left[-2\sqrt{C}\frac{d\widetilde{L}_1(i_{C_1 R_1})}{di_{C_1 R_1}}\frac{dR_1(i_{C_1 R_1})}{di_{R_1 L_1}} - \left(\bar{W}(t) - J(t) - 2\sqrt{C}R_1(i_{C_1 R_1})\right)\frac{d^2\widetilde{L}_1(i_{C_1 R_1})}{di_{C_1 R_1}^{\ 2}}\right];$$

$$\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial J} = \frac{\partial}{\partial J}\left(\frac{\bar{W}(t) - J(t) - 2\sqrt{C} R_1(i_{C_1 R_1})}{2\sqrt{C}\ d\widetilde{L}_1(i_{C_1 R_1})/di_{C_1 R_1}}\right) = \frac{-1}{2\sqrt{C}\ d\,\widetilde{L}_1(i_{C_1 R_1})/di_{C_1 R_1}};$$

$$\left|\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial W}\right| \leq \frac{1}{2\sqrt{C}\ \hat{L}_1^{\ 1}};\quad \left|\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial J}\right| \leq \frac{1}{2\sqrt{C}\ \hat{L}_1^{\ 1}};$$

$$\left|\frac{\partial I_{C_1}(W, i_{C_1 R_1}, J)}{\partial i_{C_1 R_1}}\right| \leq \frac{2\sqrt{C}}{\hat{L}_1^{\ 1}}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{\breve{L}_1^{\ 2}\left(W_0 e^{-\beta} + J_0 + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^{n}\right)}{\left(\hat{L}_1^{\ 1}\right)^2}.$$

For the fourth component we obtain

$$\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial W} = \frac{\partial}{\partial W}\left(\frac{d\bar{W}(t)}{dt} + \frac{1}{Z_0}\frac{\bar{W}(t) + J(t) + 2\sqrt{L}i_{C_1 R_1}(t)}{d\widetilde{C}_1\left(\dfrac{\bar{W}(t) - J(t)}{2\sqrt{C}}\right)/du}\right) =$$

$$= \frac{2\sqrt{C}\dfrac{d\widetilde{C}_1(u(\Lambda, t))}{du} - \left(\bar{W}(t) + J(t) + 2\sqrt{L}i_{C_1 R_1}(t)\right)\dfrac{d^2\widetilde{C}_1(u(\Lambda, t))}{du^2}}{2\sqrt{C}Z_0\left(d\widetilde{C}_1(u(\Lambda, t))/du\right)^2} \quad\Rightarrow$$

$$\left|\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial W}\right| \leq \frac{2\sqrt{C}\left|d\widetilde{C}_1(u(\Lambda, t))/du\right|}{2\sqrt{C}Z_0\left(d\widetilde{C}_1(u(\Lambda, t))/du\right)^2} + \frac{\left(\left|\bar{W}(t)\right| + \left|J(t)\right| + 2\sqrt{L}\left|i_{C_1 R_1}(t)\right|\right)\left|d^2\widetilde{C}_1(u(\Lambda, t))/du^2\right|}{2\sqrt{C}Z_0\left(d\widetilde{C}_1(u(\Lambda, t))/du\right)^2} \leq$$

$$\leq \frac{1}{Z_0 \hat{C}_1^{\ 1}} + \frac{\breve{C}_1^{\ 2}\left(W_0 e^{-\beta} + J_0 + 2\sqrt{L}I_{C_1}\right)}{2\sqrt{C}Z_0\left(\hat{C}_1^{\ 1}\right)^2};$$

$$\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial i_{C_1 R_1}} = \frac{\partial}{\partial i_{C_1 R_1}}\left(\frac{d\bar{W}(t)}{dt} - \frac{\bar{W}(t) - J(t) + 2\sqrt{L}\ i_{R_1 L_1}(t)}{Z_0 d\widetilde{C}_1\left(\dfrac{\bar{W}(t) - J(t)}{2\sqrt{C}}\right)/du}\right) = \frac{2\sqrt{L}}{Z_0 d\widetilde{C}_1(u(\Lambda, t))/du} \quad\Rightarrow$$

$$\left|\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial i_{C_1 R_1}}\right| \le \frac{2\sqrt{L}}{Z_0 \widehat{C}_1^{\,1}};$$

$$\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial J} = \frac{\partial}{\partial J}\left(\frac{d\bar{J}(t)}{dt} - \frac{\bar{W}(t) - J(t) + 2\sqrt{L}\, i_{R_1 L_1}(t)}{Z_0 d\widetilde{C}_1\left(\frac{\bar{W}(t) - J(t)}{2\sqrt{C}}\right)/du}\right) = \frac{-\dfrac{d\widetilde{C}_1(u(\Lambda, t))}{du} - \dfrac{\bar{W}(t) - J(t) + 2\sqrt{L}\, i_{C_1 R_1}(t)}{2\sqrt{C}}\dfrac{d^2\widetilde{C}_1(u(\Lambda, t))}{du^2}}{Z_0\left(d\widetilde{C}_1(u(\Lambda, t))/du\right)^2} \Rightarrow$$

$$\left|\frac{\partial I(W, i_{C_1 R_1}, J)}{\partial J}\right| \le \frac{1}{\widehat{C}_1^{\,1} Z_0} + \frac{\breve{C}_1^{\,2}\left(W_0 e^{-\beta} + J_0 + 2\sqrt{L}\, I_{C_1}\right)}{2\sqrt{C}\left(\widehat{C}_1^{\,1}\right)^2 Z_0}.$$

7. EXISTENCE-UNIQUENESS OF AN OSCILLATORY SOLUTION

The main result of the paper is:

Theorem 7.1. Let conditions **(IN-1)**, **(IN-2)**, **(L), (C), (R)** be fulfilled and the following inequalities are valid:

$$\frac{2L}{R\widehat{L}_0^{\,1}\sqrt{C}}\sinh\frac{RT_0}{L}\left[W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{C_0}^{\,n}\right] \le I_{C_0};$$

$$J_0 e^{-\beta} + \frac{4L\left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0}\right)}{Z_0 \widehat{C}_0^{\,1} R}\sinh\frac{RT_0}{L} \le W_0;$$

$$\frac{2L}{R\widehat{L}_1^{\,1}\sqrt{C}}\sinh\frac{RT_0}{L}\left(W_0 e^{-\beta} + J_0 + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^{\,n}\right) \le I_{C_1};$$

$$W_0 e^{-\beta} + \frac{4L}{R}\frac{W_0 e^{-\beta} + J_0 + 4\sqrt{L}I_{C_1}}{Z_0 \widehat{C}_1^{\,1}}\sinh\frac{RT_0}{L} \le J_0.$$

Then there exists a unique oscillatory solution of (18).

Proof: One can show (cf. [20]) that the functions

$$B_0(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t),\ B_W(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t), B_1(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t),\ B_J(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t)$$

are continuous ones. They are also differentiable on every interval (t_k, t_{k+1}). We prove the following inequalities:

$$\left|B_0^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)\right| \le I_{C_0} e^{-\frac{R}{L}t};\quad \left|B_W^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t)\right| \le W_0 e^{-\frac{R}{L}t};$$

$$\left|B_1^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)\right| \le I_{C_1} e^{-\frac{R}{L}t};\quad \left|B_J^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t)\right| \le J_0 e^{-\frac{R}{L}t}.$$

Indeed,

$$\left|B_0^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)\right| \le \int_{t_k}^{t}\left|I_{C_0}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(s)\right|ds + \left|\int_{t_k}^{t_{k+1}}I_{C_0}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(s)ds\right| \equiv J_1 + J_2$$

and then

$$J_1 \le \frac{1}{2\sqrt{C}}\left|\int_{t_k}^{t}\frac{W(s) - \bar{J}(s) - 2\sqrt{C}R_0(i_{C_0 R_0}(s))}{d\widetilde{L}_0(i_{R_0 L_0}(s))/di_{C_0 R_0}}ds\right| \le \frac{1}{2\widehat{L}_0^{\,1}\sqrt{C}}\int_{t_k}^{t}\left(\left|W(s)\right| + \left|\bar{J}(s)\right| + 2\sqrt{C}\left|R_0(i_{C_0 R_0}(s))\right|\right)ds \le$$

$$\le \frac{1}{2\widehat{L}_0^{\,1}\sqrt{C}}\int_{t_k}^{t}\left(W_0 e^{-\frac{R}{L}s} + J_0 e^{-\beta}e^{-\frac{R}{L}s} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|\left|i_{C_0 R_0}(s)\right|^n\right)ds \le$$

$$\leq \frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left[W_0\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t}\right)+J_0e^{-\beta}\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t}\right)+2\sqrt{C}\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t}\right)\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right]\leq$$

$$\leq \frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t}\right)\left[W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right]\leq$$

$$\leq e^{-\frac{R}{L}t}\frac{1}{\widehat{L}_0^{-1}\sqrt{C}}\frac{L}{R}\sinh\frac{RT_0}{L}\left[W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right].$$

In view of the inequalities

$$e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\leq e^{-\frac{R}{L}t}\left(e^{\frac{RT_0}{L}}-e^{-\frac{RT_0}{L}}\right)\quad (k=0,1,2,...)$$

we have

$$J_2 \leq \frac{1}{2\sqrt{C}}\left|\int_{t_k}^{t_{k+1}}\frac{W(s)-\bar{J}(s)-2\sqrt{C}R_0(i_{C_0R_0}(s))}{d\widetilde{L}_0(i_{C_0R_0}(s))/di_{C_0R_0}}ds\right|\leq$$

$$\leq \frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left(W_0\int_{t_k}^{t_{k+1}}e^{-\frac{R}{L}s}ds+J_0e^{-\beta}\int_{t_k}^{t_{k+1}}e^{-\frac{R}{L}s}ds+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\int_{t_k}^{t_{k+1}}e^{-n\frac{R}{L}s}ds\right)\leq$$

$$\leq \frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left[W_0\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)+J_0e^{-\beta}\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\frac{L}{nR}\left(e^{-n\frac{R}{L}t_k}-e^{-n\frac{R}{L}t_{k+1}}\right)\right]\leq$$

$$\leq \frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left[W_0\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)+J_0e^{-\beta}\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)\right]\leq$$

$$\leq \frac{L}{R}\left(e^{-\frac{R}{L}t_k}-e^{-\frac{R}{L}t_{k+1}}\right)\frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left[W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right]\leq$$

$$\leq e^{-\frac{R}{L}t}\left(e^{\frac{RT_0}{L}}-e^{-\frac{RT_0}{L}}\right)\frac{L}{R}\frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left(W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right)\leq$$

$$\leq e^{-\frac{R}{L}t}\left(e^{\frac{RT_0}{L}}-e^{-\frac{RT_0}{L}}\right)\frac{L}{R}\frac{1}{2\widehat{L}_0^{-1}\sqrt{C}}\left(W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right)\leq$$

$$\leq e^{-\frac{R}{L}t}\frac{L}{\widehat{L}_0^{-1}R\sqrt{C}}\sinh\frac{RT_0}{L}\left(W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right).$$

