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Ideals of Almost Distributive Lattices with respect to a Congruence

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Abstract: *The concept of θ-ideals is introduced in an Almost Distributive lattice(ADL) with respect to a congruence and the properties of θ-ideals are studied. Derived a set of equivalent conditions for a θ-ideal to become a θ-prime ideal.*

Keywords: *Almost Distributive Lattice (ADL), congruence, ideal , Prime filter, θ-ideal; θ-Prime ideal*

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1. Introduction

After Booles axiomatization of two valued propositional calculus as a Boolean algebra, a number of generalizations both ring theoretically and lattice theoretically have come into being. The concept of an Almost Distributive Lattice (ADL) was introduced by Swamy and Rao [6] as a common abstraction of many existing ring theoretic generalizations of a Boolean algebra on one hand and the class of distributive lattices on the other. In that paper, the concept of an ideal in an ADL was introduced analogous to that in a distributive lattice and it was observed that the set PI(L) of all principal ideals of L forms a distributive lattice. This enables us to extend many existing concepts from the class of distributive lattices to the class of ADLs. Swamy, G.C. Rao and G.N. Rao introduced the concept of Stone ADL and characterized it in terms of its ideals. In [3], N. Rafi, G.C. Rao and Ravi kumar Bandaru introduced θ-filters in an Almost Distributive Lattices and proved their properties. The usual lattice theoretic duality principle doesn't hold in ADLs. For example, in an ADL L, $\wedge$ is right distributive over $\vee$ but $\vee$ is not right distributive over $\wedge$. So that in this paper, the concept of θ-ideals is introduced in an ADL and then characterized in terms of ADL congruences. Also the concept of θ-prime ideals is introduced and established a set of equivalent conditions for every θ-ideal to become a θ-prime ideal. Some properties of θ-ideals and θ-prime ideals are studied. The class of all θ-ideals of an ADL can be made into a bounded distributive lattice. Finally, the prime ideal theorem is generalized in the case of θ-prime ideals in an ADL.

2. Preliminaries

Definition 2.1.[6] An Almost Distributive Lattice with zero or simply ADL is an algebra $(L, \vee, \wedge, 0)$ of type (2, 2, 0) satisfying

1. $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
2. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
3. $(x \vee y) \wedge y = y$
4. $(x \vee y) \wedge x = x$
5. $x \vee (x \wedge y) = x$
6. $0 \wedge x = 0$
7. $x \vee 0 = x$, for any $x, y, z \in L$.

Every non-empty set X can be regarded as an ADL as follows. Let $x_0 \in X$. Define the binary operations $\vee$, $\wedge$ on X by

$$x \vee y = \begin{cases} x & \text{if } x \neq x_0 \\ y & \text{if } x = x_0 \end{cases} \qquad x \wedge y = \begin{cases} y & \text{if } x \neq x_0 \\ x_0 & \text{if } x = x_0. \end{cases}$$

Then $(X, \vee, \wedge, x_0)$ is an ADL (where x_0 is the zero) and is called a discrete ADL. If $(L, \vee, \wedge, 0)$ is an ADL, for any a, b $\in$ L, define $a \leq b$ if and only if $a = a \wedge b$ (or equivalently, $a \vee b = b$), then $\leq$ is a partial ordering on L.

Theorem 2.2: ([6]) If $(L, \vee, \wedge, 0)$ is an ADL, for any a, b, c $\in$ L, we have the following:

(1) $a \vee b = a \Leftrightarrow a \wedge b = b$
(2) $a \vee b = b \Leftrightarrow a \wedge b = a$
(3) $\wedge$ is associative in L
(4) $a \wedge b \wedge c = b \wedge a \wedge c$
(5) $(a \vee b) \wedge c = (b \vee a) \wedge c$
(6) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$
(7) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(8) $a \wedge (a \vee b) = a$, $(a \wedge b) \vee b = b$ and $a \vee (b \wedge a) = a$
(9) $a \leq a \vee b$ and $a \wedge b \leq b$
(10) $a \wedge a = a$ and $a \vee a = a$
(11) $0 \vee a = a$ and $a \wedge 0 = 0$
(12) If $a \leq c$, $b \leq c$ then $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$
(13) $a \vee b = (a \vee b) \vee a$.

It can be observed that an ADL L satisfies almost all the properties of a distributive lattice except the right distributivity of $\vee$ over $\wedge$, commutativity of $\vee$, commutativity of $\wedge$. Any one of these properties make an ADL L a distributive lattice.

