

Some Designs with Association Schemes Arising from Some Certain Corona Graphs

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Abstract: In this paper, we obtain PBIB designs with association schemes which are arising from the minimum dominating sets of $(C_3 \circ K_1)$, $(C_4 \circ K_1)$ and $(C_5 \circ K_1)$, then we generalize the results to the graph $(C_n \circ K_1)$. Further

we generalize the result to $(C_n \circ K_m)$ and $(G \circ K_m)$.

Keywords: Minimum dominating sets, association schemes, PBIB designs.

1. INTRODUCTION

In this paper by a graph, we mean a finite undirected graph without loops or multiple lines. For a graph G , let $V(G)$ and $E(G)$ respectively denote the point set and the line set of graph G . We say that u and v dominate each other. A set D subset of V is dominating set of G , if every vertex in $V - D$ is adjacent to some vertex in D . The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

Many authors have been studied PBIBD with m -association scheme which are arising from some dominating sets of some graphs. H.B. Walikar and et al.[8], have studied PBIBD arising from minimum dominating set of paths and cycles, Anwar and Soner [1], have studied Partially balanced incomplete block designs arising from some minimal dominating sets of SRNT graphs, Sharada and Soner [6], have studied relation between Partially balanced incomplete block designs arising from minimum efficient dominating sets of graph. Any undefined terms and notation, reader may refer to F.Harary [4]. We refer the reader to see [2], for more details about PBIBD and dominating set. We concern here to study PBIBD and the association scheme which can be obtained from the minimum dominating sets in some certain

$(C_n \circ K_1)$ graph, then we generalize the graph $(C_n \circ K_n)$ and it is open area to study the same things for the other graphs.

We can obtain different PBIBD association scheme from the $(C_n \circ K_1)$ graphs by using different definitions as we will see in next sections.

2. SOME PBIBD ARISING FROM MINIMUM DOMINATING SETS OF $(C_N \circ K_1)$

Definition 2.1

Given v objects a relation satisfying the following conditions is said to be an association scheme with m classes:

- i. Any two objects are either first associates, or second associates, ..., or m^{th} associates, the relation of association being symmetric.
- ii. Each object α has n_i i th associates, the number n_i being independent of α .
- iii. If two objects α and β are i th associates, then the number of objects which are j th associates of α and k th associates of β is p_{jk}^i and is independent of the pair of i th associates α and β . Also $p_{jk}^i = p_{kj}^i$.

If we have association scheme for the v objects we can define a PBIBD as the following definition.

Definition 2.2

The PBIBD design is arrangement of v objects into b sets (called blocks) of size k where $k < v$ such that

- i. Every object is contained in exactly r blocks.
- ii. Each block contains k distinct objects.
- iii. Any two objects which are i th associates occur together in exactly λ_i blocks.

Proposition 2.3. A PBIBD with parameters $(6, 3, 0, 4)$ can be obtained from minimum dominating sets of $(C_3 \circ K_1)$.

Proof. let $G = (V,E)$ be a corona graph $(C_3 \circ K_1)$. By labelling $\{v_1, v_2, v_3, v_1^1, v_2^1, v_3^1\}$ as in Fig (1) we can define PBIBD as follows:

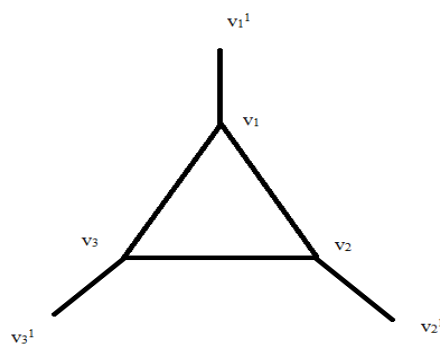


Figure1: $C_3 \circ K_1$

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3\}$, $\{v_1^1, v_2^1, v_3^1\}$, $\{v_1, v_2^1, v_3^1\}$, $\{v_2, v_1^1, v_3^1\}$, $\{v_3, v_2^1, v_1^1\}$, $\{v_1, v_2, v_3^1\}$, $\{v_2, v_3, v_1^1\}$ and $\{v_1, v_3, v_2^1\}$, and every vertex appear in 4 blocks and the size of the block is the domination number $(C_3 \circ K_1) = 3$. Any two vertices appear either exactly in zero dominating set or in two dominating sets. Then the parameters of the PBIBD is $(6,3,0,4)$.

