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Representation Theorem for the Distributional Fourier-Stieltjes Transform

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Abstract: *In the present paper our aim has to provide representation theorem for the distributional Fourier-Stieltjes Transform and with the support of the representation theorem the problems concerning the diffraction by a perfect rigid or perfectly weak screen are reduced to the solution of certain (differential) integral equation.*

Keywords: *Fourier Transform, Stieltjes Transform, Testing Function Space, Fourier-Stieltjes Transform.*

1. Introduction

The concept of integral transform originated from the celebrated Fourier integral formula. One of the impulses for the development of the operational calculus of the integral transforms was the study of differential and integral equation arising in applied mathematics, mathematical physics and engineering science, it was this setting that integral transform arose and achieved their earlier success [4].With ever greater demand for integral transform to provide theory and application for science and engineering, Zemanian [5] provide an extension of integral transform to the distribution of compact support time to time which involves some complicated analysis.

A function $\emptyset(t,x)$ defined on $t(0 \le t \le \infty), x(0 \le x \le \infty)$ is said to be member of FS_α if $\emptyset(t,x)$ is smooth and for each non-negative integerp, l, q,

$$\gamma_{k,p,l,q}\emptyset(t,x) = \operatorname*{Sup}_{I_1}\left|t^k(1+x)^p D_t^l\,(xD_x)^q\,\emptyset(t,x)\right|$$

$$\le \infty \tag{1.1}$$

The space FS_α are equiparralel with their natural Housdorff locally topologyτ_α. This topology is respectively generated by the total families of semi norm $\{\gamma_{k,p,l,q}\}$ given (1.1).

In the last few year, the theory of generalized integral transforms have been of ever increasing interest due to its application in physics especially in quantum field theory, Engineering and pure as well as applied mathematics.

The distributional Fourier-Stieltjes transform is defined as

$$FS\{f(t,x)\} = F(s,y) =< \mathrm{f}(t,x), e^{-ist}\,(x+y)^{-p>} \tag{1.2}$$

where for each fixed $t(0 \le t \le \infty), x(0 \le x \le \infty)$ the right hand side of above equation has same as an application of $f(t,x) \in FS_\alpha^*$ to $e^{-ist}\,(x+y)^{-p} \in FS_\alpha$ for some $s > 0$ and $k < Re\,p$ and FS_α^* is dual space ofFS_α [1].

In this paper we have proved Representation theorem for the Fourier-Stieltjes Transform in section 2 and Section 3 provide the conclusion of this paper. The notation and terminology will follow that Zemanian[5].

2. Representation Theorem

2.1 Theorem

Let $f(t,x)$ be an arbitrary element of FS^* and $\emptyset(t,x)$ be an element of$\mathcal{D}(\Omega)$, the space of infinitely differentiable function with compact support on Ω, Then there exist a bounded measurable functions $g_{m,n}(t,x)$ defined over Ω such that-

$$\langle f,\emptyset\rangle = \sum_{m=0}^{r+1}\sum_{n=0}^{\nu+1}(-1)^{m+n}\, t^k\,(1+x)^p x^n\, D_t^m D_x^n g_{m,n}(t,x), \emptyset(t,x) >$$

Where, k and p is fixed real number and r,ν are appropriate non- negative integers satisfying $m \le r+1, n \le \nu+1$.

Proof:-

Let $\{Y_{k,p,l,q}\}_{l,q=0}^{\infty}$ be the sequence of semi norms. Let $f(t,x)$ and $\emptyset(t,x)$ be arbitrary elements of FS_α^* and $\mathcal{D}(\Omega)$ respectively. Then boundedness property of generalized function, we have an for an appropriate constant C and a non negative integer r and ν satisfying $|l| \le r$ and$|q| \le \nu$.

$$|\square\langle f,\emptyset\rangle| \le C \max_{\substack{|l|\le r\\ |q|\le\nu}} Y_{k,p,l,q}\emptyset(t,x) \le C \max_{\substack{|l|\le r\\ |q|\le\nu}} \sup_I t^k\,(1+x)^p\left|D_t^l\,(xD_x)^q\emptyset(t,x)\right|$$

$$\le C \max_{\substack{|l|\le r\\ |q|\le\nu}} \sup_I t^k\,(1+x)^p\left|\sum_{m=0}^{l}\sum_{n=0}^{q} B_n\; x^n\, D_t^m D_x^n\, \emptyset(t,x)\right|$$

$$\le C' \max_{\substack{|l|\le r\\ |q|\le\nu}} \sup_I t^k\,(1+x)^p \max_{\substack{m\le l\\ n\le q}} x^n\, D_t^m D_x^n \emptyset(t,x)$$