Theorem 2.3. ([6]) Let $(L, \vee, \wedge, 0)$ be an ADL with 0. Then the following are equivalent:

(1) $(L, \vee, \wedge, 0)$ is a distributive lattice

(2) $a \vee b = b \vee a$, for all a, b $\in$ L

(3) $a \wedge b = b \wedge a$, for all a, b $\in$ L

(4) $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$, for all a, b, c $\in$ L.

As usual, an element m $\in$ L is called maximal if it is a maximal element in the partially ordered set $(L, \leq)$. That is, for any a $\in$ L, $m \leq a \Rightarrow m = a$.

Theorem 2.4: ([6]) Let L be an ADL and m $\in$ L. Then the following are equivalent:

(1) m is maximal with respect to $\leq$

(2) $m \vee a = m$, for all a $\in$ L

(3) $m \wedge a = a$, for all a $\in$ L

(4) $a \vee m$ is maximal, for all a $\in$ L.

As in distributive lattices [[1], [2]], a non-empty sub set I of an ADL L is called an ideal of L if $a \vee b \in I$ and $a \wedge x \in I$ for any a, b $\in$ I and x $\in$ L. Also, a non-empty subset F of L is said to be a filter of L if $a \wedge b \in F$ and $x \vee a \in F$, for a, b $\in$ F and x $\in$ L. The set I(L) of all ideals of L is a bounded distributive lattice with least element {0} and greatest element L under set inclusion in which, for any I, J $\in$ I(L), $I \cap J$ is the infimum of I and J while the supremum is given by $I \vee J := \{a \vee b \mid a \in I, b \in J\}$. A proper ideal P of L is called a prime ideal if, for any x, y $\in$ L, $x \wedge y \in P \Rightarrow x \in P$ or $y \in P$. A proper ideal M of L is said to be maximal if it is not properly contained in any proper ideal of L. It can be observed that every maximal ideal of L is a prime ideal. Every proper ideal of L is contained in a maximal ideal. For any subset S of L the smallest ideal containing S is given by

(S] := { $(\bigvee_{i=1}^{n} s_i) \wedge x$ | $s_i \in$ S, x $\in$ L and n $\in$ N}. If S = {s}, we write (s] instead of (S]. Similarly, for any S ⊆ L, [S) := { $x \vee (\bigwedge_{i=1}^{n} s_i)$ | $s_i \in$ S, x $\in$ L and n $\in$ N}. If S = {s}, we write [s) instead of [S).

Theorem 2.5 ([6]). *For any* x, y *in* L *the following are equivalent:*

1). (x] $\subseteq$ (y]
2). y $\wedge$ x = x
3). y $\vee$ x = y
4). [y) $\subseteq$ [x).

For any x, y $\in$ L, it can be verified that (x] $\vee$ (y] = (x$\vee$y] and (x] $\wedge$ (y] = (x $\wedge$ y]. Hence the set PI(L) of all principal ideals of L is a sublattice of the distributive lattice I(L) of ideals of L.

Theorem 2.6([4]). Let I be an ideal and F a filter of L such that I $\cap$ F = ϕ. Then there exists a prime ideal P such that I $\subseteq$ P and P $\cap$ F = ϕ.

Definition 2.7 (4]). An equivalence relation θ on an ADL L is called a congruence relation on L if (a$\wedge$c, b$\wedge$d), (a$\vee$c, b$\vee$d) $\in \theta$, for all (a, b), (c, d) $\in \theta$.

Definition 2.8 ([4]). For any congruence relation θ on an ADL L and a $\in$ L, we define $[a]_\theta$ = {b $\in$ L | (a, b) $\in$ θ} and it is called the congruence class containing a.

Theorem 2.9 ([4]). An equivalence relation θ on an ADL L is a congruence relation if and only if for any (a, b) $\in \theta$, x $\in$ L, (a $\vee$ x, b $\vee$ x), (x $\vee$ a, x $\vee$b), (a $\wedge$ x, b $\wedge$ x), (x $\wedge$ a, x $\wedge$ b) *are all in* θ.

3. θ-IDEALS IN AN ADL

The concept of θ-filters in an Almost Distributive Lattice was given by Rafi, Rao and Ravi Kumar [3]. The usual lattice theoretic duality principle doesn't hold in ADLs. For example, in an ADL L, ∧ is right distributive over ∨ but ∨ is not right distributive over ∧. So that we introduce the concept of θ-ideals in an ADL and study their important properties. Throughout this paper L represents an ADL with 0.