Proposition 2.4. A PBIBD with parameters $(8, 4, 0, 8)$ can be obtained from minimum dominating sets of $(C_4 \circ K_1)$.

Proof. Let $G = (V, E)$ be a Corona graph $C_4 \circ K_1$. By labelling $\{v_1, v_2, v_3, v_4, v_1^1, v_2^1, v_3^1, v_4^1\}$ as in Fig(2) we can define PBIBD as follows:

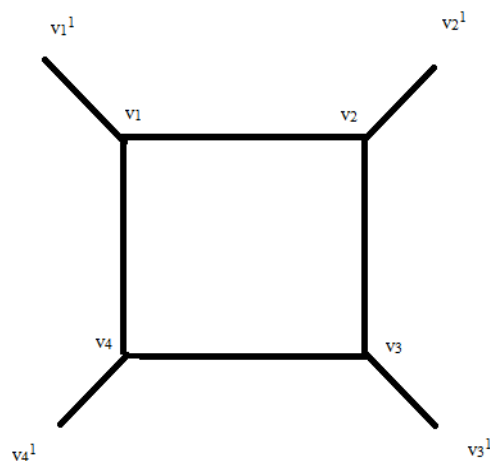


Figure2: $C_4 \circ K_1$

The point set is the vertices of and the block set is the minimum dominating sets $\{v_1, v_2, v_3, v_4\}$, $\{v_1^1, v_2^1, v_3^1, v_4^1\}$, $\{v_1, v_2^1, v_3^1, v_4^1\}$, $\{v_2, v_1^1, v_3^1, v_4^1\}$, $\{v_3, v_1^1, v_2^1, v_4^1\}$, $\{v_4, v_1^1, v_2^1, v_3^1\}$, $\{v_1, v_2, v_3^1, v_4^1\}$, $\{v_2, v_3, v_1^1, v_4^1\}$, $\{v_3, v_4, v_1^1, v_2^1\}$, $\{v_1, v_4, v_2^1, v_3^1\}$, $\{v_1, v_2, v_3, v_4^1\}$, $\{v_2, v_3, v_4, v_1^1\}$, $\{v_1, v_3, v_4, v_2^1\}$, $\{v_1, v_2, v_4, v_3^1\}$, $\{v_1, v_3, v_2^1, v_4^1\}$ and $\{v_2, v_4, v_1^1, v_3^1\}$, and every vertex appear in 8 blocks and the size of the block is the domination number $(C_4 \circ K_1) = 4$. Any two vertices appear either exactly in zero dominating set or in four dominating sets. Then the parameters of the PBIBD is $(8,4,0,8)$.

Proposition 2.5. A PBIBD with parameters $(10, 5, 0, 16)$ can be obtained from minimum dominating sets of $(C_5 \circ K_1)$.

Proof. Let $G = (V,E)$ be a Corona graph $(C_5 \circ K_1)$. By labelling $\{v_1, v_2, v_3, v_4, v_5, v_1^1, v_2^1, v_3^1, v_4^1, v_5^1\}$ as in Fig(3) we can define PBIBD as follows:

