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Pulsatile Motion of Non-Newtonian Fluid with Heat and Mass Transfer through a Porous Medium in a Solid Cylindrical Pipe in the Presence of Magnetic Field

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Abstract: *The pulsatile flow of non-Newtonian fluid which obeying biviscosity fluid (Casson model) stress-strain relation is studied. We take into consideration the porosity of medium and the unsteady motion with heat and mass transfer under the effects of magnetic field and heat source. The equations of momentum, energy and concentration have been solved by using Lightill method. The governing equations are solved analytically by using Mathematica package. The velocity, temperature and concentration distribution are obtained. The effects of various parameters of the problem on these distributions are discussed and depicted graphically through a set of figures.*

Keywords: *pulsatile flow; Non-Newtonian fluid; Porous medium; Magnetic field; Heat transfer; Mass transfer; Diffusion Thermo.*

1. INTRODUCTION

The problem of pulsatile flow have gained importance due to their immediate practical applications in biomechanical and engineering science. In physiology, pulsatile mechanism is involved in urine transport from kidney to bladder through the ureter, movement of chime in the gastrointestinal tract, transport of spermatozoa in the ductus efferentes of the male reproductive tracts, and in the cervical canal, in movement of ovum in the fallopian, transport of lymph in the lymphatic vessels. In situations like travel in vehicles, aircraft, operating jackhammer and sudden movements of body during sports activities, the human body experience external body acceleration. Prolonged exposure of a healthy human body to external acceleration may cause serious health problem like headache, increase of pulse rate and loss of vision on account of disturbances in blood flow [1].

The analysis of the mechanisms responsible for pulsatile transport have been studied by many authors. The problem of pulsatile flow with reference to stenosis in microcirculation was analysed by Bitoun and Bellet [2]. Rao and Rathna Devanathan [3] and Schneck and Ostrack[4] studied pulsatile flow through circular tubes varying cross-section at low Reynolds number. Young and Tasi[5-6] studied the steady and unsteady flows across a stenosis experimentally which can be found also in Siouffi et al. [7] . Eldabe et al. [8-9] studied pulsatile magneto hydrodynamic viscoelastic flow through a channel bounded by permeable plates and the effect of couple stresses on pulsatile hydro magnetic poiseuille flow. In these studied the tube wall is taken to be impermeable. Macey[10,11] discussed the steady flow of a viscous fluid through a circular tube with a permeable wall. The pulsatile flow of non-Newtonian fluids in pipe discussed by Edwards et al. [12]. The problem of pulsatile flow of MHD non-Newtonian fluid obeying power law model with convection heat transfer through a non –Darcy porous medium between two coaxial cylinders is studied by Abou-zeid[13]. Flow through porous media is very prevalent in nature and therefore the study of flow through a porous medium has become of principle interest in many engineering applications. Many authors have studied the effect of porous medium on the motion of the fluid. Some of these studies have been made by Abdel-hady and Kamel [14] discussed the steady MHD flow of a viscoelastic fluid through a porous medium. Numerical study of pulsatile

MHD non-Newtonian fluid flow with heat and mass transfer through a porous medium between two permeable parallel plates was studied by Eldabe et al. [15].

In this paper, the main aim is to obtain a numerical solution of the problem of unsteady plusatile flow with heat and mass transfer. The fluid used is biviscosity fluid through a uniform porous media in a solid cylindrical pipe in the presence of magnetic field. The governing equations were solved numerically by using Mathematica. The velocity, temperature and concentration distribution are obtained. The velocity, temperature and concentration distributions are calculated for different values of P_s, P_u, Re, K, M, β, ω, t, Pr, q, Sc and Sr.

2. Formulation of the Problem

Consider the unsteady flow of non-Newtonian fluid obeying Casson model through a permeable infinite cylindrical tube under the action of external uniform magnetic field of strength B_0 . Choose cylindrical coordinates (r,θ,z), where z is the axis of the tube.

The constitutive equation of the Casson fluid model is

$$\tau_{ij}=\begin{cases}2(\mu_\beta+p_y/\sqrt{2\pi})e_{ij}, & \pi\ge\pi_c\\ 2(\mu_\beta+p_y/\sqrt{2\pi_c})e_{ij}, & \pi\prec\pi_c\end{cases}$$

Where p_y is the yielding stress, $\pi=e_{ij}\,e_{ij}$, where e_{ij} is the (i,j) component of the deformation rate and $\beta=\dfrac{\mu_\beta\sqrt{2\pi_c}}{p_y}$ is the dimensionless upper limit of apparent viscosity coefficient. For ordinary Newtonian fluid ($p_y=0$).