Therefore

$$\left|B_0^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J)\right|\leq e^{-\frac{R}{L}t}\frac{L}{\widehat{L}_0^{-1}R\sqrt{C}}\sinh\frac{RT_0}{L}\left[W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right]+$$

$$+e^{-\frac{R}{L}t}\frac{L}{\widehat{L}_0^{-1}R\sqrt{C}}\sinh\frac{RT_0}{L}\left(W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right)\leq$$

$$\leq e^{-\frac{R}{L}t}\frac{2L}{\widehat{L}_0^{-1}R\sqrt{C}}\sinh\frac{RT_0}{L}\left[W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(0)}\right|{I_{C_0}}^n\right]\leq I_{C_0}e^{-\frac{R}{L}t}.$$

Further on we have

$$\left|B_W^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J)(t)\right|\leq \left|\int_{t_k}^{t}U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\right|+\left|\int_{t_k}^{t_{k+1}}U(i_{C_0R_0},W,i_{C_1R_1},J)(s)ds\right|\equiv W_1+W_2.$$

But

$$W_1 \le \left| \int_{t_k}^{t} \frac{d\bar{J}(s)}{dt} ds \right| + \frac{1}{Z_0} \int_{t_k}^{t} \left| \frac{W(s) - \bar{J}(s) + 2\sqrt{L}\, i_{C_0 R_0}(s)}{d\widetilde{C}_0(u(0,s))/du} \right| ds \le$$

$$\le \left| \bar{J}(t) \right| + \frac{1}{Z_0 \widehat{C}_0^{\,1}} \int_{t_k}^{t} \left(|W(s)| + |\bar{J}(s)| + 2\sqrt{L}\, |i_{C_0 R_0}(s)| \right) ds \le$$

$$\le J_0 e^{-\beta} e^{-\frac{R}{L}t} + \frac{1}{Z_0 \widehat{C}_0^{\,1}} \int_{t_k}^{t} \left(W_0 e^{-\frac{R}{L}s} + J_0 e^{-\frac{R}{L}s} e^{-\beta} + 2\sqrt{L}\, I_{C_0} e^{-\frac{R}{L}s} + 2\sqrt{L}\, I_{C_0} e^{-\frac{R}{L}s} \right) ds \le$$

$$\le J_0 e^{-\beta} e^{-\frac{R}{L}t} + \frac{1}{Z_0 \widehat{C}_0^{\,1}} \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \int_{t_k}^{t} e^{-\frac{R}{L}s} ds \le$$

$$\le J_0 e^{-\beta} e^{-\frac{R}{L}t} + \frac{1}{Z_0 \widehat{C}_0^{\,1}} \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \frac{L}{R} \left(e^{-\frac{R}{L}t_k} - e^{-\frac{R}{L}t} \right) \le$$

$$\le e^{-\frac{R}{L}t} \left[J_0 e^{-\beta} + \frac{L}{Z_0 \widehat{C}_0^{\,1} R} \left(e^{\frac{RT_0}{L}} - e^{-\frac{RT_0}{L}} \right) \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \right] \le$$

$$\le e^{-\frac{R}{L}t} \left[J_0 e^{-\beta} + \frac{2L \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right)}{Z_0 \widehat{C}_0^{\,1} R} \sinh \frac{RT_0}{L} \right]$$

and

$$W_2 \le \left| \int_{t_k}^{t_{k+1}} \frac{d\bar{J}(s)}{dt} ds \right| + \frac{1}{Z_0} \int_{t_k}^{t_{k+1}} \left| \frac{W(s) - \bar{J}(s) + 2\sqrt{L}\, \breve{I}_{in}(s) + 2\sqrt{L}\, i_{C_0 R_0}(s)}{d\widetilde{C}_0(u(0,s))/du} \right| ds \le$$

$$\le \frac{1}{Z_0 \widehat{C}_0^{\,1}} \int_{t_k}^{t_{k+1}} \left(|W(s)| + |\bar{J}(s)| + 2\sqrt{L}\, |\breve{I}_{in}(s)| + 2\sqrt{L}\, |i_{C_0 R_0}(s)| \right) ds \le$$

$$\le \frac{1}{Z_0 \widehat{C}_0^{\,1}} \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \frac{L}{R} \left(e^{-\frac{R}{L}t_k} - e^{-\frac{R}{L}t_{k+1}} \right) \le$$

$$\le e^{-\frac{R}{L}t} \left(e^{\frac{RT_0}{L}} - e^{-\frac{RT_0}{L}} \right) \frac{L}{Z_0 \widehat{C}_0^{\,1} R} \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \le e^{-\frac{R}{L}t} \frac{2L}{Z_0 \widehat{C}_0^{\,1} R} \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right) \sinh \frac{RT_0}{L}.$$

Therefore

$$\left| B_W^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J)(t) \right| \le e^{-\frac{R}{L}t} \left[J_0 e^{-\beta} + \frac{4L \left(W_0 + J_0 e^{-\beta} + 4\sqrt{L}\, I_{C_0} \right)}{Z_0 \widehat{C}_0^{\,1} R} \sinh \frac{RT_0}{L} \right] \le e^{-\frac{R}{L}t} W_0.$$

Further on we have

$$\left| B_1^{(k)}(i_{C_0 R_0}, W, i_{C_1 R_1}, J) \right| \le \int_{t_k}^{t} \left| I_{C_1}(W, i_{C_1 R_1}, J)(s) \right| ds + \left| \int_{t_k}^{t_{k+1}} I_{C_1}(W, i_{C_1 R_1}, J)(s) ds \right| \equiv I_1 + I_2.$$

Since

$$I_1 \le \frac{1}{2\sqrt{C}} \int_{t_k}^{t} \left| \frac{\bar{W}(s) - J(s) - 2\sqrt{C} R_1(i_{C_1 R_1}(s))}{d\widetilde{L}_1(i_{C_1 R_1}(s))/di_{C_1 R_1}} \right| ds \le$$

$$\le \frac{1}{2\widehat{L}_1^{\,1} \sqrt{C}} \left[W_0 e^{-\beta} \int_{t_k}^{t} e^{-\frac{R}{L}s} ds + J_0 \int_{t_k}^{t} e^{-\frac{R}{L}s} ds + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{\,n} \int_{t_k}^{t} e^{-n\frac{R}{L}s} ds \right] \le$$

$$\le \frac{1}{2\widehat{L}_1^{\,1} \sqrt{C}} \frac{L}{R} \left(e^{-\frac{R}{L}t_k} - e^{-\frac{R}{L}t} \right) \left[W_0 e^{-\beta} + J_0 + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{\,n} \right] \le$$

$$\le e^{-\frac{R}{L}t} \frac{L}{\widehat{L}_1^{\,1} \sqrt{C} R} \sinh \frac{RT_0}{L} \left(W_0 e^{-\beta} + J_0 + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{\,n} \right)$$

and

$$I_2 \leq \frac{1}{2\sqrt{C}} \left| \int_{t_k}^{t_{k+1}} \frac{\vec{W}(s) - J(s) - 2\sqrt{C}R_1(i_{C_1R_1}(s))}{d\widetilde{L}_1(i_{C_1R_1}(s))/di_{C_1R_1}} ds \right| \leq$$

$$\leq \frac{1}{2\widehat{L}_1^1\sqrt{C}} \frac{L}{R} \left(e^{-\frac{R}{L}t_k} - e^{-\frac{R}{L}t_{k+1}} \right) \left[W_0 e^{-\beta} + J_0 + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| I_{C_1}^n \right] \leq$$

$$\leq e^{-\frac{R}{L}t} \frac{L}{\widehat{L}_1^1\sqrt{C}R} \sinh\frac{RT_0}{L} \left(W_0 e^{-\beta} + J_0 + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| I_{C_1}^n \right)$$

then

$$\left| B_1^{(k)}(i_{C_0R_0}, W, i_{C_1R_1}, J) \right| \leq e^{-\frac{R}{L}t} \frac{2L}{\widehat{L}_1^1\sqrt{C}R} \sinh\frac{RT_0}{L} \left(W_0 e^{-\beta} + J_0 + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(1)}| I_{C_1}^n \right) \leq I_{C_1} e^{-\frac{R}{L}t}.$$

Finally

$$\left| B_J^{(k)}(i_{C_0R_0}, W, i_{C_1R_1}, J)(t) \right| \leq \int_{t_k}^{t} |I(W, i_{C_1R_1}, J)(s)| ds + \left| \int_{t_k}^{t_{k+1}} I(W, i_{C_1R_1}, J)(s) ds \right| \equiv A_1 + A_2.$$

Since

$$A_1 \leq \left| \int_{t_k}^{t} \frac{d\vec{W}(s)}{dt} ds \right| + \frac{1}{Z_0} \int_{t_k}^{t} \left| \frac{\vec{W}(s) - J(s) + 2\sqrt{L}\,i_{C_1R_1}(s)}{d\widetilde{C}_1(u(\Lambda,s))/du} \right| ds \leq$$

$$\leq e^{-\frac{R}{L}t} \left[W_0 e^{-\beta} + \frac{2L}{R} \frac{W_0 e^{-\beta} + J_0 + 4\sqrt{L}I_{C_1}}{Z_0\widehat{C}_1^1} \sinh\frac{RT_0}{L} \right]$$

and

$$A_2 \leq \left| \int_{t_k}^{t_{k+1}} \frac{d\vec{W}(s)}{dt} ds \right| + \frac{1}{Z_0} \int_{t_k}^{t_{k+1}} \left| \frac{\vec{W}(s) - J(s) + 2\sqrt{L}\,i_{C_1R_1}(s)}{d\widetilde{C}_1(u(\Lambda,s))/du} \right| ds \leq e^{-\frac{R}{L}t} \left[\frac{2L}{R} \frac{W_0 e^{-\beta} + J_0 + 4\sqrt{L}I_{C_1}}{Z_0\widehat{C}_1^1} \sinh\frac{RT_0}{L} \right]$$

we have

$$\left| B_J^{(k)}(i_{C_0R_0}, W, i_{C_1R_1}, J)(t) \right| \leq e^{-\frac{R}{L}t} \left[W_0 e^{-\beta} + \frac{4L}{R} \frac{W_0 e^{-\beta} + J_0 + 4\sqrt{L}I_{C_1}}{Z_0\widehat{C}_1^1} \sinh\frac{RT_0}{L} \right] \leq e^{-\frac{R}{L}t} J_0.$$

Therefore B maps $M_0 \times M_W \times M_1 \times M_J$ into itself.