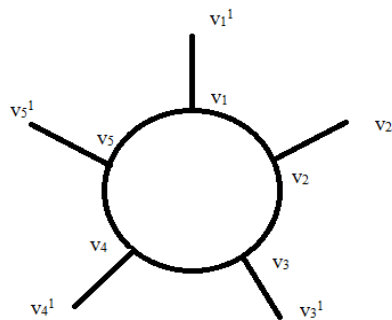


Figure3: $C_5 \circ K_1$

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3, v_4, v_5\}$, $\{v_1^1, v_2^1, v_3^1, v_4^1, v_5^1\}$, $\{v_1, v_2^1, v_3^1, v_4^1, v_5^1\}$, $\{v_1, v_2, v_3^1, v_4^1, v_5^1\}$, $\{v_2, v_1^1, v_3^1, v_4^1, v_5^1\}$, $\{v_3, v_2^1, v_1^1, v_4^1, v_5^1\}$, $\{v_4, v_1^1, v_2^1, v_3^1, v_5^1\}$, $\{v_5, v_1^1, v_2^1, v_3^1, v_4^1\}$, $\{v_1, v_2, v_3^1, v_4^1, v_5^1\}$, $\{v_2, v_3, v_1^1, v_4^1, v_5^1\}$, $\{v_3, v_4, v_1^1, v_2^1, v_5^1\}$, $\{v_4, v_5, v_1^1, v_2^1, v_3^1\}$, $\{v_1, v_5, v_2^1, v_3^1, v_4^1\}$, $\{v_1, v_2, v_3, v_4^1, v_5^1\}$, $\{v_2, v_3, v_4, v_1^1, v_5^1\}$, $\{v_3, v_4, v_5, v_1^1, v_2^1\}$, $\{v_1, v_4, v_5, v_2^1, v_3^1\}$, $\{v_1, v_2, v_5, v_3^1, v_4^1\}$, $\{v_1, v_2, v_3, v_4, v_5^1\}$, $\{v_2, v_3, v_4, v_5, v_1^1\}$, $\{v_1, v_3, v_4, v_5, v_2^1\}$, $\{v_1, v_2, v_4, v_5, v_3^1\}$, $\{v_1, v_2, v_3, v_5, v_4^1\}$, $\{v_1, v_2, v_4, v_3^1, v_5^1\}$, $\{v_2, v_3, v_5, v_1^1, v_4^1\}$, $\{v_1, v_3, v_4, v_2^1, v_5^1\}$, $\{v_2, v_4, v_5, v_1^1, v_3^1\}$, $\{v_1, v_3, v_5, v_2^1, v_4^1\}$, $\{v_1, v_3, v_2^1, v_4^1, v_5^1\}$, $\{v_2, v_4, v_1^1, v_3^1, v_5^1\}$, $\{v_3, v_5, v_1^1, v_2^1, v_4^1\}$, $\{v_1, v_4, v_2^1, v_3^1, v_5^1\}$ and $\{v_2, v_5, v_1^1, v_3^1, v_4^1\}$ such that every vertex appear in 16 blocks and the size of the block is the domination number $(C_5 \circ K_1) = 5$. Any two vertices appear either exactly in zero dominating set or in eight dominating sets. Then the parameters of the PBIBD is $(10,5,0,16)$.

Theorem 2.6. For any corona graph $(C_n \circ K_1)$, where $n \geq 3$, we can define PBIBD with the following parameters $(2n, n, 0, 2^{n-1})$.

Proof. The above theorem follows by propositions 2.3, 2.4 and 2.5.

Theorem 2.7. Let $G \cong (C_n \circ K_1)$. Then the number of minimum dominating sets are 2^n .

Proof. By labelling the vertices of the graph G , $\{v_1, v_1^1, v_2, v_2^1, \dots, v_n, v_n^1\}$ as in Fig(4).

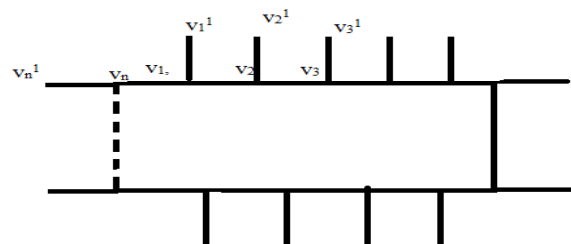


Figure4: $C_n \circ K_1$

Let $A = \{v_1, v_2, \dots, v_n\}$ and $B = \{v_1^1, v_2^1, \dots, v_n^1\}$. It is obvious that $|G| = n$. Let $D(G)$ be the number of minimum dominating set of G , we have 2 dominating sets $S_1 = A$ and $S_2 = B$ are minimum dominating sets. Now to select the minimum dominating sets we have to select x vertices from A and y vertices from B . But from the definition of minimum dominating set, we have option only for x , the other vertices from B will be compulsory in the minimum dominating set. So if we select one vertex from A , then $(n-1)$ vertices from B have to be selected i.e., if we select v_i from A , then $(n-1)$ vertices from B appear in the dominating set, such that all the vertices in B except v_i^1 . By using the probability theory we can select 1 or 2 or up to $(n - 1)$ elements from A the other will appear.