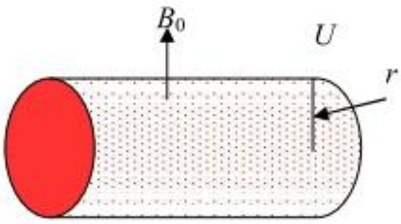


Fig 1. *Schematic of the Problem*

The governing equations used in this problem can be written as follows:

Continuity equation:

$$\frac{\partial\rho}{\partial t}+\rho\nabla.\underline{V}=0, \tag{1}$$

Momentum equation in porous medium under the effect of uniform external magnetic field

$$\rho\left(\frac{\partial\underline{V}}{\partial t}+\underline{V}.\nabla\underline{V}\right)=-\nabla P+\nabla.\underline{\tau}-\frac{\mu_\beta}{k_p}\underline{V}+\underline{J}\times\underline{B}, \tag{2}$$

Temperature equation with heat generation

$$\rho c_p\frac{dT}{dt}=k_c\nabla^2T+QT, \tag{3}$$

Concentration equation with thermal diffusion

$$\frac{dC}{dt}=D_m\nabla^2C+\frac{D_m k_T}{T_m}\nabla^2T, \tag{4}$$

Where $\underline{J} = \sigma(\underline{E} + \underline{V} \wedge \underline{B})$ is the current density vector, σ is the electrical conductivity, $\underline{B} = \mu_e \underline{H}$ is the magnetic field strength, μ_e is the magnetic permeability, $\underline{H}$ is the applied magnetic field, $\underline{E}$ is the electric field, $\underline{V}$ is the velocity fluid, ρ is the density, $\tau(\tau_{ij})$ is the stress tensor, p is the pressure, t is the time, μ_β is the plastic viscosity of the fluid, k_p is the permeability porous medium, T is the fluid temperature, k_c is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, Q is the heat generation, C is the fluid concentration, D_m is the coefficient of mass diffusivity, k_T is the thermal diffusion rate, T_m is the mean fluid temperature.

Since the flow parameters are independent of the azimuthal coordinate θ, the velocity is given by $\underline{V} = (0,0,\ u)$ and the magnetic field vector is $\underline{B} = (B_0,0,0)$.

Now, we shall consider the magnetic Reynolds number is very small, therefore the induced magnetic field and external electric field are neglected. Under these assumptions, the equations (2-4) can be written as:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu_\beta(1+\beta^{-1})}{\rho}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) - \frac{1}{\rho}(\frac{\mu_\beta}{k_p} + \sigma B_0^2)u, \tag{5}$$

$$\frac{\partial T}{\partial t} = k_T(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}) + \frac{Q}{\rho c_p}T, \tag{6}$$

$$\frac{\partial C}{\partial t} = D_m(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}) + \frac{D_m k_T}{T_m}(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}), \tag{7}$$

The approximate boundary conditions are

$$\left.\begin{array}{llll} \frac{\partial u}{\partial r} = 0,\ \frac{\partial T}{\partial r} = 0 & \text{and} & \frac{\partial C}{\partial r} = 0 & \text{at} \quad r = 0 \\ u = 0,\quad T = T_1 & \text{and} & C = C_1 & \text{at} \quad r = a \end{array}\right\}, \tag{8}$$

Let us introduce the following dimensionless quantities as follows:

$$\left.\begin{array}{llll} r^* = \frac{r}{a}, & z^* = \frac{z}{a}, & u^* = \frac{u a}{\upsilon}, & t^* = \frac{\upsilon}{a^2}t, \\ P^* = \frac{a^2}{\rho\upsilon^2}P, & \theta^* = \frac{T}{T_1}, & \varphi^* = \frac{C}{C_1}, & \omega^* = \frac{a^2}{\upsilon}\omega \end{array}\right\}, \tag{9}$$

After substituting from equation (9), equations (5), (6) and (7) may be written in dimensionless form after dropping star mark.

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial z} + (1+\beta^{-1})\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) - (\frac{1}{K} + M)u, \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}) + q\theta, \tag{11}$$

$$\frac{\partial \varphi}{\partial t} = \frac{1}{Sc}(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}) + Sr(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r}), \tag{12}$$

The dimensionless boundary conditions are

$$\left.\begin{array}{llll}\dfrac{\partial u}{\partial r}=0,\ \dfrac{\partial \theta}{\partial r}=0 \quad \text{and} \quad \dfrac{\partial \varphi}{\partial r}=0 & \text{at} & r=0 \\ u=0,\quad \theta=1 \quad \text{and} \quad \varphi=1 & \text{at} & r=1\end{array}\right\}, \tag{13}$$

Where $M=\dfrac{\sigma\beta_0^2 a^2}{\rho\upsilon}$ (the magnetic field parameter), $K=\dfrac{k_p}{a^2}$ (the porosity parameter) , $\Pr=\dfrac{\upsilon}{k_T}$(Prandtl number), $q=\dfrac{a^2 Q}{\upsilon\rho c_p}$ (heat source), $Sc=\dfrac{\upsilon}{D_m}$ (Schmidt number), $Sr=\dfrac{D_m k_T \upsilon T_1}{a^2 T_m C_1}$(Soret number).