Now we begin with the definition of a θ-ideal in an ADL L.

Definition 3.1: Let θ be a congruence relation on an ADL L. An ideal I of L is called a θ-ideal of L, if for any $a \in I$ that implies $[a]_\theta \subseteq I$.

For any congruence θ on a ADL L, it can be easily observed that the zero ideal {0} is a θ-ideal if and only if $[0]_\theta$ = {0}.

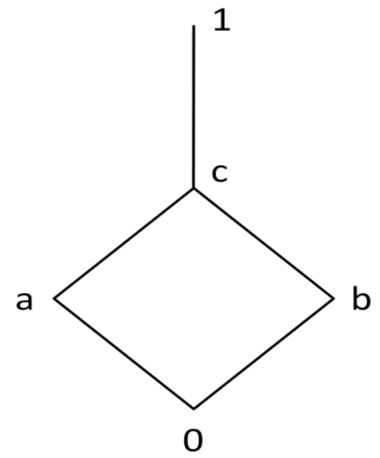
The following lemma can be verified easily.

Lemma 3.2. Let θ be a congruence on L and m be any maximal element of L. For any ideal I of L, the following hold:

1. If I is a θ-ideal, then $[0]_\theta \subseteq$ I;
2. If I is a proper θ-ideal, then I $\cap [m]_\theta = \phi$.
3. If θ is the smallest congruence then every ideal is a θ-ideal;

Example 3.3: Let D = {0', a'} be a discrete ADL and R a distributive lattice whose Hasse diagram is given in the figure. Then

L = D ×R = {(0', 0), (0', a), (0', b), (0', c), (0' , 1), (a', 0), (a', a), (a', b), (a', c), (a', 1)} is an ADL under point-wise operations.



Distributive Lattice

Take

θ= {((0', 0), (0' , 0)), ((0' , a), (0' , a)), ((0' , b), (0', b)), ((0' , c), (0' , c)),
((0', 1), (0' , 1)), ((a' , 0), (a', 0)), ((a' , a), (a', a)), ((a' , b), (a', b)),
((a', c), (a', c)), ((a' , 1), (a', 1)), ((0' , c), (0', 1)), ((0' , 1), (0' , c)) }. Clearly θ is a congruence relation on L. Consider the ideal I = {(0', 0), (0' , a)}. Clearly I is a θ-ideal of L.

But J = {(0, 0), (0, a), (0, b), (0', c)} is not a θ-ideal, because $[(0', c)]_{\theta} \not\subset$ J.

Theorem3.4: Let θ be a congruence relation on an ADL L. For any ideal I of L, the following conditions are equivalent:

1. I is a θ-ideal
2. For any $x, y \in L,\ (x, y) \in \theta$ and $x \in I \Rightarrow y \in I$
3. $I = \bigcup_{x \in I} [x]_{\theta}$.

Proof: $\underline{(1) \Rightarrow (2)}$: Assume that I is a θ-ideal of L. Let x, y ∈ L be such that (x, y) ∈ θ. Suppose x ∈ I. Therefore we get that y ∈ $[x]_{\theta}$ ⊆ I.

$\underline{(2) \Rightarrow (3)}$: Assume the condition (2). Let x ∈ I. Since x ∈ $[x]_{\theta}$, we get I ⊆ $\bigcup_{x \in I} [x]_{\theta}$. Conversely, let a ∈ $\bigcup_{x \in I} [x]_{\theta}$. Then (a, x) ∈ θ, for some x ∈ I. By the condition (2), we get that a ∈ I. Therefore I = $\bigcup_{x \in I} [x]_{\theta}$.

$\underline{(3) \Rightarrow (1)}$: Assume that the condition (3) holds. Let a ∈ I. Then we get (x, a) ∈ θ, for some x ∈ I. Let t ∈ $[a]_{\theta}$. Then we get (t, a) ∈ θ. Hence (x, t) ∈ θ. Thus it yields that t ∈ $[x]_{\theta}$ ⊆ I. Therefore I is a θ-ideal of L. The concept of θ-prime ideals is now introduced in an ADL.

Defination 3.5: Let θ be a congruence relation on an ADL L. A proper θ-ideal P of an ADL L is called a θ-prime ideal of L if for any $a, b \in L$ with $a \wedge b \in [0]_{\theta} \Rightarrow$ either $a \in P$ or $b \in P$.