Where C' is constant which depend only on m, n and hence l,q. So,

$$|<\langle f,\emptyset\rangle| \le\le C'' \max_{\substack{m\le r\\ n\le\nu}} \sup_I t^k\,(1+x)^p\, x^n\, D_t^m D_x^n \emptyset(t,x) \tag{1.1}$$

Now let us set-

$$\emptyset_{r,\nu}(t,x) = t^k\,(1+x)^p\, x^n\,\emptyset(t,x), \qquad m\le r, n\le\nu \tag{1.2}$$

Then clearly$\emptyset_{r,\nu}(t,x) \in \mathcal{D}(\Omega)$, on differentiating (2) partially w.r.to t and x successively, we get-

$$D_t D_x\emptyset = (1+x)^{-p}x^{-n}\,t^{-k}\left\{\frac{k}{t}\emptyset_{r,\nu}(t,x)\left[\frac{p}{(1+x)} + \frac{n}{x}\right] - \left[\frac{p}{(1+x)} + \frac{n}{x}\right]\frac{\partial\emptyset_{r,\nu}}{\partial t} - \frac{k}{t}\frac{\partial\emptyset_{r,\nu}}{\partial x} + \frac{\partial^2\emptyset_{r,\nu}}{\partial t\partial x}\right\}$$

Let suppose that in Ω,

$$\sup\emptyset = \sup\emptyset_{r,\nu} = \Omega = [A,B]. \text{Then since, } (1+x)^{-p}x^{-n}\,t^{-k} > 0$$

$$|D_t D_x\emptyset| \le (1+x)^{-p}t^{-k}x^{-n}\left\{\frac{1}{B}\left[\frac{|kp|}{1+A} + \frac{|kn|}{A}\right]|\emptyset_{r,\nu}(t,x)| + \left[\frac{|p|}{1+A} + \frac{|n|}{A}\right]\left|\frac{\partial\emptyset_{r,\nu}}{\partial t}\right| + \frac{k}{B}\left|\frac{\partial\emptyset_{r,\nu}}{\partial x}\right| + \left|\frac{\partial^2\emptyset_{r,\nu}}{\partial t\partial x}\right|\right\}$$

$$\le C'''(1+x)^{-p}t^{-k}x^{-n}\left\{|\emptyset_{r,\nu}(t,x)| + \left|\frac{\partial\emptyset_{r,\nu}}{\partial t}\right| + \left|\frac{\partial\emptyset_{r,\nu}}{\partial x}\right| + \left|\frac{\partial^2\emptyset_{r,\nu}}{\partial t\partial x}\right|\right\}$$

Where, $C''' = \max\left[\frac{1}{B}\left[\frac{|kp|}{1+A}+\frac{|kn|}{A}\right],\left[\frac{|p|}{1+A}+\frac{|n|}{A}\right],\frac{k}{B}\,,\ 1\right]$

Hence by induction we can prove that in Ω for obvious constant C^{iv} which depend on p, n and k. Then,

$$|D_t^m\, D_x^n\, \emptyset| \le C^{iv}(1+x)^{-p}t^{-k}x^{-n}\ \sum_{\substack{c\le m\\ d\le n}}\left|D_t^c\, D_x^d\, \emptyset_{r,v}\right|$$

Substituting in equation (2.1), we have-

$$|\langle f,\emptyset\rangle| \le C^{v}\ \max_{\substack{m\le r\\ n\le v}} \sup_I\left|D_t^c\, D_x^d\, \emptyset_{r,v}(t,x)\right| \tag{1.3}$$

Where $c \le m$ and $d \le n$ and C^v is suitable constant.

Now we can write-

$$\sup_I|\emptyset(t,x)| \le \sup_I\left|\int_t^\infty\int_x^\infty D_t\, D_x\, \emptyset_{r,v}(t,x)\ dt\, dx\right|$$
$$\le \left\|D_t\, D_x\, \emptyset_{r,v}(t,x)\right\|_{L'\times L'}$$

Hence from equation (1),

Equation (3) implies-

$$|\langle f,\emptyset\rangle| \le C^{vi}\ \max_{\substack{m\le r+1\\ n\le v+1}} \sup_I\left\|D_t^m\, D_x^n\, \emptyset_{r,v}(t,x)\right\|_{L'\times L'} \tag{1.4}$$

Let the product space $L' \times L'$ be denoted by$\left(L'\right)^2$. We consider the linear one to one mapping-

$\tau\colon \emptyset \to \left\{D_t^m\, D_x^n\, \emptyset_{r,v}(t,x)\right\}_{\substack{m\le r+1\\ n\le v+1}}$ of $\mathcal{D}(\Omega)$ into $\left(L'\right)^2$.