3. Method of Solution

For pulsation pressure gradient, let

$$-\frac{\partial P}{\partial z}=P_s+P_o e^{i\omega t}, \tag{14}$$

Where ω is the frequency, P_s and P_o are the steady component and the oscillatory component of pressure gradient, respectively

The equations (10), (11) and (12) can be solved by using the following perturbation technique:

$$\left.\begin{array}{l} u=u_s+u_o e^{i\omega t} \\ \theta=\theta_s+\theta_o e^{i\omega t} \\ \varphi=\varphi_s+\varphi_o e^{i\omega t}\end{array}\right\}, \tag{15}$$

Substituting from (14) and (15) in (10), (11) and (12) and equating the like terms on both sides, we get the following system equations:

$$\frac{d^2u_s}{dr^2}+\frac{1}{r}\frac{du_s}{dr}-\frac{(1/K+M)}{(1+\beta^{-1})}u_s=\frac{-P_s}{(1+\beta^{-1})}, \tag{16}$$

$$\frac{d^2u_o}{dr^2}+\frac{1}{r}\frac{du_o}{dr}-\frac{(i\omega+1/K+M)}{(1+\beta^{-1})}u_o=\frac{-P_o}{(1+\beta^{-1})}, \tag{17}$$

$$\frac{1}{\Pr}\left(\frac{d^2\theta_s}{dr^2}+\frac{1}{r}\frac{d\theta_s}{dr}\right)+q\theta_s=0, \tag{18}$$

$$\frac{1}{\Pr}\left(\frac{d^2\theta_o}{dr^2}+\frac{1}{r}\frac{d\theta_o}{dr}\right)-(i\omega-q)\theta_o=0, \tag{19}$$

$$\frac{1}{Sc}\left(\frac{d^2\varphi_s}{dr^2}+\frac{1}{r}\frac{d\varphi_s}{dr}\right)+Sr\left(\frac{d^2\theta_s}{dr^2}+\frac{1}{r}\frac{d\theta_s}{dr}\right)=0, \tag{20}$$

$$\frac{1}{Sc}\left(\frac{d^2\varphi_o}{dr^2}+\frac{1}{r}\frac{d\varphi_o}{dr}\right)-i\varpi C_o+Sr\left(\frac{d^2\varphi_o}{dr^2}+\frac{1}{r}\frac{d\varphi_o}{dr}\right)=0, \tag{21}$$

Subject to the following boundary conditions

$$\left.\begin{array}{lll} u_s=\text{finite},\ u_o=0, \theta_s=\text{finite},\ \theta_o=0, \varphi_s=\text{finite} \ \text{and} \ \varphi_o=\text{finite} & \text{at} & r=0 \\ u_s=0,\ u_o=0,\ \theta_s=1, \theta_o=0,\ \varphi_s=1 \ \text{and} \ \varphi_o=0 & \text{at} & r=1\end{array}\right\}, \tag{22}$$

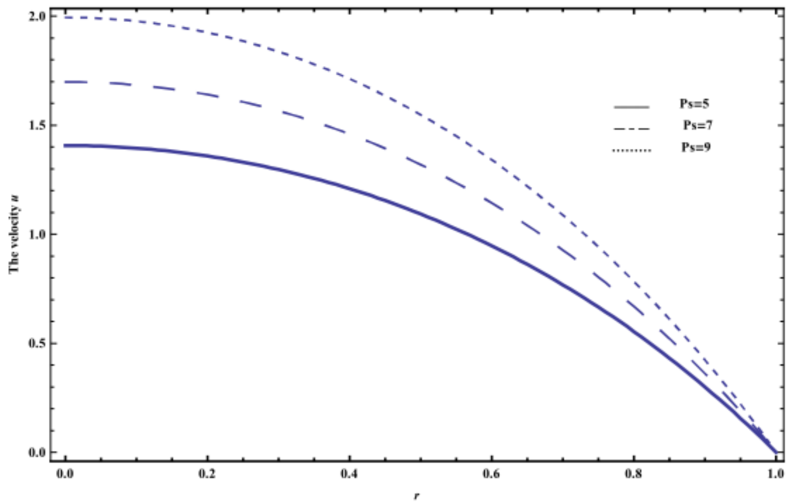
4. NUMERICAL RESULTS AND DISCUSSION

The system of equations that governs the non-Newtonian fluid flow in a solid cylindrical pipe in the presence of magnetic field and porous medium are solved analytically by using Lightill method. The formulas for the velocity, temperature and concentration distributions are obtained, and calculated for different values of P_s, P_u, Re, K, M, β, ω, t, Pr, q, Sc and Sr in figures (2-14).