Hence

$$D(G) = \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n-1} + 2$$

As we know that

$$(x + a)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i} \rightarrow (1)$$

If $a = b = 1$, then $2^n = \sum_{i=0}^n \binom{n}{i}$

from equation (1), we get

$$\begin{aligned} \sum_{i=0}^n \binom{n}{i} &= \binom{n}{0} + \sum_{i=1}^n \binom{n}{i} \\ \sum_{i=0}^n \binom{n}{i} &= \binom{n}{0} + \sum_{i=0}^{n-1} \binom{n}{i} + 1 \\ \sum_{i=0}^n \binom{n}{i} &= 2 + \sum_{i=0}^{n-1} \binom{n}{i} \end{aligned}$$

Therefore, $\sum_{i=0}^{n-1} \binom{n}{i} = 2^n - 2$

Implies $D(G) = 2^n - 2 + 2$

Hence $D(G) = 2^n$.

Lemma 2.8. Let $G \cong C_n \circ K_1$. Then every vertex v contained in 2^{n-1} minimum dominating sets.

Proof. Let u be any vertex in G , there are 2 cases:

Case(1): Let $u \in A = \{v_1, v_2, \dots, v_n\}$. To count the number of minimum dominating set which contains u , the first minimum dominating sets is A itself and the other selecting x vertices from

inside and $(n-1)$ vertices from outside which is $\sum_{i=1}^{n-1} \binom{n-1}{i} = 2^{n-1} - 1$.

Hence, there is 2^{n-1} different minimum dominating set containing u .

Case(2): Let $u \in B = \{v_1^1, v_2^1, v_3^1, \dots, v_n^1\}$. To count the number of minimum dominating

sets which contains u , the first minimum dominating sets is B itself and the other selecting y

vertices from A and $(n - 1)$ vertices from B which is $\sum_{i=1}^{n-1} \binom{n-1}{i} = 2^{n-1} - 1$.

Hence, there are 2^{n-1} different minimum dominating sets containing u .

3. SOME ASSOCIATION SCHEME OBTAINED FROM MINIMUM DOMINATING SETS OF $C_N \circ K_1$

Theorem 3.1. From $C_3 \circ K_1$ we can get PBIBD with parameters (6, 3, 0, 2) and association scheme of 2-classes with

$$P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} .$$

Proof. Let $G = (V,E)$ be a corona graph $C_3 \circ K_1$. By labelling $\{v_1, v_2, v_3, v_1^1, v_2^1, v_3^1\}$ we can define PBIBD as follows:

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3\}, \{v_1^1, v_2^1, v_3^1\}, \{v_1, v_2^1, v_3^1\}, \{v_2, v_1^1, v_3^1\}, \{v_3, v_2^1, v_1^1\}, \{v_1, v_2, v_3^1\}, \{v_2, v_3, v_1^1\}$ and $\{v_1, v_3, v_2^1\}$. We define the association scheme as follows, for any $\alpha\beta \in V(G)$, α is first associate of β if α and β appear in zero block and α is second associate of β if α and β appear in 2 blocks.

Table 1.