In what follows we show that operator B is contractive one. Indeed

$$\left| B_0^{(k)}(i_{C_0R_0}, W, J)(t) - B_0^{(k)}(\bar{i}_{C_0R_0}, \overline{W}, \bar{J})(t) \right| \leq \int_{t_k}^{t} \left| I_{C_0}(i_{C_0R_0}, W, J)(s) - I_{C_0}(\bar{i}_{C_0R_0}, \overline{W}, \bar{J})(s) \right| ds +$$

$$+ \left| \int_{t_k}^{t_{k+1}} \left(I_{C_0}(i_{C_0R_0}, W, J)(s) - I_{C_0}(\bar{i}_{C_0R_0}, \overline{W}, \bar{J})(s) \right) ds \right| \equiv I_1 + I_2.$$

Since

$$I_1 \leq \frac{1}{2\sqrt{C}\,\widehat{L}_0^1} \int_{t_k}^{t} |\bar{J}(s) - \bar{J}(s)| ds +$$

$$+ \int_{t_k}^{t} e^{\mu(s-t_k)} |i_{C_0R_0}(s) - \bar{i}_{C_0R_0}(s)| ds \left[\frac{2\sqrt{C}}{\widehat{L}_0^1} \sum_{n=1}^{m} n|r_n^{(0)}| I_{C_0}^{n-1} + \frac{\breve{L}_0^2 \left(W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m} |r_n^{(0)}| I_{C_0}^n e^{(n-1)\mu_0} \right)}{2\sqrt{C}\left(\breve{L}_0^1\right)^2} \right] +$$

$$+ \frac{1}{2\sqrt{C}\,\widehat{L}_0^1} \int_{t_k}^{t} |W(s) - \overline{W}(s)| ds \leq$$

$$
\leq \rho^{(k)}(i_{C_0R_0}, \bar{i}_{C_0R_0}) \frac{e^{\mu(t-t_k)}-1}{\mu} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{\breve{L}_0^{\ 2} \left(W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n} \right)}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \right) +
$$

$$
+ \frac{1}{2\sqrt{C}\widehat{L}_0^{\ 1}} \rho^{(k)}(W,\overline{W}) \frac{e^{\mu(t-t_k)}-1}{\mu} \leq
$$

$$
\leq \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{i}_{C_0R_0}, \dot{\bar{i}}_{C_0R_0})}{\mu} \frac{e^{\mu(t-t_k)}-1}{\mu} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{\breve{L}_0^{\ 2} \left(W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n} \right)}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \right) +
$$

$$
+ \frac{1}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{W}, \dot{\overline{W}})}{\mu} \frac{e^{\mu(t-t_k)}-1}{\mu} \leq
$$

$$
\leq \hat{\rho}_\mu^{(k)} \left((i_{C_0R_0}, W, i_{C_1R_1}, J, \dot{i}_{C_0R_0}, \dot{W}, \dot{i}_{C_1R_1}, \dot{J}), (\bar{i}_{C_0R_0}, \overline{W}, \bar{i}_{C_1R_1}, \overline{J}, \dot{\bar{i}}_{C_0R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1R_1}, \dot{\overline{J}}) \right) \times
$$

$$
\times e^{\mu(t-t_k)} \frac{e^{\mu_0}}{\mu^2} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{\breve{L}_0^{\ 2} \left(W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n} \right)}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} + \frac{1}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} \right);
$$

$$
I_2 \leq \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} \left| i_{C_0R_0}(s) - \bar{i}_{C_0R_0}(s) \right| ds \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \breve{L}_0^{\ 2} \right) +
$$

$$
+ \frac{1}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} \int_{t_k}^{t_{k+1}} \left| W(s) - \overline{W}(s) \right| ds \leq
$$

$$
\leq e^{\mu_0} \rho_\mu^{(k)}(i_{R_0L_0}, \bar{i}_{R_0L_0}) \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \breve{L}_0^{\ 2} \right)
$$

$$
+ \frac{1}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} e^{\mu_0} \rho_\mu^{(k)}(W,\overline{W}) \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu} \leq
$$

$$
\leq \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{i}_{C_0R_0}, \dot{\bar{i}}_{C_0R_0})}{\mu} \frac{e^{\mu_0}-1}{\mu} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \breve{L}_0^{\ 2} \right) +
$$

$$
+ \frac{1}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{W}, \dot{\overline{W}})}{\mu} \frac{e^{\mu_0}-1}{\mu} \leq
$$

$$
\leq \hat{\rho}_\mu^{(k)} \left((i_{C_0R_0}, W, i_{C_1R_1}, J, \dot{i}_{C_0R_0}, \dot{W}, \dot{i}_{C_1R_1}, \dot{J}), (\bar{i}_{C_0R_0}, \overline{W}, \bar{i}_{C_1R_1}, \overline{J}, \dot{\bar{i}}_{C_0R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1R_1}, \dot{\overline{J}}) \right) \times
$$

$$
\times \frac{e^{\mu_0}-1}{\mu^2} e^{\mu_0} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \left(\widehat{L}_0^{\ 1} \right)^2} \breve{L}_0^{\ 2} + \frac{1}{2\widehat{L}_0^{\ 1}\sqrt{C}} \right);
$$

$$\left| B_0^{(k)}(i_{C_0 R_0}, W, J)(t) - B_0^{(k)}(\bar{i}_{C_0 R_0}, \overline{W}, \overline{J})(t) \right| \le$$

$$\le e^{\mu(t-t_k)} \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \right) \times$$

$$\times \frac{e^{2\mu_0}}{\mu^2} \left(\frac{2\sqrt{C}}{\check{L}_0^1} \sum_{n=1}^m n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{\check{L}_0^2 \left(W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^m \left| r_n^{(0)} \right| I_{C_0}^n \right)}{2\sqrt{C}\left(\hat{L}_0^1\right)^2} + \frac{1}{2\hat{L}_0^1 \sqrt{C}} \right) \equiv$$

$$\equiv e^{\mu(t-t_k)} K_0 \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \right) \le$$

$$\le e^{\mu_0} K_0 \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \right).$$

It follows

$$\hat{\rho}^{(k)}(B_0(i_{C_0 R_0}, W, J), B_0(\bar{i}_{C_0 R_0}, \overline{W}, \overline{J})) \le$$

$$\le e^{\mu_0} K_0 \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{R_1 L_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{R_1 L_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{R_1 L_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{R_1 L_1}, \dot{\overline{J}}) \right).$$

For the second component we get

$$\left| B_W^{(k)}(i_{C_0 R_0}, W, J)(t) - B_W^{(k)}(\bar{i}_{C_0 R_0}, \overline{W}, \overline{J})(t) \right| \le \int_{t_k}^t \left| U(s) - \overline{U}(s) \right| ds + \int_{t_k}^{t_{k+1}} \left| U(s) - \overline{U}(s) \right| ds \equiv W_1 + W_2.$$

Further on we have

$$W_1 \le \left| \int_{t_k}^t \left(\frac{dJ(t-T)}{dt} - \frac{d\overline{J}(t-T)}{dt} \right) ds \right| + \frac{2\sqrt{L}}{Z_0 \hat{C}_0^1} \int_{t_k}^t \left| i_{C_0 R_0}(s) - \bar{i}_{C_0 R_0}(s) \right| ds +$$

$$+ \left(\frac{1}{Z_0 \hat{C}_0^1} + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2Z_0 \sqrt{C}\left(\hat{C}_0^1\right)^2} \check{C}_0^2 \right) \int_{t_k}^t \left| W(s) - \overline{W}(s) \right| ds +$$

$$+ \left(\frac{1}{Z_0 \hat{C}_0^1} + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2\sqrt{C}\left(\hat{C}_0^1\right)^2 Z_0} \check{C}_0^2 \right) \int_{t_k}^t \left| J(s-T) - \overline{J}(s-T) \right| ds \le$$

$$\le \int_{t_k}^t e^{\mu(s-t_k)} ds \left[\frac{2\sqrt{L}}{Z_0 \hat{C}_0^1} e^{\mu_0} \rho_\mu^{(k)}(i_{C_0 R_0}, \bar{i}_{C_0 R_0}) + \left(\frac{1}{Z_0 \hat{C}_0^1} + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2Z_0 \sqrt{C}\left(\hat{C}_0^1\right)^2} \check{C}_0^2 \right) e^{\mu_0} \rho_\mu^{(k)}(W, \overline{W}) \right] \le$$