Now, we have the following.

Lemma3.6: If θ is the smallest congruence relation on an ADL, then every prime ideal of L is a θ-prime ideal.

Proof: Suppose that θ is the smallest congruence on L. Let P be a prime ideal of L. Then by above lemma 3.2, P is a θ-ideal of L. Let a, b ∈ L be such that a∧b ∈ $[0]_{\theta}$. Then we get that

$[a \wedge b]_\theta = [0]_\theta$. Since θ-is the smallest congruence on L, it can be concluded that a ∧ b = 0 ∈ P. Therefore P is a θ-prime ideal of L.

Lemma 3.7: Let θ be a congruence relation on an ADL L. Then every prime θ-ideal of L is a θ-prime ideal of L.

Proof: Let P be a prime θ-ideal of an ADL L. Let x, y ∈ L be such that $x \wedge y \in [0]_\theta$. Since P is a θ-ideal of L, we get that $x \wedge y \in [0]_\theta \subseteq P$. Since P is a prime ideal of L, we get that either x ∈ P or y ∈ P. Therefore P is a θ-prime ideal of L.

Theorem 3.8: Let θ be a congruence relation on an ADL L and P, a θ-ideal of L. Then the following conditions are equivalent:

1. P is a θ-prime ideal of L
2. For any ideals I, J of L with $I \cap J \subseteq [0]_\theta$ implies that $I \subseteq P$ or $J \subseteq P$
3. For any $a, b \in L$, $[a]_\theta \cap [b]_\theta = [0]_\theta$ implies that either $a \in P$ or $b \in P$.

Proof: $\underline{(1) \Rightarrow (2):}$ Assume that P is a θ-prime ideal of L. Let I, J be two ideals of L such that $I \cap J \subseteq [0]_\theta$. Let a ∈ I and b ∈ J. Then $a \wedge b \in I \cap J \subseteq [0]_\theta$. Since P is θ-prime, we get that either a ∈ P or b ∈ P. Thus we get that either I ⊆ P or J ⊆ P.

$\underline{(2) \Rightarrow (3):}$ Assume the condition (2). Suppose that $[a]_\theta \cap [b]_\theta = [0]_\theta$ for any a, b ∈ L. Then we get $[a \wedge b]_\theta = [0]_\theta$. Thus it yields that $a \wedge b \in [0]_\theta$ and hence $(a] \cap (b] \subseteq [0]_\theta$. Therefore by the assumed condition (2), we get that either a ∈ (a] ⊆ P or b ∈ (b] ⊆ P.

$\underline{(3) \Rightarrow (1):}$ Assume that the condition (3) holds. Let a, b ∈ L be such that $a \wedge b \in [0]_\theta$. Hence we get $[a]_\theta \cap [b]_\theta = [a \wedge b]_\theta = [0]_\theta$. Thus by condition (3), we get that either a ∈ P or b ∈ P. Therefore P is a θ-prime ideal of L.

Now, we prove the following.

Lemma 3.9: Let θ be a congruence relation on an ADL L and m be any maximal element of L. Then every maximal ideal disjoint from $[m]_\theta$ is a θ-ideal of L.

Proof: Let M be a maximal ideal of L and m be any maximal element of L such that $M \cap [m]_\theta = \phi$. Let x, y ∈ L be such that $(x, y) \in \theta$ and x ∈ M. Suppose y ∉ M. Then M ∨ (y] = L. That implies a∨y is a maximal element of L for some a ∈ M. Since $(x, y) \in \theta$, we get that $(a \vee x, a \vee y) \in \theta$. Thus we can obtain that $a \vee x \in [a \vee y]_\theta$. Since a ∨ x ∈ M, we get that $M \cap [a \vee y]_\theta = \phi$, which is a contradiction. Therefore y ∈ M, which yields that M is a θ-ideal of L.

The following Corollary is a direct consequence of the above.

Corollary 3.10: Let θ be a congruence relation on an ADL L and m be any maximal element of L. If $[m]_\theta = \{m\}$, then every maximal ideal of L is a θ-ideal of L.

Now, we have the following definition.

Definition 3.11: Let θ be a congruence relation on an ADL L. For any ideal I of L, define the set I^θ as given by $I^\theta = \{x \in L \mid (x, a) \in \theta, \text{ for some } a \in I\}$.

Lemma 3.12: Let θ be a congruence relation on an ADL L. For any ideal I of L, the set I^θ is an ideal of L.