Elements	First Associates	Second Associates
v_1	v_1^1	v_2, v_3, v_2^1, v_3^1
v_2	v_2^1	v_1, v_3, v_1^1, v_3^1
v_3	v_3^1	v_1, v_2, v_1^1, v_2^1
v_1^1	v_1	v_2, v_3, v_2^1, v_3^1
v_2^1	v_2	v_1, v_3, v_1^1, v_3^1
v_3^1	v_3	v_1, v_2, v_1^1, v_2^1

$$P_1 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} .$$

Theorem 3.2. From $C_4 \circ K_1$ we can get PBIBD with parameters (8, 4, 0, 4) and association scheme of

$$2\text{-classes with } P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} .$$

Proof. Let $G = (V, E)$ be a corona graph $C_4 \circ K_1$. By labelling $\{v_1, v_2, v_3, v_4, v_1^1, v_2^1, v_3^1, v_4^1\}$ we can define PBIBD as follows:

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3, v_4\}, \{v_1^1, v_2^1, v_3^1, v_4^1\}, \{v_1, v_2^1, v_3^1, v_4^1\}, \{v_2, v_1^1, v_3^1, v_4^1\}, \{v_3, v_1^1, v_2^1, v_4^1\}, \{v_4, v_1^1, v_2^1, v_3^1\}, \{v_1, v_2, v_3^1, v_4^1\}, \{v_2, v_3, v_1^1, v_4^1\}, \{v_3, v_4, v_1^1, v_2^1\}, \{v_1, v_4, v_2^1, v_3^1\}, \{v_1, v_2, v_3, v_4^1\}, \{v_2, v_3, v_4, v_1^1\}, \{v_1, v_3, v_4, v_2^1\}, \{v_1, v_2, v_4, v_3^1\}, \{v_1, v_3, v_2^1, v_4^1\}$ and $\{v_2, v_4, v_1^1, v_3^1\}$. We define the association scheme as follows, for any

$\alpha\beta \in V(G)$, α is first associate of β if α and β appear in zero block and α is second associate of β if α and β appear in 3 blocks.

Table 2.

Elements	First Associates	Second Associates
v_1	v_1^1	$v_2, v_3, v_4, v_2^1, v_3^1, v_4^1$
v_2	v_2^1	$v_1, v_3, v_4, v_1^1, v_3^1, v_4^1$
v_3	v_3^1	$v_1, v_2, v_4, v_1^1, v_2^1, v_4^1$
v_4	v_4^1	$v_1, v_2, v_3, v_1^1, v_2^1, v_3^1$
v_1^1	v_1	$v_2, v_3, v_4, v_2^1, v_3^1, v_4^1$
v_2^1	v_2	$v_1, v_3, v_4, v_1^1, v_3^1, v_4^1$
v_3^1	v_3	$v_1, v_2, v_4, v_1^1, v_2^1, v_4^1$
v_4^1	v_4	$v_1, v_2, v_3, v_1^1, v_2^1, v_3^1$

$$P_1 = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} .$$

Theorem 3.3. For any corona graph $C_n \circ K_1$, there is PBIBD with the parameters $(n, k, r, \lambda_1, \lambda_2)$.where n is number of points, k is number of minimum dominating sets, r is the size of the block λ_1 is 2^{n-1} .

Proof. This Theorem follows from 2.7 and 2.8.

From the previous Theorems we can conclude that for any corona graph $C_n \circ K_1$, where $k \geq 3$, we can define PBIBD from the minimum dominating sets with $2n$ points and also n blocks also it is clear that the size of any block is the domination number of C_n and for any $\alpha\beta \in V(G)$, α is first associate of β if

α and β appear in zero block and α is second associate of β if α and β appear in 2^{n-1} block with parameters $(2n, n, 0, 2^{n-1})$ and association scheme of 2-classes with

$$P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 2^{n-1} \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2^{n-1} \end{bmatrix} .$$

Theorem 3.4. From $C_3 \circ K_1$ we can get PBIBD with parameters $(6, 3, 0, 2)$ and association scheme of

$$\text{2-classes with } P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} .$$

Proof. let $G = (V,E)$ be a corona graph $C_3 \circ K_1$. By labelling $\{v_1, v_2, v_3, v_1^1, v_2^1, v_3^1\}$ we can define PBIBD as follows:

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3\}, \{v_1^1, v_2^1, v_3^1\}, \{v_1, v_2^1, v_3^1\}, \{v_2, v_1^1, v_3^1\}, \{v_3, v_2^1, v_1^1\}, \{v_1, v_2, v_3^1\}, \{v_2, v_3, v_1^1\}$ and $\{v_1, v_3, v_2^1\}$. We define the association scheme as follows, for any $\alpha\beta \in V(G)$, α is first associate of β if α and β appear in a cycle and α is second associate of β if otherwise.