$$\le e^{\mu(t-t_k)} \frac{1}{\mu} \left[\frac{2\sqrt{L}}{Z_0 \hat{C}_0^1} \frac{e^{\mu_0} \rho_\mu^{(k)}(i_{C_0 R_0}, \dot{\bar{i}}_{C_0 R_0})}{\mu} + \left(\frac{1}{Z_0 \hat{C}_0^1} + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2Z_0 \sqrt{C}\left(\hat{C}_0^1\right)^2} \check{C}_0^2 \right) \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{W}, \dot{\overline{W}})}{\mu} \right] \le$$

$$\le e^{\mu(t-t_k)} \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \right) \times$$

$$\times \frac{1}{\mu^2} \frac{e^{\mu_0}}{Z_0 \hat{C}_0^1} \left[2\sqrt{L} + 1 + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2\sqrt{C}\hat{C}_0^1} \check{C}_0^2 \right];$$

$$W_2 \le \left| \int_{t_k}^{t_{k+1}} \left(\frac{dJ(t-T)}{dt} - \frac{d\overline{J}(t-T)}{dt} \right) ds \right| + \frac{2\sqrt{L}}{Z_0 \hat{C}_0^1} \int_{t_k}^{t_{k+1}} \left| i_{R_0 L_0}(s) - \bar{i}_{R_0 L_0}(s) \right| ds + \frac{1}{Z_0 \hat{C}_0^1} \int_{t_k}^{t_{k+1}} \left| W(s) - \overline{W}(s) \right| ds +$$

$$+ \left(\frac{1}{Z_0 \hat{C}_0^1} + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2\sqrt{C}\left(\hat{C}_0^1\right)^2 Z_0} \check{C}_0^2 \right) \int_{t_k}^{t_{k+1}} \left| J(s-T) - \overline{J}(s-T) \right| ds \le$$

$$\le e^{\mu(t-t_k)} \hat{\rho}_\mu^{(k)} \left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \right) \times$$

$$\times \frac{\left(e^{\mu_0}-1\right)}{\mu^2}\frac{e^{\mu_0}}{Z_0\widehat{C}_0^{\,1}}\left(2\sqrt{L}+1+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2\sqrt{C}\,\widehat{C}_0^{\,1}}\breve{C}_0^{\,2}\right);$$

$$\left|B_W^{(k)}U(i_{C_0R_0},W,J)(t)-B_W^{(k)}U(\bar{i}_{C_0R_0},\overline{W},\overline{J})(t)\right|\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big)\times$$

$$\times\frac{e^{2\mu_0}}{\mu^2 Z_0\widehat{C}_0^{\,1}}\left(2\sqrt{L}+1+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2\sqrt{C}\,\widehat{C}_0^{\,1}}\breve{C}_0^{\,2}\right)\equiv$$

$$\equiv e^{\mu(t-t_k)}K_W\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big).$$

It follows

$$\hat{\rho}^{(k)}\Big(B_W(i_{C_0R_0},W,J),B_W(\bar{i}_{C_0R_0},\overline{W},\overline{J})\Big)\le$$

$$\le e^{\mu_0}K_W\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big).$$

For the third component we have

$$\left|B_1^{(k)}(W,i_{C_1R_1},J)(t)-B_1^{(k)}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(t)\right|\le\int_{t_k}^{t}\left|I_{C_1}(W,i_{C_1R_1},J)(s)-I_{C_1}(\overline{W},\bar{i}_{C_1R_1}\overline{J})(s)\right|ds+$$

$$+\int_{t_k}^{t_{k+1}}\left|I_{C_1}(W,i_{C_1R_1},J)(s)-I_{C_1}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(s)\right|ds\equiv P_1+P_2.$$

Since

$$P_1\le\frac{1}{2\sqrt{C}\,\widehat{L}_1^{\,1}}\int_{t_k}^{t}\left|\overline{W}(s)-\overline{W}(s)\right|ds+\left(\frac{2\sqrt{C}}{\widehat{L}_1^{\,1}}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{\left(\widehat{L}_1^{\,1}\right)^2}\breve{L}_1^{\,2}\right)\times$$

$$\times\int_{t_k}^{t}\left|i_{C_1R_1}(s)-\bar{i}_{C_1R_1}(s)\right|ds+\frac{1}{2\sqrt{C}\,\widehat{L}_1^{\,1}}\int_{t_k}^{t}\left|J(s)-\overline{J}(s)\right|ds\le$$

$$\le\left(\frac{2\sqrt{C}}{\widehat{L}_1^{\,1}}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\left(\widehat{L}_1^{\,1}\right)^2}\breve{L}_1^{\,2}\right)\frac{e^{\mu_0}\rho_\mu^{(k)}(i_{C_1R_1},\dot{\bar{i}}_{C_1R_1})}{\mu}\frac{e^{\mu(t-t_k)}-1}{\mu}+$$

$$+\frac{e^{\mu_0}\rho_\mu^{(k)}(\dot{J},\dot{\overline{J}})}{2\mu\sqrt{C}\,\widehat{L}_1^{\,1}}\frac{e^{\mu(t-t_k)}-1}{\mu}\le$$

$$e^{\mu(t-t_k)}e^{\mu_0}\frac{1}{\mu^2\widehat{L}_1^{\,1}}\left(2\sqrt{C}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\widehat{L}_1^{\,1}}\breve{L}_1^{\,2}+\frac{1}{2\sqrt{C}}\right)$$

$$\times\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big)$$

and

$$P_2\le\frac{1}{2\sqrt{C}\,\widehat{L}_1^{\,1}}\int_{t_k}^{t_{k+1}}\left|\overline{W}(s)-\overline{W}(s)\right|ds+$$

$$+ \left(\frac{2\sqrt{C}}{\widehat{L}_1^1} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^n}{2\sqrt{C}\left(\widehat{L}_1^1\right)^2} \breve{L}_1^{\;2} \right) \int_{t_k}^{t_{k+1}} \left| i_{C_1 R_1}(s) - \bar{i}_{C_1 R_1}(s) \right| ds +$$

$$+ \frac{1}{2\sqrt{C}\;\widehat{L}_1^1} \int_{t_k}^{t_{k+1}} \left| J(s) - \bar{J}(s) \right| ds \le \left(\frac{2\sqrt{C}}{\widehat{L}_1^1} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^n}{2\sqrt{C}\left(\widehat{L}_1^1\right)^2} \breve{L}_1^{\;2} \right) \times$$

$$\times \rho^{(k)}(i_{C_1 R_1}, \bar{i}_{C_1 R_1}) \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} ds + \frac{\rho^{(k)}(J, \bar{J})}{2\sqrt{C}\;\widehat{L}_1^1} \int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} ds \le$$

$$\le \left(\frac{2\sqrt{C}}{\widehat{L}_1^1} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^n}{2\sqrt{C}\left(\widehat{L}_1^1\right)^2} \breve{L}_1^{\;2} \right) \frac{e^{\mu_0} \rho_\mu^{(k)}(i_{C_1 R_1}, \dot{\bar{i}}_{C_1 R_1})}{\mu} \frac{e^{\mu T_0} - 1}{\mu} +$$

$$+ \frac{e^{\mu_0} \rho_\mu^{(k)}(\dot{J}, \dot{\bar{J}})}{2\mu\sqrt{C}\;\widehat{L}_1^1} \frac{e^{\mu T_0} - 1}{\mu} \le$$

$$\le e^{\mu(t-t_k)} e^{\mu_0} \frac{e^{\mu_0} - 1}{\mu^2 \widehat{L}_1^1} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^n}{2\sqrt{C}\widehat{L}_1^1} \breve{L}_1^{\;2} + \frac{1}{2\sqrt{C}} \right) \times$$

$$\times \hat{\rho}_\mu^{(k)}\left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \bar{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\bar{J}}) \right)$$

we obtain

$$\left| B_1^{(k)}(W, i_{C_1 R_1}, J)(t) - B_1^{(k)}(\overline{W}, \bar{i}_{C_1 R_1}, \bar{J})(t) \right| \le$$

$$\le e^{\mu(t-t_k)} \frac{e^{2\mu_0}}{\mu^2 \widehat{L}_1^1} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^n}{2\sqrt{C}\widehat{L}_1^1} \breve{L}_1^{\;2} + \frac{1}{2\sqrt{C}} \right) \times$$

$$\times \hat{\rho}_\mu^{(k)}\left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \bar{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\bar{J}}) \right) \equiv$$

$$\equiv e^{\mu_0} K_1 \hat{\rho}_\mu^{(k)}\left((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \bar{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\bar{J}}) \right).$$

Further on we have

$$\left| B_J^{(k)}(W, i_{C_1 R_1}, J)(t) - B_J^{(k)}(\overline{W}, \bar{i}_{C_1 R_1}, \bar{J})(t) \right| \le \int_{t_k}^{t} \left| I(W, i_{C_1 R_1}, J)(s) - I(\overline{W}, \bar{i}_{C_1 R_1}, \bar{J})(s) \right| ds +$$

$$+ \frac{t - t_k}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} \left| I(W, i_{C_1 R_1}, J)(s) - I(\overline{W}, \bar{i}_{C_1 R_1}, \bar{J})(s) \right| ds \equiv J_1 + J_2.$$