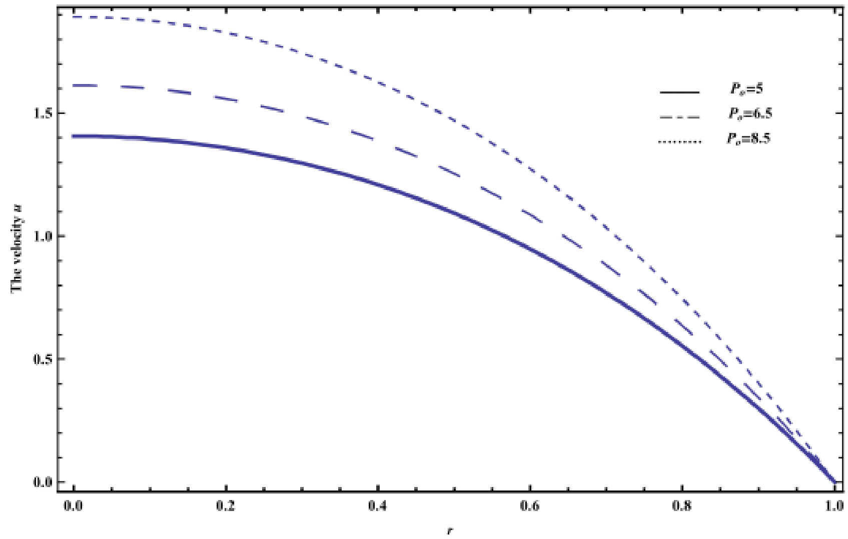
The effects of physical parameters on the velocity distribution are indicated through figures 1-8. In these figures the velocity distribution u is plotted versus the coordinate r. Figures (2) and (3) show the behavior of the velocity u for different values of the steady component of pressure gradient P_s and the oscillatory component of pressure gradient P_o respectively. It is seen from Figs. (2) and (3), that the velocity u increases with increase both of P_s and P_o i.e. the increase of the pressure gradient caused increment in the movement of fluid. Figure (4) illustrates the change of the velocity u for several values of the non-Newtonian fluid parameter β. It is observed from this figure that the velocity u increases with the increase of β. The result in figure (4) is in agreement with the result which is obtained by Abdelnaby et al. [15]. The effect of the magnetic parameter M on the velocity u is shown in Fig. (5). It is clear that the velocity u decreases with the increase of the magnetic parameter M, because the uniform magnetic field is in resistant direction to the movement of the fluid. Similarly, if we draw the velocity for different values of the permeabilty parameter K, we will obtain a figure in which the behavior of the curves are the same as those obtained in figure (6). The velocity u for various values of the time t and the frequency of the oscillating ω are exhibited in Figs. (7) and (8), respectively. From these figures, we observed that the velocity u decreases with increase both of t and ω.

The effect of the heat generation q on the temperature θ is shown in Fig. (9), and it shown that the temperature θ increases by increasing q. Because the fluid is exposed to heat and this is shown form the heat equation and that this result is agreement with those obtained by Abou-zeid [13]. In figure (10) the effect of Prandtl number Pr on the temperature θ is presented. From this figure, we observed that the effect of Pr on θ is similar to the effect of q on θ illustrated in Fig. (10).

Figures (11) gives the change of the concentration φ for several values the heat source parameter q. It is noted from this figures that the concentration decreases with increasing q. We observed that this result is a proven fact because the heat source is inversely proportional to concentration. The effects of Schmidt number Sc, Soret number Sr and Prandtl number Pr on the concentration φ are elucidate in figures (12), (13) and (14), respectively. We observed from these figures that the effects of these parameters are found to be similar to the effect of the heat source parameter q on φ illustrated in Fig. (11).



Figure(2)