Proof: Clearly, $0 \in I^\theta$. Let $x, y \in I^\theta$. Then we get $(x, a) \in \theta$ and $(y, b) \in \theta$, for some a, b ∈ I. Hence we get $(x \vee y, a \vee b) \in \theta$. That implies $x \vee y \in I^\theta$. Again, let $x \in I^\theta$ and r ∈ L. Then $(x, a) \in \theta$, for some a ∈ I. Since θ is a congruence, we get $(x \wedge r, a \wedge r) \in \theta$. Since a ∧ r ∈ I, we get $x \wedge r \in I^\theta$. Therefore I^θ is an ideal of L.

Lemma 3.13: Let θ be a congruence relation on an ADL L. For any two ideals I, J of L, we have the following:

1. $I \subseteq I^{\theta}$
2. $I \subseteq J$ implies $I^{\theta} \subseteq J^{\theta}$
3. $(I \cap J)^{\theta} = I^{\theta} \cap J^{\theta}$
4. $(I^{\theta})^{\theta} = I^{\theta}$.

Proof: 1. Let $a \in I$. We have $(a,a) \in \theta$ and hence $a \in I^{\theta}$. Therefore $I \subseteq I^{\theta}$.

2. Suppose that $I \subseteq J$. Let $x \in I^{\theta}$. Then $(x,a) \in \theta$, for some $a \in I$. Since $I \subseteq J$, we get $(x,a) \in \theta$ and $a \in J$. Therefore $x \in J^{\theta}$. Hence $I^{\theta} \subseteq J^{\theta}$.

3. Clearly $(I \cap J)^{\theta} \subseteq I^{\theta} \cap J^{\theta}$. Conversely let $x \in I^{\theta} \cap J^{\theta}$. This implies $(x,a),(x,b) \in \theta$ for some $a \in I$ and $b \in J$. So that $(x, a \wedge b) \in \theta$ and $a \wedge b \in I \cap J$. Implies that $x \in (I \cap J)^{\theta}$. Therefore $(I \cap J)^{\theta} = I^{\theta} \cap J^{\theta}$.

4. Clearly $I^{\theta} \subseteq (I^{\theta})^{\theta}$. Again, let $x \in (I^{\theta})^{\theta}$. Then $(x,a) \in \theta$, for some $a \in I^{\theta}$. Since $a \in I^{\theta}$, we have $(a,b) \in \theta$, for some $b \in I$. This implies $(x,b) \in \theta$, $b \in I$ and hence $x \in I^{\theta}$. Therefore $(I^{\theta})^{\theta} \subseteq I^{\theta}$. Thus $(I^{\theta})^{\theta} = I^{\theta}$.

Proposition 3.14: Let θ be congruence relation on an ADL L. For any ideal I of L , I^{θ} is the smallest θ-ideal of L such that $I \subseteq I^{\theta}$.

Proof: From Lemma 3.12 and Lemma 3.13(1), we get that I^{θ} is a θ-ideal of L containing the ideal I. Let K be a θ-ideal of L such that I ⊆ K. Let x ∈ I^{θ}. Then we get (x, a) ∈ θ for some a ∈ I ⊆ K. Hence x ∈ $[x]_{\theta} = [a]_{\theta}$ ⊆K. Therefore I^{θ} ⊆ K.

Let R be a distributive lattice whose Hasse diagram is given in the example 3.3. For any congruence relation θ on a distributive lattice R, one can easily observe that the set $\Im_{\theta}$(R) of all θ-ideals of R is not a sublattice of the ideal lattice $\Im$ (R). For, consider the ideal I = {0, a} and J = {0, b}. Now, for the congruence relation θ whose partition is {{0}, {a}, {b}, {c, 1}}, we can observe that I and J are both the θ-ideals of the distributive lattice R. But the ideal I ∨ J is a not a θ-ideal of R. Keeping in view of the operation depicted in the Definition 3.11, it can be observed that $\Im_{\theta}$ (L) can be made into a distributive lattice with respect to the following operations for any I, J ∈ $\Im_{\theta}$ (L), I∧J = I∩J and I⊔J = $(I \vee J)^{\theta}$

Theorem 3.15: Let θ be a congruence relation on an ADL L. For any proper θ-ideal I of L, we have $I = \bigcap\{P \mid P$ is a θ-prime ideal and $I \subseteq P\}$.