Since

$$J_1 \le \left| \int_{t_k}^{t} \left(\frac{dW(t-T)}{dt} - \frac{d\overline{W}(t-T)}{dt} \right) ds \right| + \frac{2\sqrt{L}}{Z_0 \widehat{C}_1^1} \int_{t_k}^{t} \left| i_{C_1 R_1}(s) - \bar{i}_{C_1 R_1}(s) \right| ds +$$

$$+ \left(\frac{1}{\widehat{C}_1^1 Z_0} + \frac{W_0 e^{-\beta} + J_0 + 2\sqrt{L} I_{C_1}}{2\sqrt{C}\left(\widehat{C}_1^1\right)^2 Z_0} \breve{C}_1^{\;2} \right) \int_{t_k}^{t} \left| J(s) - \bar{J}(s) \right| ds +$$

$$+\left(\frac{1}{Z_0\widehat{C}_1^{\;1}}+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}I_{C_1}}{2\sqrt{C}Z_0\left(\widehat{C}_1^{\;1}\right)^2}\breve{C}_1^{\;2}\right)\int\limits_{t_k}^{t}\left|W(s-T)-\overline{W}(s-T)\right|ds\le$$

$$\le\frac{e^{\mu(t-t_k)}-1}{\mu}e^{\mu_0}\left[\frac{2\sqrt{L}}{Z_0\widehat{C}_1^{\;1}}\frac{\rho_\mu^{(k)}(\dot{i}_{R_1L_1},\dot{\overline{i}}_{R_1L_1})}{\mu}+\left(\frac{1}{\widehat{C}_1^{\;1}Z_0}+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}\,I_{C_1}}{2\sqrt{C}\left(\widehat{C}_1^{\;1}\right)^2Z_0}\breve{C}_1^{\;2}\right)\frac{\rho^{(k)}(\dot{J},\dot{\overline{J}})}{\mu}\right]\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\overline{i}_{C_0R_0},\overline{W},\overline{i}_{C_1R_1},\overline{J},\dot{\overline{i}}_{C_0R_0},\dot{\overline{W}},\dot{\overline{i}}_{C_1R_1},\dot{\overline{J}})\Big)\times$$

$$\times\frac{e^{\mu_0}}{\mu^2Z_0\widehat{C}_1^{\;1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}\,I_{C_1}}{2\sqrt{C}\,\widehat{C}_1^{\;1}}\breve{C}_1^{\;2}\right)$$

and

$$J_2\le\left|\int\limits_{t_k}^{t_{k+1}}\left(\frac{dW(t-T)}{dt}-\frac{d\overline{W}(t-T)}{dt}\right)ds\right|+\frac{2\sqrt{L}}{Z_0\widehat{C}_1^{\;1}}\int\limits_{t_k}^{t_{k+1}}\left|i_{C_1R_1}(s)-\overline{i}_{C_1R_1}(s)\right|ds+\frac{1}{Z_0\widehat{C}_1^{\;1}}\int\limits_{t_k}^{t_{k+1}}\left|J(s)-\overline{J}(s)\right|ds+$$

$$+\left(\frac{W_0e^{-\beta}+J_0+2\sqrt{L}\,I_{C_1}}{2\sqrt{C}\left(\widehat{C}_1^{\;1}\right)^2Z_0}\breve{C}_1^{\;2}\right)\int\limits_{t_k}^{t_{k+1}}\left|W(s-T)-\overline{W}(s-T)\right|ds\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\overline{i}_{C_0R_0},\overline{W},\overline{i}_{C_1R_1},\overline{J},\dot{\overline{i}}_{C_0R_0},\dot{\overline{W}},\dot{\overline{i}}_{C_1R_1},\dot{\overline{J}})\Big)\times$$

$$\times\left(e^{\mu_0}-1\right)\frac{e^{\mu_0}}{\mu^2Z_0\widehat{C}_1^{\;1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}\,I_{C_1}}{2\sqrt{C}\left(\widehat{C}_1^{\;1}\right)^2Z_0}\breve{C}_1^{\;2}\right)$$

then

$$\left|B_J^{(k)}(W,i_{C_1R_1},J)(t)-B_J^{(k)}(\overline{W},\overline{i}_{C_1R_1},\overline{J})(t)\right|\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\overline{i}_{C_0R_0},\overline{W},\overline{i}_{C_1R_1},\overline{J},\dot{\overline{i}}_{C_0R_0},\dot{\overline{W}},\dot{\overline{i}}_{C_1R_1},\dot{\overline{J}})\Big)\times$$

$$\times\frac{e^{2\mu_0}}{\mu^2Z_0\widehat{C}_1^{\;1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}\,I_{C_1}}{2\sqrt{C}\left(\widehat{C}_1^{\;1}\right)^2Z_0}\breve{C}_1^{\;2}\right)\le$$

$$\le e^{\mu_0}K_J\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\overline{i}_{C_0R_0},\overline{W},\overline{i}_{C_1R_1},\overline{J},\dot{\overline{i}}_{C_0R_0},\dot{\overline{W}},\dot{\overline{i}}_{C_1R_1},\dot{\overline{J}})\Big).$$

It follows

$$\hat{\rho}^{(k)}\Big(B_J(i_{C_0R_0},W,J),B_J(\overline{i}_{C_0R_0},\overline{W},\overline{J})\Big)\le$$

$$\le e^{\mu_0}K_J\hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\overline{i}_{C_0R_0},\overline{W},\overline{i}_{C_1R_1},\overline{J},\dot{\overline{i}}_{C_0R_0},\dot{\overline{W}},\dot{\overline{i}}_{C_1R_1},\dot{\overline{J}})\Big).$$

For the derivative of the first component we have

$$\left|\dot{B}_0^{(k)}(i_{C_0R_0},W,J)(t)-\dot{B}_0^{(k)}(\overline{i}_{C_0R_0},\overline{W},\overline{J})(t)\right|\le\left|I_{C_0}(i_{C_0R_0},W,J)(t)-I_{C_0}(\overline{i}_{C_0R_0},\overline{W},\overline{J})(t)\right|+$$

$$+\frac{1}{t_{k+1}-t_k}\left|\int\limits_{t_k}^{t_{k+1}}\Big(I_{C_0}(i_{C_0R_0},W,J)(s)-I_{C_0}(\overline{i}_{C_0R_0},\overline{W},\overline{J})(s)\Big)ds\right|\equiv\dot{I}_1+\dot{I}_2.$$

But

$$\dot{I}_1\le\left|i_{C_0R_0}(t)-\overline{i}_{C_0R_0}(t)\right|\left(\frac{2\sqrt{C}}{\widehat{L}_0^{\;1}}\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum\limits_{n=1}^{m}\left|r_n^{(0)}\right|I_{C_0}^n}{2\sqrt{C}\left(\widehat{L}_0^{\;1}\right)^2}\breve{L}_0^{\;2}+\right.$$

$$+\frac{1}{2\sqrt{C}\widehat{L}_0^{\;1}}\left|W(t)-\overline{W}(t)\right|+\frac{1}{2\sqrt{C}\widehat{L}_0^{\;1}}\left|J(t-T)-\overline{J}(t-T)\right|\le$$

$$\le e^{\mu(t-t_k)} \rho_\mu^{(k)}(i_{C_0R_0},\bar{i}_{C_0R_0}) \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\left(\widehat{L}_0^{\ 1}\right)^2} \breve{L}_0^2 \right) + e^{\mu(t-t_k)} \frac{\rho_\mu^{(k)}(W,\overline{W})}{2\sqrt{C}\widehat{L}_0^{\ 1}} \le$$

$$\le \frac{\rho_\mu^{(k)}(\dot{i}_{C_0R_0},\dot{\bar{i}}_{C_0R_0})}{\mu} \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\left(\widehat{L}_0^{\ 1}\right)^2} \breve{L}_0^2 \right) + \frac{\rho_\mu^{(k)}(\dot{W},\dot{\overline{W}})}{2\sqrt{C}\mu\widehat{L}_0^{\ 1}} \le$$

$$\le \hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big) \times$$

$$\times \frac{e^{\mu(t-t_k)}}{\mu\widehat{L}_0^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\ \widehat{L}_0^{\ 1}} \breve{L}_0^2 + \frac{1}{2\sqrt{C}} \right)$$

and

$$\dot{I}_2 \le \frac{1}{t_{k+1}-t_k} \left[\int_{t_k}^{t_{k+1}} e^{\mu(s-t_k)} \left| i_{C_0R_0}(s) - \bar{i}_{C_0R_0}(s) \right| ds \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\left(\widehat{L}_0^{\ 1}\right)^2} \breve{L}_0^2 \right) + \right.$$

$$\left. + \frac{1}{2\sqrt{C}\widehat{L}_0^{\ 1}} \int_{t_k}^{t_{k+1}} \left| W(s) - \overline{W}(s) \right| ds + \frac{1}{2\sqrt{C}\widehat{L}_0^{\ 1}} \int_{t_k}^{t_{k+1}} \left| J(s-T) - \overline{J}(s-T) \right| ds \right] \le$$

$$\le \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu(t_{k+1}-t_k)} \left[\rho_\mu^{(k)}(i_{C_0R_0},\bar{i}_{C_0R_0}) \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\left(\widehat{L}_0^{\ 1}\right)^2} \breve{L}_0^2 \right) + \frac{1}{2\sqrt{C}\widehat{L}_0^{\ 1}} \rho_\mu^{(k)}(W,\overline{W}) \right] \le$$

$$\le \frac{e^{\mu_0}-1}{\mu_0\mu} \left[\rho^{(k)}(\dot{i}_{C_0R_0},\dot{\bar{i}}_{C_0R_0}) \left(\frac{2\sqrt{C}}{\widehat{L}_0^{\ 1}} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\left(\widehat{L}_0^{\ 1}\right)^2} \breve{L}_0^2 \right) + \frac{\rho_\mu^{(k)}(\dot{W},\dot{\overline{W}})}{2\sqrt{C}\widehat{L}_0^{\ 1}} \right] \le$$

$$\le \hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big) \times$$

$$\times \frac{e^{\mu_0}-1}{\mu_0\mu\widehat{L}_0^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\widehat{L}_0^{\ 1}} \breve{L}_0^2 + \frac{1}{2\sqrt{C}} \right)$$

then

$$\left| \dot{B}_0^{(k)}(i_{C_0R_0},W,J)(t) - \dot{B}_0^{(k)}(\bar{i}_{C_0R_0},\overline{W},\overline{J})(t) \right| \le$$

$$\le e^{\mu(t-t_k)} \hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big) \times$$

$$\times \left(1 + \frac{e^{\mu_0}-1}{\mu_0} \right) \frac{1}{\widehat{L}_0^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum\limits_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C}\widehat{L}_0^{\ 1}} \breve{L}_0^2 + \frac{1}{2\sqrt{C}} \right) \equiv$$

$$\equiv e^{\mu(t-t_k)} \dot{K}_0 \hat{\rho}_\mu^{(k)}\Big((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\Big).$$

It follows

$$\rho_\mu^{(k)}\left(\dot{B}_0^{(k)}(i_{C_0R_0},W,J),\dot{B}_0^{(k)}(\bar{i}_{C_0R_0},\overline{W},\overline{J})\right)\le$$
$$\le \dot{K}_0\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right).$$