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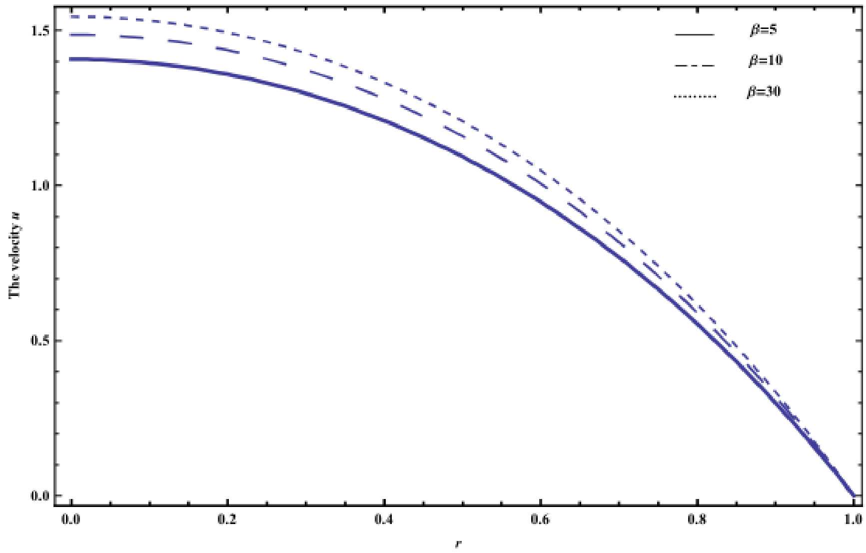

Figure (4)

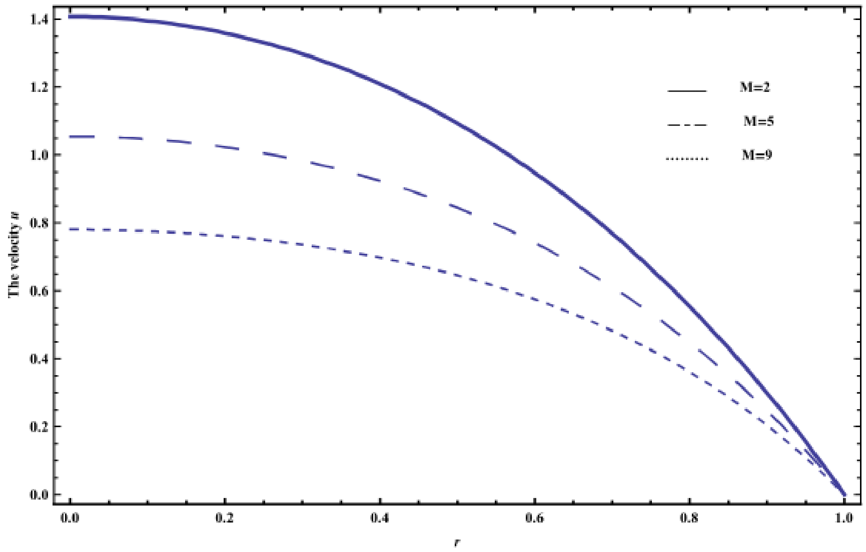

Figure (5)

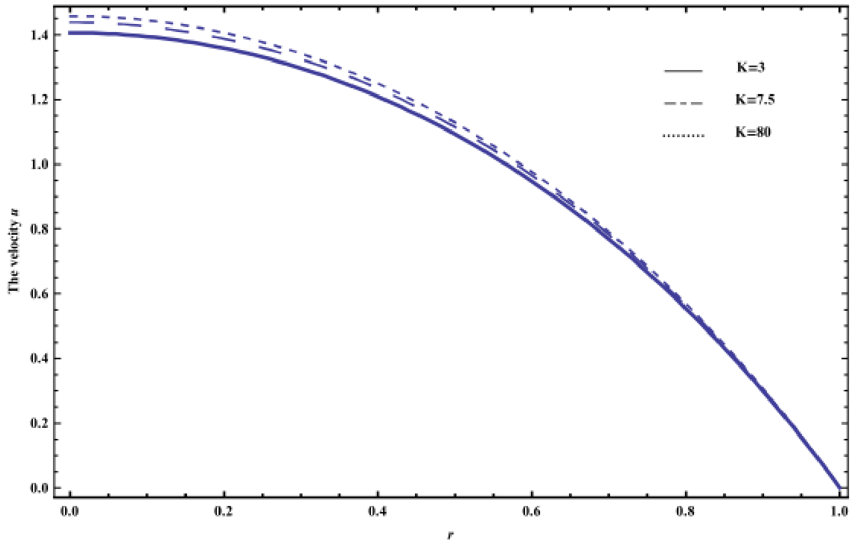

Figure (6)