Proof: Take $I_0 = \bigcap\{P \mid P$ is a θ-prime ideal, $I \subseteq P\}$. Clearly $I \subseteq I_0$. Let $a \notin I$. Consider $\mathfrak{F} = \{J \mid J$ is a θ-ideal, $I \subseteq J$ and $a \notin I\}$. Clearly $I \in \mathfrak{F}$. Let $\{J_{\alpha}\}_{\alpha \in \Delta}$ be a chain of θ-ideals in $\mathfrak{F}$.

Clearly, $\bigcup_{\alpha \in \Delta} J_{\alpha}$ is a θ-ideal of L such that $I \subseteq \bigcup_{\alpha \in \Delta} J_{\alpha}$ and $a \notin \bigcup_{\alpha \in \Delta} J_{\alpha}$. Hence by the Zorn's lemma, $\mathfrak{F}$ has a maximal element M, say. That means M is a θ-ideal, $I \subseteq M$ and $a \notin M$. Suppose $x, y \in L$ such that $x \notin M$ and $y \notin M$. Then $M \subset M \vee (x] \subseteq (M \vee (x])^{\theta}$ and $M \subset M \vee (y] \subseteq (M \vee (y])^{\theta}$. By the maximality of M, we get that $a \in (M \vee (x])^{\theta} \cap (M \vee (y])^{\theta} = (M \vee (x \wedge y]).^{\theta}$ If x ∧ y ∈ $[0]_{\theta}$, then x ∧ y ∈ $[0]_{\theta}$ ⊆ M. Hence a ∈ M,
which is a contradiction. Hence M is θ-prime. Therefore for any a ∉ I, there exists a θ-prime ideal M of L such that I ⊆ M and a ∉ M. Thus a ∉ I_0. Therefore I_0 ⊆ I. Hence I_0 = I.

Corollary 3.16: $[0]_\theta = \bigcap\{P \mid P$ is a θ-prime ideal$\}$.

Corollary 3.17. If θ is the smallest congruence on L, then we have $\{0\} = \bigcap\{P \mid P$ is a θ-prime ideal$\}$.

Corollary 3.18: Let L θ be a congruence relation on an ADL L. If $a \notin [0]_\theta$ then there exists a θ-prime ideal P of L such that $a \notin P$.

Finally, we conclude this paper with the following theorem.

Theorem 3.19: Let L θ be a congruence on L. Suppose I is a θ-ideal and F is a filter of L such that $I \cap F = \phi$. Then there exists a θ-prime ideal P of L such that $I \subseteq P$ and $F \cap P = \phi$.

Proof: Let I be a θ-ideal and F, a filter of L such that I ∩ F = ϕ. Consider

$\Im$ = {J | J is a θ-ideal, I ⊆ J and J ∩ F = ϕ }. Clearly I ∈ $\Im$. Let $\{ J_i \mid i \in \Delta \}$ be a chain of θ-ideals in $\Im$. Clearly, $\bigcup_{i\in\Delta} J_i$ is a θ-ideal such that I ⊆ $\bigcup_{i\in\Delta} J_i$ and $(\bigcup_{i\in\Delta} J_i)$ ∩ F = ϕ. Let M be a maximal element of $\Im$. Suppose x, y ∈ L such that $x \notin M$ and $y \notin M$. Then M ⊂ M ∨ (x] ⊆$\{M \vee (x]\}^\theta$ and M ⊂ M ∨ (y] ⊆ $\{M \vee (y]\}^\theta$. By the maximality of M, we get that $\{M \vee (x]\}^\theta \cap F \neq \phi$ and $\{M \vee (y]\}^\theta \cap F \neq \phi$. Choose a ∈ $\{M \vee (x]\}^\theta \cap F$ and b ∈ $\{M \vee (y]\}^\theta \cap F$. Hence a∧b ∈ $\{M \vee (x]\}^\theta \cap \{M \vee (y]\}^\theta = \{M \vee (x \wedge y]\}^\theta$.
If x ∧ y ∈ $[0]_\theta$, then x ∧ y ∈ $[0]_\theta$ ⊆ M. Since M is a θ-ideal, we get that a ∧ b ∈ M^θ = M. Hence a ∧ b ∈ M ∩ F, which is a contradiction. Therefore M is a θ-prime ideal of L.

4. Conclusion

Some remarkable results have been established on θ-ideals by using congruence in an Almost Distributive Lattice(ADL).The change of θ-ideal into a θ-Prime ideal is achieved with the help of a set of equivalent conditions.

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