For the derivative of the second component on we have

$$\left|\dot{B}_W^{(k)}(i_{C_0R_0},W,J)(t)-\dot{B}_W^{(k)}(\bar{i}_{C_0R_0},\overline{W},\overline{J})(t)\right|\le\left|U(t)-\overline{U}t\right|+\int_{t_k}^{t_{k+1}}\left|U(s)-\overline{U}(s)\right|ds\equiv\dot{W}_1+\dot{W}_2.$$

Since

$$\dot{W}_1\le\left|\frac{dJ(t-T)}{dt}-\frac{d\overline{J}(t-T)}{dt}\right|+\frac{2\sqrt{L}}{Z_0\hat{C}_0^1}\left|i_{C_0R_0}(t)-\bar{i}_{C_0R_0}(t)\right|+$$

$$+\left(\frac{1}{Z_0\hat{C}_0^1}+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2Z_0\sqrt{C}\left(\hat{C}_0^1\right)^2}\breve{C}_0^2\right)\left|W(t)-\overline{W}(t)\right|+$$

$$+\left(\frac{1}{Z_0\hat{C}_0^1}+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2Z_0\sqrt{C}\left(\hat{C}_0^1\right)^2}\breve{C}_0^2\right)\left|J(t-T)-\overline{J}(t-T)\right|\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\times$$

$$\times\left[\frac{1}{\mu Z_0\hat{C}_0^1}\left(2\sqrt{L}+1+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2Z_0\sqrt{C}\,\hat{C}_0^1}\breve{C}_0^2\right)\right]$$

and

$$\dot{W}_2\le\frac{1}{t_{k+1}-t_k}\left|\int_{t_k}^{t_{k+1}}\left(\frac{dJ(s-T)}{dt}-\frac{d\overline{J}(s-T)}{dt}\right)ds\right|+$$

$$+\frac{2\sqrt{L}}{Z_0\hat{C}_0^1}\int_{t_k}^{t_{k+1}}\left|i_{C_0R_0}(s)-\bar{i}_{C_0R_0}(s)\right|ds+\frac{1}{Z_0\hat{C}_0^1}\int_{t_k}^{t_{k+1}}\left|W(s)-\overline{W}(s)\right|ds+\frac{1}{Z_0\hat{C}_0^1}\int_{t_k}^{t_{k+1}}\left|J(s-T)-\overline{J}(s-T)\right|ds\le$$

$$\le e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\times$$

$$\times\frac{\left(e^{\mu_0}-1\right)}{\mu_0}\frac{1}{\mu Z_0\hat{C}_0^1}\left(2\sqrt{L}+1+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2Z_0\sqrt{C}\,\hat{C}_0^1}\breve{C}_0^2\right)$$

we have

$$\left|\dot{B}_W^{(k)}(i_{C_0R_0},W,J)(t)-\dot{B}_W^{(k)}(\bar{i}_{C_0R_0},\overline{W},\overline{J})(t)\right|\le$$

$$\le\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\frac{1}{\mu Z_0\hat{C}_0^1}\left(2\sqrt{L}+1+\frac{W_0+J_0e^{-\beta}+4\sqrt{L}\,I_{C_0}}{2Z_0\sqrt{C}\,\hat{C}_0^1}\breve{C}_0^2\right)\times$$

$$\times e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\equiv$$

$$\equiv e^{\mu(t-t_k)}\dot{K}_W\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right).$$

It follows

$$\hat{\rho}^{(k)}\left(\dot{B}_W(i_{C_0R_0},W,J)(t) - \dot{B}_W(\bar{i}_{C_0R_0},\overline{W},\overline{J})\right) \le$$
$$\le \dot{K}_W \hat{\rho}_{\mu}^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right).$$

For the derivative of the third component we obtain

$$\left|\dot{B}_1^{(k)}(W,i_{C_1R_1},J)(t) - \dot{B}_1^{(k)}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(t)\right| \le$$
$$\le \left|I_{C_1}(W,i_{C_1R_1},J)(t) - I_{C_1}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(t)\right| + \frac{1}{t_{k+1}-t_k}\int_{t_k}^{t_{k+1}}\left|I_{C_1}(W,i_{C_1R_1},J)(s) - I_{C_1}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(s)\right|ds \equiv \dot{P}_1 + \dot{P}_2.$$

We have

$$\dot{P}_1 \le \frac{1}{2\sqrt{C}\ \hat{L}_1}\left|W(t-T) - \overline{W}(t-T)\right| +$$

$$+ \left(\frac{2\sqrt{C}}{\hat{L}_1^1}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\left(\hat{L}_1^1\right)^2}\breve{L}_1^2\right)\left|i_{C_1R_1}(t) - \bar{i}_{C_1R_1}(t)\right| + \frac{1}{2\sqrt{C}\ \hat{L}_1^1}\left|J(t) - \overline{J}(t)\right| \le$$

$$\le \left(\frac{2\sqrt{C}}{\hat{L}_1^1}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\left(\hat{L}_1^1\right)^2}\breve{L}_1^2\right)\frac{\rho_{\mu}^{(k)}(i_{C_1R_1},\dot{i}_{C_1R_1})}{\mu} + \frac{\rho_{\mu}^{(k)}(J,\dot{J})}{2\mu\sqrt{C}\ \hat{L}_1^1} \le$$

$$\le \frac{1}{\mu\hat{L}_1^1}\left(2\sqrt{C}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\hat{L}_1^1}\breve{L}_1^2 + \frac{1}{2\sqrt{C}}\right)\right) \times$$

$$\times e^{\mu(t-t_k)}\hat{\rho}_{\mu}^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},\dot{i}_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)$$

and

$$\dot{P}_2 \le \frac{1}{t_{k+1}-t_k}\left\{\frac{1}{2\sqrt{C}\ \hat{L}_1^1}\int_{t_k}^{t_{k+1}}\left|\dot{W}(s) - \dot{\overline{W}}(s)\right|ds +\right.$$

$$+ \left(\frac{2\sqrt{C}}{\hat{L}_1^1}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\left(\hat{L}_1^1\right)^2}\breve{L}_1^2\right)\int_{t_k}^{t_{k+1}}\left|i_{C_1R_1}(s) - \bar{i}_{C_1R_1}(s)\right|ds +$$

$$\left. + \frac{1}{2\sqrt{C}\ \hat{L}_1^1}\int_{t_k}^{t_{k+1}}\left|J(s) - \overline{J}(s)\right|ds\right\} \le$$

$$\le \left(\frac{2\sqrt{C}}{\hat{L}_1^1}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1} + \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^n}{2\sqrt{C}\left(\hat{L}_1^1\right)^2}\breve{L}_1^2\right)\rho^{(k)}(i_{C_1R_1},\bar{i}_{C_1R_1})\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds +$$

$$+ \frac{\rho^{(k)}(J,\overline{J})}{2\sqrt{C}\ \hat{L}_1^1}\int_{t_k}^{t_{k+1}}e^{\mu(s-t_k)}ds \le$$

$$\leq \frac{e^{\mu(t_{k+1}-t_k)}-1}{\mu(t_{k+1}-t_k)}\left[\left(\frac{2\sqrt{C}}{\hat{L}_1^{\,1}}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^{n}}{2\sqrt{C}\left(\hat{L}_1^{\,1}\right)^2}\breve{L}_1^{\,2}\right)\frac{\rho_\mu^{(k)}(i_{C_1R_1},\dot{i}_{C_1R_1})}{\mu}+\frac{\rho_\mu^{(k)}(\dot{J},\dot{\bar{J}})}{2\mu\sqrt{C}\ \hat{L}_1^{\,1}}\right]\leq$$

$$\leq e^{\mu(t-t_k)}\frac{e^{\mu_0}-1}{\mu_0}\frac{1}{\mu\hat{L}_1^{\,1}}\left[2\sqrt{C}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^{n}}{2\sqrt{C}\hat{L}_1^{\,1}}\breve{L}_1^{\,2}+\frac{1}{2\sqrt{C}}\right]\times$$

$$\times\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},i_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right).$$

Thus

$$\left|\dot{B}_1^{(k)}(W,i_{C_1R_1},J)(t)-\dot{B}_1^{(k)}(\overline{W},\bar{i}_{C_1R_1},\overline{J})(t)\right|\leq$$

$$\leq e^{\mu(t-t_k)}\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\frac{1}{\mu\hat{L}_1^{\,1}}\left(2\sqrt{C}\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{C_1}^{n-1}+\frac{W_0+J_0e^{-\beta}+2\sqrt{C}\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{C_1}^{n}}{2\sqrt{C}\hat{L}_1^{\,1}}\breve{L}_1^{\,2}+\frac{1}{2\sqrt{C}}\right)\times$$

$$\times\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},i_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\equiv$$

$$\equiv e^{\mu(t-t_k)}\dot{K}_1\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},i_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right).$$

Finally for the derivative of the fourth component we obtain

$$\left|\dot{B}_J^{(k)}(i_{C_0R_0},W,i_{C_1R_1},J)(t)-\dot{B}_J^{(k)}(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J})(t)\right|\leq\dot{I}_1+\dot{I}_2.$$