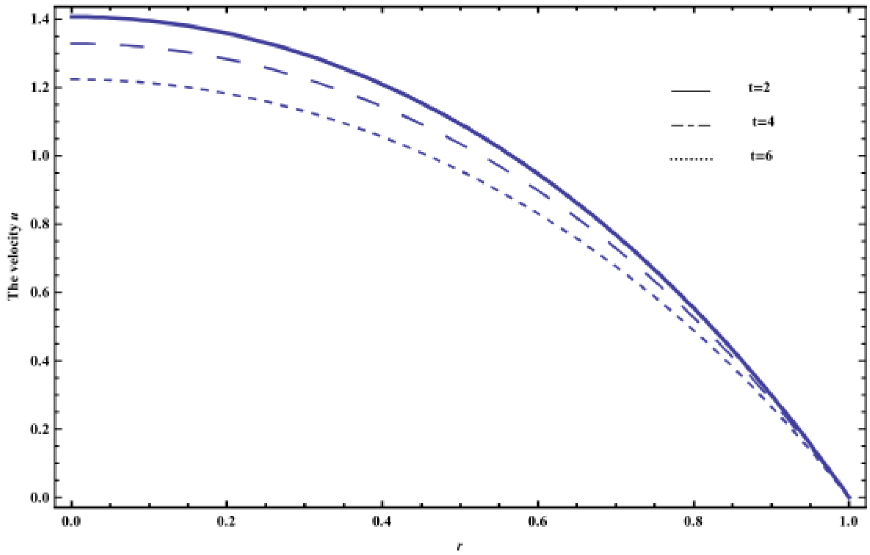

Figure (7)

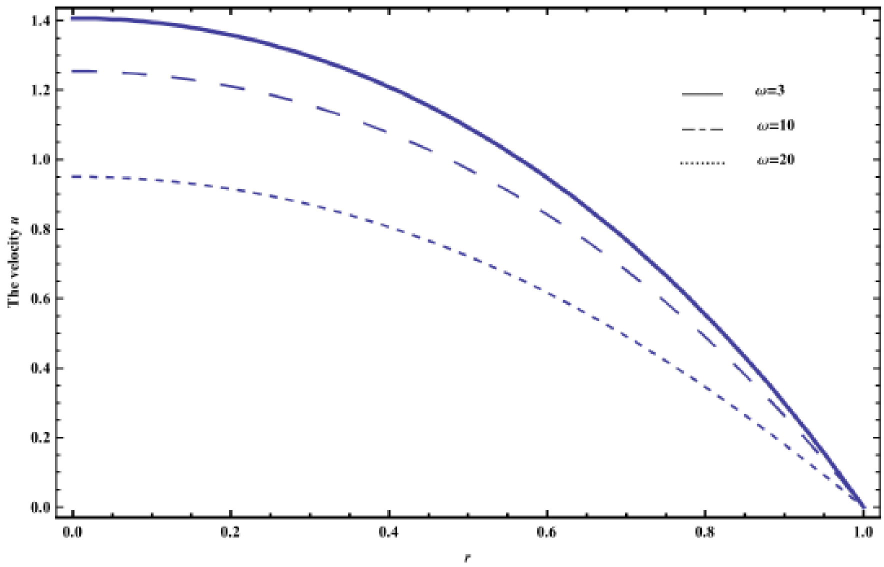

Figure (8)

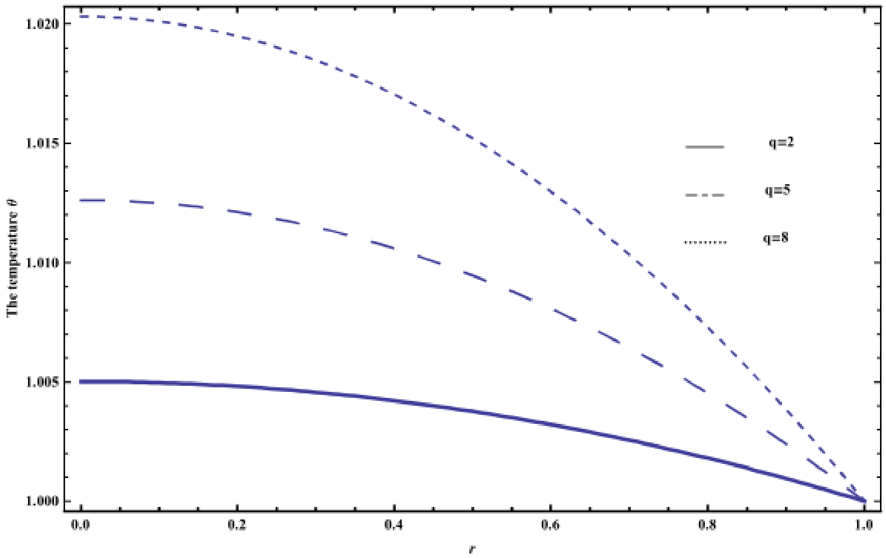

Figure (9)