Table 3.

Elements	First Associates	Second Associates
v_1	v_2, v_3	v_1^1, v_2^1, v_3^1
v_2	v_1, v_3	v_1^1, v_2^1, v_3^1
v_3	v_1, v_2	v_1^1, v_2^1, v_3^1
v_1^1	v_2^1, v_3^1	v_1, v_2, v_3
v_2^1	v_1^1, v_3^1	v_1, v_2, v_3
v_3^1	v_1^1, v_2^1	v_1, v_2, v_3

$$P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} .$$

Theorem 3.5. From $C_4 \circ K_1$ we can get PBIBD with parameters $(8, 4, 0, 4)$ and association scheme of

$$\text{2-classes with } P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix} .$$

Proof. let $G = (V,E)$ be a corona graph $C_4 \circ K_1$. By labelling $\{v_1, v_2, v_3, v_4, v_1^1, v_2^1, v_3^1, v_4^1\}$ we can define PBIBD as follows:

The point set is the vertices and the block set is the minimum dominating sets $\{v_1, v_2, v_3, v_4\}, \{v_1^1, v_2^1, v_3^1, v_4^1\}, \{v_1, v_2^1, v_3^1, v_4^1\}, \{v_2, v_1^1, v_3^1, v_4^1\}, \{v_3, v_1^1, v_2^1, v_4^1\}, \{v_4, v_1^1, v_2^1, v_3^1\}, \{v_1, v_2, v_3^1, v_4^1\}, \{v_2, v_3, v_1^1, v_4^1\}, \{v_3, v_4, v_1^1, v_2^1\}, \{v_1, v_4, v_2^1, v_3^1\}, \{v_1, v_2, v_3, v_4^1\}, \{v_2, v_3, v_4, v_1^1\}, \{v_1, v_3, v_4, v_2^1\}, \{v_1, v_2, v_4, v_3^1\}, \{v_1, v_3, v_2^1, v_4^1\}$ and $\{v_2, v_4, v_1^1, v_3^1\}$. We define the association scheme as follows, for any

$\alpha\beta \in V(G)$, α is first associate of β if α and β appear in cycle and α is second associate of β if otherwise.

Table 4.

Elements	First Associates	Second Associates
v_1	v_2, v_3, v_4	v_1, v_2, v_3, v_4
v_2	v_1, v_3, v_4	v_1, v_2, v_3, v_4
v_3	v_1, v_2, v_4	v_1, v_2, v_3, v_4
v_4	v_1, v_2, v_3	v_1, v_2, v_3, v_4
v_1^1	v_2^1, v_3^1, v_4^1	v_1, v_2, v_3, v_4
v_2^1	v_1^1, v_3^1, v_4^1	v_1, v_2, v_3, v_4
v_3^1	v_1^1, v_2^1, v_4^1	v_1, v_2, v_3, v_4
v_4^1	v_1^1, v_2^1, v_3^1	v_1, v_2, v_3, v_4

$$P_1 = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}.$$

Theorem 3.6. From the previous theorems we can conclude that for any corona graph $C_n \circ K_1$, where

$k \geq 3$, we can define PBIBD from the minimum dominating sets with $2n$ points and also n blocks also it is clear that the size of any block is the domination number of C_n and for any $\alpha\beta \in V(G)$, α is first associate of β if α and β appear in cycle and α is second associate of β if otherwise with parameters

$(2n, n, 0, 2^{n-1})$ and association scheme of 2-classes with

$$P_1 = \begin{bmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{bmatrix} = \begin{bmatrix} n-2 & 0 \\ 0 & n \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & n-1 \\ n-1 & 0 \end{bmatrix}.$$

4. CONCLUSION

We obtain PBIB designs with association schemes which are arising from the minimum dominating sets and then we generalize the results to the graph $(C_n \circ K_1)$.

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