But

$$\dot{I}_1\leq\left|\frac{dW(t-T)}{dt}-\frac{d\overline{W}(t-T)}{dt}\right|+\frac{1}{Z_0\hat{C}_1^{\,1}}\left(2\sqrt{L}\left|i_{C_1R_1}(t)-\bar{i}_{C_1R_1}(t)\right|+\left|J(t)-\overline{J}(t)\right|+\left|W(t-T)-\overline{W}(t-T)\right|\right)\leq$$

$$\leq e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},i_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\times$$

$$\times\frac{1}{\mu Z_0\hat{C}_1^{\,1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}I_{C_1}}{2\sqrt{C}\hat{C}_1^{\,1}}\breve{C}_1^{\,2}\right)$$

and

$$\dot{I}_2\leq\frac{1}{t_{k+1}-t_k}\left[\left|\int_{t_k}^{t_{k+1}}\left(\frac{d\overline{W}(s)}{ds}-\frac{d\overline{W}(s)}{ds}\right)ds\right|+\frac{2\sqrt{L}}{Z_0\hat{C}_1^{\,1}}\int_{t_k}^{t_{k+1}}\left|i_{C_1R_1}(s)-\bar{i}_{C_1R_1}(s)\right|ds+\right.$$

$$+\frac{1}{Z_0\hat{C}_1^{\,1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}I_{C_1}}{2\sqrt{C}\hat{C}_1^{\,1}}\breve{C}_1^{\,2}\right)\int_{t_k}^{t_{k+1}}\left|J(s)-\overline{J}(s)\right|ds+\frac{1}{Z_0\hat{C}_1^{\,1}}\int_{t_k}^{t_{k+1}}\left|\overline{W}(s)-\overline{W}(s)\right|ds\right]\leq$$

$$\leq e^{\mu(t-t_k)}\hat{\rho}_\mu^{(k)}\left((i_{C_0R_0},W,i_{C_1R_1},J,\dot{i}_{C_0R_0},\dot{W},i_{C_1R_1},\dot{J}),(\bar{i}_{C_0R_0},\overline{W},\bar{i}_{C_1R_1},\overline{J},\dot{\bar{i}}_{C_0R_0},\dot{\overline{W}},\dot{\bar{i}}_{C_1R_1},\dot{\overline{J}})\right)\times$$

$$\times\frac{e^{\mu_0}-1}{\mu_0}\frac{1}{\mu Z_0\hat{C}_1^{\,1}}\left(2\sqrt{L}+1+\frac{W_0e^{-\beta}+J_0+2\sqrt{L}I_{C_1}}{2\sqrt{C}\hat{C}_1^{\,1}}H_1\right)$$

then

$$\left| \dot{B}_J^{(k)} U(W, i_{C_1 R_1}, J)(t) - \dot{B}_J^{(k)} U(\overline{W}, \bar{i}_{C_1 R_1}, \overline{J})(t) \right| \le$$

$$\le e^{\mu(t-t_k)} \hat{\rho}_\mu^{(k)} \Big((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \Big) \times$$

$$\times \left(1 + \frac{e^{\mu_0} - 1}{\mu_0} \right) \frac{1}{\mu Z_0 \widehat{C}_1^1} \left(2\sqrt{L} + 1 + \frac{W_0 e^{-\beta} + J_0 + 2\sqrt{L} I_{C_1}}{2\sqrt{C} \widehat{C}_1^1} H_1 \right) \equiv$$

$$\equiv e^{\mu(t-t_k)} \dot{K}_J \hat{\rho}_\mu^{(k)} \Big((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \Big).$$

It follows

$$\hat{\rho}_\mu^{(k)} \Big(\dot{B}_J(i_{C_0 R_0}, W, J), \dot{B}_J(\bar{i}_{C_0 R_0}, \overline{W}, \overline{J}) \Big) \le$$

$$\le \dot{K}_J \hat{\rho}_\mu^{(k)} \Big((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \Big).$$

Let us denote by

$$K = \max\left\{ e^{\mu_0} K_0, e^{\mu_0} K_W, e^{\mu_0} K_1, e^{\mu_0} K_J, \dot{K}_0, \dot{K}_W \dot{K}_1, \dot{K}_J \right\} < 1.$$

Then

$$\hat{\rho}_\mu^{(k)} \Big((B_0, B_W, B_1, B_J, \dot{B}_0, \dot{B}_W, \dot{B}_1, \dot{B}_J), (\overline{B}_0, \overline{B}_W, \overline{B}_1, \overline{B}_J, \dot{\overline{B}}_0, \dot{\overline{B}}_W, \dot{\overline{B}}_1, \dot{\overline{B}}_J) \Big) \le$$

$$\le K \hat{\rho}_\mu^{(k)} \Big((i_{C_0 R_0}, W, i_{C_1 R_1}, J, \dot{i}_{C_0 R_0}, \dot{W}, \dot{i}_{C_1 R_1}, \dot{J}), (\bar{i}_{C_0 R_0}, \overline{W}, \bar{i}_{C_1 R_1}, \overline{J}, \dot{\bar{i}}_{C_0 R_0}, \dot{\overline{W}}, \dot{\bar{i}}_{C_1 R_1}, \dot{\overline{J}}) \Big).$$

The operator B has a unique fixed point. It is an oscillatory solution of the above problem on the interval $[T, 2T]$.
Theorem 7.1 is thus proved.

Remark 7.1. The obtained solution on the interval $[T, 2T]$ might be taken as an initial function and by the same way to obtain a unique solution on $[2T, 3T]$ and so on.

8. NUMERICAL EXAMPLE

We collect all inequalities guaranteeing an existence-uniqueness of an oscillatory solution:

$$\frac{W_0 + J_0}{2\sqrt{C}} \le \phi_0; \tag{I}$$

$$\frac{2L}{R\widehat{L}_0^1 \sqrt{C}} \sinh\frac{RT_0}{L}\left[W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{\ n} \right] \le I_{C_0}; \tag{II}$$

$$J_0 e^{-\beta} + \frac{4L\left(W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0} \right)}{Z_0 \widehat{C}_0^1 R} \sinh\frac{RT_0}{L} \le W_0; \tag{III}$$

$$\frac{2L}{R\widehat{L}_1^1 \sqrt{C}} \sinh\frac{RT_0}{L}\left(W_0 e^{-\beta} + J_0 + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{\ n} \right) \le I_{C_1}; \tag{IV}$$

$$W_0 e^{-\beta} + \frac{4L}{R} \frac{W_0 e^{-\beta} + J_0 + 4\sqrt{L} I_{C_1}}{Z_0 \widehat{C}_1^1} \sinh\frac{RT_0}{L} \le J_0; \tag{V}$$

$$e^{\mu_0} K_0 = \frac{e^{3\mu_0}}{\mu^2 \hat{L}_0^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \breve{L}_0^{\ 2} \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \hat{L}_0^{\ 1}} + \frac{1}{2\sqrt{C}} \right) < 1; \qquad \text{(VI)}$$

$$e^{\mu_0} K_W = \frac{e^{3\mu_0}}{\mu^2 Z_0 \hat{C}_0^{\ 1}} \left(2\sqrt{L} + 1 + \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2\sqrt{C} \hat{C}_0^{\ 1}} \breve{C}_0^{\ 2} \right) < 1; \qquad \text{(VII)}$$

$$e^{\mu_0} K_1 = \frac{e^{3\mu_0}}{\mu^2 \hat{L}_1^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \breve{L}_1^{\ 2} \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{n}}{2\sqrt{C} \hat{L}_1^{\ 1}} + \frac{1}{2\sqrt{C}} \right) < 1; \qquad \text{(VIII)}$$

$$e^{\mu_0} K_J = \frac{e^{3\mu_0}}{\mu^2 Z_0 \hat{C}_1^{\ 1}} \left(2\sqrt{L} + 1 + \breve{C}_1^{\ 2} \frac{W_0 e^{-\beta} + J_0 + 2\sqrt{L} I_{C_1}}{2\sqrt{C} \hat{C}_1^{\ 1}} \right) < 1; \qquad \text{(IX)}$$

$$\dot{K}_0 = \left(1 + \frac{e^{\mu_0} - 1}{\mu_0} \right) \frac{1}{\mu \hat{L}_0^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(0)} \right| I_{C_0}^{n-1} + \breve{L}_0^{\ 2} \frac{W_0 + J_0 e^{-\beta} + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(0)} \right| I_{C_0}^{n}}{2\sqrt{C} \hat{L}_0^{\ 1}} + \frac{1}{2\sqrt{C}} \right) < 1; \qquad \text{(X)}$$

$$\dot{K}_W = \left(1 + \frac{e^{\mu_0} - 1}{\mu_0} \right) \frac{1}{\mu Z_0 \hat{C}_0^{\ 1}} \left(2\sqrt{L} + 1 + \breve{C}_0^{\ 2} \frac{W_0 + J_0 e^{-\beta} + 4\sqrt{L} I_{C_0}}{2\sqrt{C} \hat{C}_0^{\ 1}} \right) < 1; \qquad \text{(XI)}$$

$$\dot{K}_1 = \left(1 + \frac{e^{\mu_0} - 1}{\mu_0} \right) \frac{1}{\mu \hat{L}_1^{\ 1}} \left(2\sqrt{C} \sum_{n=1}^{m} n \left| r_n^{(1)} \right| I_{C_1}^{n-1} + \breve{L}_1^{\ 2} \frac{W_0 e^{-\beta} + J_0 + 2\sqrt{C} \sum_{n=1}^{m} \left| r_n^{(1)} \right| I_{C_1}^{n}}{2\sqrt{C} \hat{L}_1^{\ 1}} + \frac{1}{2\sqrt{C}} \right) < 1; \qquad \text{(XII)}$$

$$\dot{K}_J = \left(1 + \frac{e^{\mu_0} - 1}{\mu_0} \right) \frac{1}{\mu Z_0 \hat{C}_1^{\ 1}} \left(2\sqrt{L} + 1 + \breve{C}_1^{\ 2} \frac{W_0 e^{-\beta} + J_0 + 2\sqrt{L} I_{C_1}}{2\sqrt{C} \hat{C}_1^{\ 1}} \right) < 1. \qquad \text{(XIII)}$$

Consider a transmission line with length $\Lambda = 100\,m$ and cross-section area $S = 4\ mm^2$. Specific resistance for the cuprum is $\rho_c = 0{,}0175$, the resistance per-unit length is $R = (\rho_c \Lambda)/S \approx 0{,}44\,\Omega$. Let $L = 0{,}45\,\mu H/m$, $C = 80\,pF/m$, $v = 1/\sqrt{LC} = 1/(6.10^{-9}) = 1{,}66.10^8$; $Z_0 = \sqrt{L/C} = 75\,\Omega$. Then $T = \Lambda\sqrt{LC} = 6.10^{-7}$, $R/L = \approx 10^6$, $RT/L = R\Lambda/Z_0 \approx 0{,}59$; $e^{0,59} \approx 1{,}8$. For waves with length $\lambda_0 = \frac{10^{-3}}{6}\,m$ we have $f_0 = \frac{1}{\lambda_0 \sqrt{LC}} = 10^{12}\,Hz \Rightarrow T_0 = \frac{1}{f_0} = 10^{-12}$, $\frac{RT_0}{L} = 10^6$. We choose $\mu = 10^{12}$.