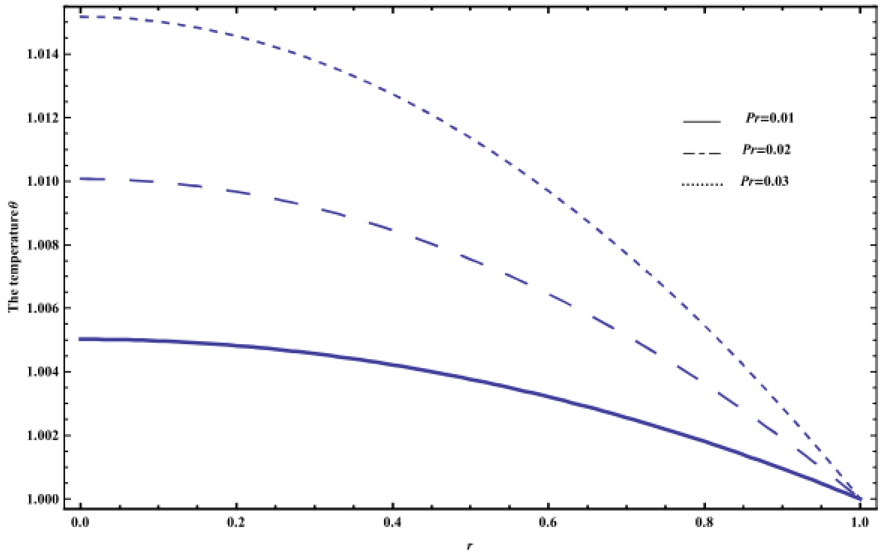

Figure (10)

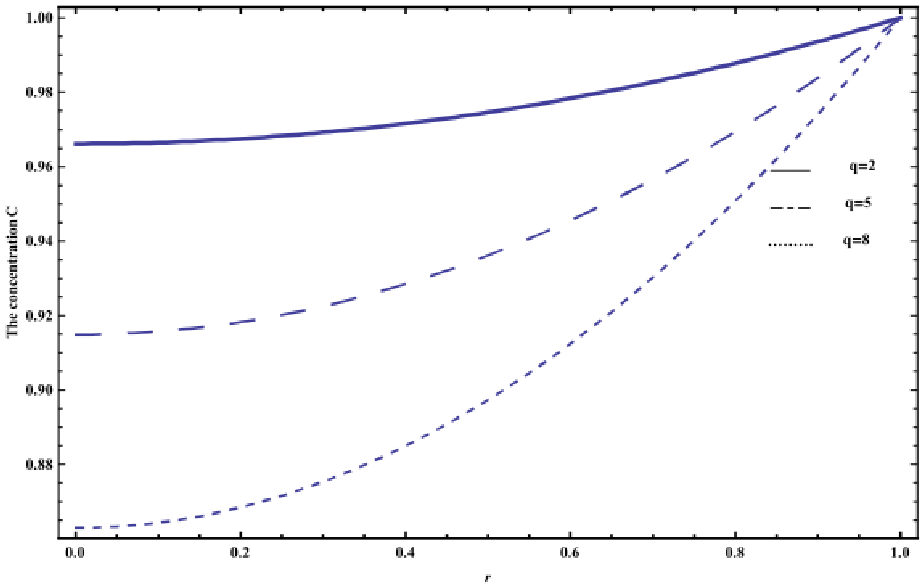

Figure (11)

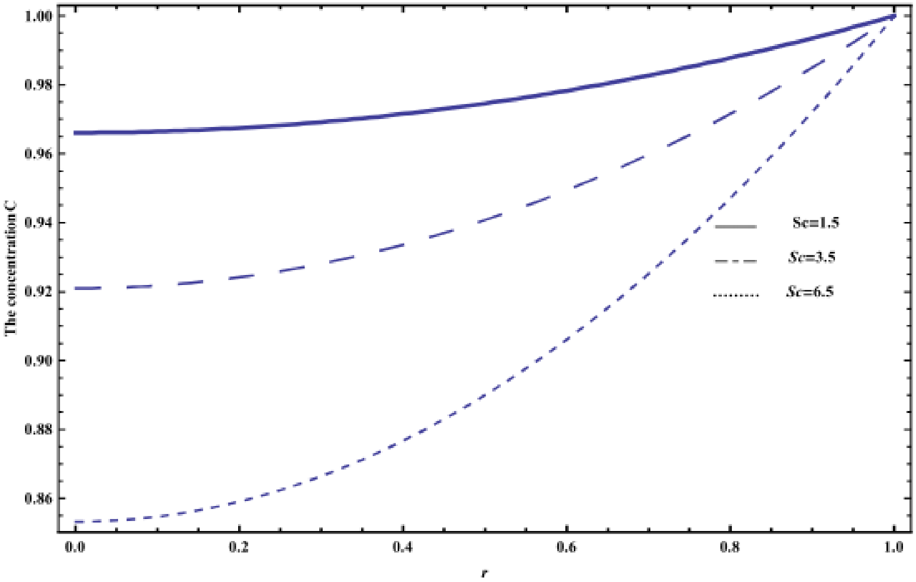

Figure (12)