In view of (4), we see that the linear functional $\tau\colon \emptyset_{r,v} \to \langle f,\emptyset\rangle$ is continuous on $\tau\mathcal{D}(\Omega)$ for Topology induced by$\left(L'\right)^2$. Hence by Hahn-Banach theorem, it can be continuous linear functional in the whole of $\left(L'\right)^2$. But the dual space of $\left(L'\right)^2$ is isomorphic with $(L^\infty)^2$ according to [9].

Therefore there exist two L^∞ functions $g_{m,n}(m \le r+1, n \le v+1)$ such that-

$$\langle f,\emptyset\rangle = \sum_{\substack{m\le r+1\\ n\le v+1}}\langle g_{m,n}(t,x), D_t^m\, D_x^n\, \emptyset_{r,v}(t,x)\rangle$$

By equation (2) we have

$$\langle f,\emptyset\rangle = \sum_{\substack{m\le r+1\\ n\le v+1}}\langle g_{m,n}(t,x), D_t^m\, D_x^n\, \mathrm{t}^{\mathrm{k}}\,(1+\mathrm{x})^{\mathrm{p}}\,\mathrm{x}^{\mathrm{n}}\,\emptyset(\mathrm{t},\mathrm{x})\rangle$$

By using the property differentiation of a distribution by an infinitely smooth function we get-

$$\langle f,\emptyset\rangle = \sum_{\substack{m\le r+1\\ n\le v+1}}\langle(-1)^{m+n}\mathrm{t}^{\mathrm{k}}\,(1+\mathrm{x})^{\mathrm{p}}\,\mathrm{x}^{\mathrm{n}}\, D_t^m\, D_x^n\, g_{m,n}(t,x), \emptyset(\mathrm{t},\mathrm{x})\rangle$$

Where, $g_{m,n}(t,x)$ are bounded measurable functions defined over$\Omega = (0,\infty)$.

Therefore,

$$f(t,x) = \sum_{\substack{m\le r+1\\ n\le v+1}}(-1)^{m+n}\mathrm{t}^{\mathrm{k}}\,(1+\mathrm{x})^{\mathrm{p}}\,\mathrm{x}^{\mathrm{n}}\, D_t^m\, D_x^n\, g_{m,n}(t,x)$$

It follows theorem.

3. CONCLUSION

In this paper, we proved the Representation theorem for the distributional Fourier-Stieltjes transform. The Representation theorem support the problems concerning the diffraction by a perfectly rigid or perfectly weak screen are reduced to the solution.

REFERENCES

[1]. Sharma V.D. and Dolas P.D.:-"Analyticity of Distribution Generalized FS transform", Int. Jr. of Math. Analysis, Vol. 6, 2012, No. 9, 447-451.

[2]. Sharma V.D. and Dolas P.D.;-"Some S-type spaces of Fourier Stieltjes transform", Int. Jr. of Engineering and Innovative technology (IJEIT), Vol.3, Issue 3, Sept. 2013.

[3]. Sharma V.D. and Dolas P.D.:-"Generalization of Fourier-Stieltjes transform". American Jr. of Mathematics and Sciences, Vol.2, No.1, January 13, ISSN No. 2250-3102.

[4]. Debnath Loknath :-" Integral Transform and their application", Second edition, London, New York, 2007.

[5]. Gulhane P.A. and Gudadhe A.S.:-"Representation theorem for the Distributional Fourier-Stieltjes Transform", Science Journal of GVISH, Vol. II, 2005, 29-32.

[6]. Zemanian A.H.:-"Generalized integral transformation", Inter science publisher, New York, 1968.

[7]. Pathak R.S.: "A Representation theorem for class of Stieltjes Transformable Generalized function", Indian Mathematical Society, (1974), pp 392-342.

[8]. Zemanian A.H.: "Distribution Theory and Transform analysis", Mc Graw-Hill, New York,1965.

[9]. Treves f: Topological vector space, distribution and kernels, Academic Press, New York, (1967).

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