Then $\mu T_0 = \mu_0 = 1$. Consequently $\mu - (R/L) \approx 10^{12}$; $\sinh RT_0/L \approx RT_0/L = 10^{-6}$. For resistive elements with V-I characteristics $R_0(i) = R_1(i) = 0{,}028i - 0{,}125i^3$ and inductive elements with characteristics $\widetilde{L}_0(i) = \widetilde{L}_1(i) = 10^{-7} \tanh(10^2 i)$ (cf. [2]) for $\left| i_{C_p R_p}(x,t) \right| \le I_{C_p} \le i_0 = 0{,}01$ we have:

$$\frac{d\widetilde{L}_p(i)}{di} = \frac{10^{-5}}{\cosh^2(10^2 i)} \ge \frac{10^{-5}}{\cosh^2 1} = \frac{10^{-5}}{2{,}38} \equiv \hat{L}_p^{\ 1} > 0 \Rightarrow \frac{1}{d\widetilde{L}_p(i)/di} \le \frac{1}{\hat{L}_p^{\ 1}} = \frac{2{,}38}{10^{-5}};$$

$$\frac{d^2 \widetilde{L}_p(i)}{di^2} = 10^{-5}(-2)\cosh^{-3}(10^2 i)\sinh(10^2 i).10^2 = -2.10^{-3} \frac{\sinh(10^2 i)}{\cosh^3(10^2 i)};$$

$$\left| \frac{d^2 \widetilde{L}_p(i)}{di^2} \right| = 2.10^{-3} \frac{\sinh\left(10^2 i\right)}{\cosh^3(10^2 i)} \le 2.10^{-3} \sinh\left(10^2 i_0\right) = 0{,}002 = \breve{L}_p^{\ 2}.$$

Let us take $h = 2$. Then for $c_p = 50\,pF$, $\Phi_p = 0{,}5\,V$ $(p = 0{,}1)$ and $\phi_0 = 0{,}16$ we have

$$\widehat{C}_0^{\,1} = \widehat{C}_1^{\,1} = \frac{2c_0\sqrt{\Phi_0}}{\sqrt{(\Phi_0 + \phi_0)^5}}\,\frac{\Phi_0 - 2{,}5.\phi_0}{2} = \frac{2.50.10^{-12}\sqrt{0{,}5}}{\sqrt{(0{,}5 + 0{,}16)^5}}\,\frac{0{,}5 - 2{,}5.0{,}16}{2} \approx 1{,}45.10^{-11};$$

$$\breve{C}_0^{\,2} = \breve{C}_1^{\,2} = \frac{2c_0\sqrt{\Phi_0}\,(2\Phi_0 + \phi_0)}{4\sqrt{(\Phi_0 - \phi_0)^5}} = \frac{2.50.10^{-12}\sqrt{0{,}5}\,(1 + 0{,}16)}{4\sqrt{(0{,}5 - 0{,}16)^5}} \approx 3.10^{-10}.$$

Finally for $W_0 = J_0 = 10^{-7}$; $I_{C_0} = I_{C_1} = 0{,}001$; $\beta > 0$ sufficiently large we obtain

$$\frac{W_0}{2\sqrt{C}} = \frac{10^{-6}}{2\sqrt{80.10^{-12}}} \le 0{,}16\,.$$ We have to check only (II), (III) and (X), (XI):

$$\frac{4{,}76.10^{-12}}{10^{-6}\sqrt{80}}\,\frac{1}{10^{-5}}\left[10^{-2} + 2\sqrt{80.10^{-12}}\,(0{,}028 + 0{,}125.10^{-6})\right] \le 1; \tag{II}$$

$$\frac{4.10^{-12}\left(1 + 4\sqrt{0{,}45.}\right)}{108{,}75.10^{-11}} \le 1; \tag{III}$$

$$\dot{K}_0 = \frac{2{,}38e}{10^7}\left(0{,}056\sqrt{80.10^{-12}} + \frac{10^{-7}2{,}38 + 4{,}76\sqrt{80.10^{-12}}\,(0{,}028.10^{-3} + 0{,}125.10^{-9})}{10^{-14}\sqrt{80}} + \frac{1}{2\sqrt{80.10^{-12}}}\right) \approx 0{,}17 < 1 \tag{X}$$

$$\dot{K}_W = 2{,}5.10^{-2}\left(1{,}34.10^{-3} + 1 + 3.10^{-10}\frac{10^{-7} + 4\sqrt{0{,}45.10^{-6}}\,.10^{-3}}{2\sqrt{80.10^{-12}}\,1{,}45.10^{-11}}\right) \approx 0{,}10 < 1; \tag{XI}$$

$$K = \max\{\dot{K}_0, \dot{K}_W, \dot{K}_1, \dot{K}_J\} = 0{,}17.$$

The successive approximations could be obtained similarly as in [20].

9. CONCLUSION

For the applications one needs only to verify a system of inequalities (cf. 8. Numerical example) to make sure that there exists a unique oscillatory solution vanishing exponentially at infinity. The solution obtained $W(x,t), J(x,t)$ might be approximated by an advance prescribed accuracy. In fact we find actual voltage and current by the formulas

$$u(x,t) = \frac{e^{-\frac{R}{L}t}W(x,t)}{2\sqrt{C}} - \frac{e^{-\frac{R}{L}t}J(x,t)}{2\sqrt{C}}, \quad i(x,t) = \frac{e^{-\frac{R}{L}t}W(x,t)}{2\sqrt{L}} + \frac{e^{-\frac{R}{L}t}J(x,t)}{2\sqrt{L}}.$$

One can obtain successive approximations to the solution beginning with simple trigonometric functions. It must be noted that

REFERENCES

[1]. L. A. Weinstein and W. A. Solntzev, *Lectures on RF Electronics*, Sovetskoe Radio, Moscow, 1973 (in Russian).

[2]. C. A. Desoer and E. S. Kuh, *Basic Circuit Theory,* McGraw Hill Book Company, New York, 1969.

[3]. D. Pozar, *Microwave Engineering*, J. Wiley & Sons Inc., New York, 1998.

[4]. C. R. Paul, *Analysis of Multi-Conductor Transmission Lines*, J. Wiley & Sons Inc., New York, 1994.

[5]. S. J. Ramo, R. Whinnery and T. van Duzer, *Fields and Waves in Communication Electronics*, J. Wiley & Sons, Inc. New York, 1994.

[6]. S. Rosenstark, *Transmission Lines in Computer Engineering*, Mc Grow-Hill, New York, 1994.

[7]. P. Vizmuller, *RF Design Guide Systems, Circuits and Equations*, Artech House, Inc., Boston London, 1995.

[8]. P. C. Magnusson, G. C. Alexander and V. K. Tripathi, *Transmission Lines and Wave Propagation*, 3rd ed. CRC Press. Boka Raton, 1992.

[9]. J. Dunlop and D. G. Smith, *Telecommunications Engineering*, Chapman & Hall, London, 1994.

[10]. S. A. Maas, *Nonlinear Microwave and RF Circuits*, 2nd ed. Artech House Boston London, 2003.

[11]. D. K. Misra, *Radio-Frequency and Microwave Communication Circuits, Analysis and Design*. 2nd ed. University of Wisconsin – Milwaukee, J. Wiley & Sons, Inc., 2004.

[12]. G. Miano and A. Maffucci, *Transmission Lines and Lumped Circuits*, 2nd ed. Academic Press, New York, 2010.

[13]. Angelov V. G. and Hristov M., Distortionless lossy transmission lines terminated by in series connected *RCL*-loads, Circuits and Systems, 2 (Oct. 2011), 297-310 (2011).

[14]. Angelov V. G., Periodic regimes for distortionless lossy transmission lines terminated by parallel connected *RCL*-loads. In "Transmission Lines: Theory, Types and Applications", Nova Science Publishers, Inc., 259-293 (2011).

[15]. Angelov V. G., Oscillations in lossy transmission lines terminated by in series connected nonlinear *RCL*-loads, Int. J. Theoretical and Mathematical Physics, 2 (5), 143-16 (2012).

[16]. Angelov V. G., Various applications of fixed point theorems in uniform spaces, Conference Proceedings, 10th Int. Conference on Fixed Point Theory and its Applications, Cluj-Napoca, (2012).

[17]. Angelov V. G., Lossless transmission lines terminated by in series connected *RL*-loads parallel to *C*-load, Int. J. Theoretical and Mathematical Physics. 3 (1), 10-25 (2013).

[18]. Angelov V. G. and Zahariev A., Lossy transmission lines terminated by parallel connected *RC*-loads and in series connected *L*-load (I). Int. J. of Modern Eng. Res. 3 (May-June, 2013), 1410-1418 (2013).

[19]. Angelov V. G., Lossless transmission lines terminated by *L*-load in series connected to parallel connected *GL*-loads. British J. of Mathematics & Computer Science, 3 (3), 352-389, (2013).

[20]. V. G. Angelov, *A Method for Analysis of Transmission Lines Terminated by Nonlinear Loads*. Nova Science, New York, 2014.

[21]. V. G. Angelov, *Fixed Points in Uniform Spaces and Applications*. Cluj University Press "Babes-Bolyia University", Romania, 2009.

AUTHOR'S BIOGRAPHY

Born - 1950, Bulgaria. PhD in Mathematics - 1981; Assoc. Professor - 1985; Doctor of Technical Sciences - 2004; Professor - 2007. Head of Department of Mathematics. Research subjects: Delay differential equations, Fixed point theory, Classical electrodynamics, Electrodynamics and radio technical devices, Transmission line theory and nonlinear circuit theory. Awards: Who's Who in the World - since 1992; Gold Medal of the American Biographical Institute - 2010; International Peace Prize, United Cultural Convention, USA - 2012.