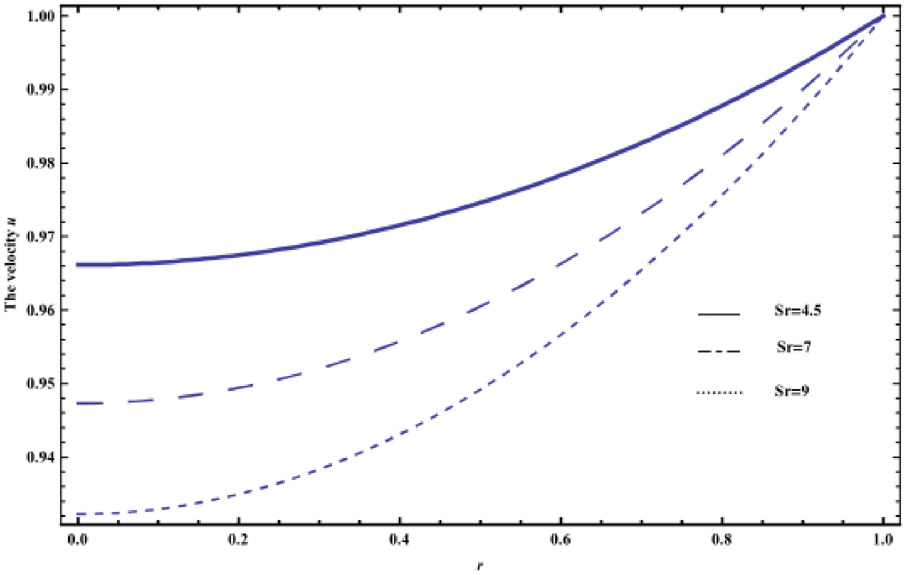

Figure (13)

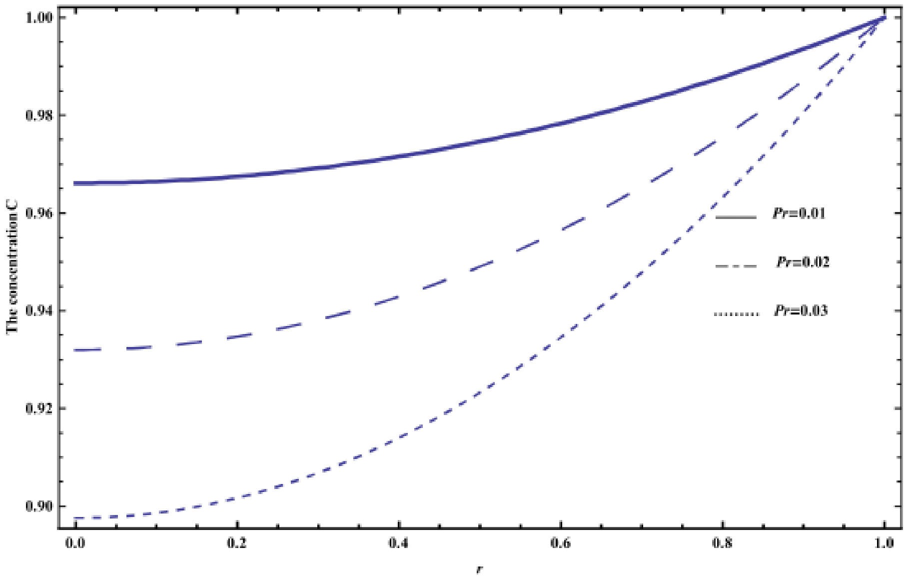

Figure (14)

5. CONCLUSION

In this paper, we have studied the unsteady pulsatile motion of non-Newtonian Casson fluid through a porous medium in a permeable cylindrical pipe in the presence of magnetic field. The equations of momentum, energy and concentration have been solved by using perturbation technique. The governing equations were solved numerically by using Mathematic. The velocity, temperature and concentration distribution are obtained. The effects of various parameters of the problem on these distributions are discussed and depicted graphically through a set of figures. Hence, this paper deals with an important branch of fluid mechanics which has many important applications in many fields, such as biology, medicine and chemistry and also in the space science. For example:

1. The rheology of blood has received much study. Blood is rheologically complex on two counts: it is a suspension because erythrocytes with characteristic dimensions of several micrometers are present in excess of 40 vol. and the suspension fluid itself exhibits non-Newtonian behavior because of the presence of high molecular weight protein. The importance of rheological properties of other body fluids is now recognized. In particular, the rheological response of mucous is respiratory system of both infants and adults are an important factor for proper respiratory system of both infants and adults are an important factor for proper respiratory behavior. For engineering purpose, one is more interested in the values of the velocity and heat transfer than in the shape of the velocity and temperature profiles.

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