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Maximal Square Sum Subgraph of a Complete Graph

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Abstract: *A (p, q) graph G is said to be square sum, if there exists a bijection f : V (G) →{0, 1, 2, . . . , p − 1} such that the induced function f ∗ : E(G) → N defined by f ∗ (uv) = (f (u))2 + (f (v))2, for every uv ϵ E(G) is injective. In [2] it is proved that complete graph Kn is square sum if and only if n ≤ 5. In this paper we consider the problem of square sum labeling of maximal square sum subgraph of Kn , n ≥ 6. The minimum number of edges to be deleted from Kn , n ≥ 6 so that the graph is square sum is defined as critical number of Kn . We call this maximal square sum subgraph of Kn as critically square sum subgraph of Kn. In this paper we calculate the critical number of Kn for 6 ≤ n ≤ 50 and developed an algorithm to decide the critical number of a critically square sum subgraph of Kn .*

Key words: *Square sum graphs, Critical number*

1. INTRODUCTION

Graph labeling, where the vertices and edges are assigned real values or subset of a set are subject to certain conditions, have often been motivated by their utility to various applied fields. Several practical problems in real life situations have motivated the study of labeling of graphs which are required to obey a variety of conditions depending on the structure

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graphs. Graph labeling has a strong communication between number theory [4] and structure of graphs [7] and [6]. Here we are interested in the study of vertex functions

$f : V(G) \to A$, $A \subseteq N$ for which the induced edge function $f^*(uv) = (f(u))^2 + (f(v))^2$, for all uv $E(G)$ is injective.

The wide-angular history of sum of squares of numbers motivated the authors to study the particular graphs named square sum graphs. Unless mentioned otherwise, by a graph we shall mean in this paper a finite, undirected, connected graph without loops or multiple edges. Terms not defined here are used in the sense of Harary[7]. Square sum graphs are vertex labeled graphs with the labels from the set $\{0, 1, 2, . . . , p-1\}$ such that the induced edge labels as the sum of the squares of the labels of the end vertices are all distinct. Not every graph is square sum. For example, any complete graph K_n, $n \geq 6$ is not square sum

[2]. It is natural to inquire the size (measured by the number of edges) of the largest square sum subgraph of K_n.If the nodes of K_n are numbered from the consecutive integers from 0, 1, . . . , $n-1$, some edges e_i receives the same labels. Removing all the edges with same labels except one from each collection e_i of same labels, we obtain a square sum subgraph of K_n, $n \geq 6$. The resulting graph is the maximal square sum subgraph of K_n.

We are interested to find out the minimum number of edges e_i to be deleted from K_n so that the edge labels of $K_n - e_i$ are distinct or $K_n - e_i$ is square sum. The minimum number of edges to be deleted from K_n so that the graph is square sum is defined as critical number. We call this maximal square sum subgraph as critically square sum subgraph of K_n. In this paper we compute the critical number of K_n, $n \geq 6$ and hence determine the size of critically square sum subgraph of K_n for $n \leq 50$. We have developed an algorithm to decide the critical number of a critically square sum subgraph of K_n and to find the size of the critically square sum subgraph of K_n.

2. Square Sum Graphs

Acharya and Germina [1] defined a square sum labeling of a (p, q)-graph G [2] as follows.

Definition 2.1. A (p, q) graph G is said to be square sum, if there exists a bijection $f: V(G) \rightarrow \{0, 1, 2, \ldots, p-1\}$ such that induced function $f^* : E(G) \rightarrow N$ defined by $f^*(uv) = (f(u))^2 + (f(v))^2$, for every $uv \in E(G)$ is injective.

Theorem 2.2. [2] Complete graph K_n is square sum if and only if $n \leq 5$.

Following algorithm determine the edges having same labels.

Algorithm 2.2.1.

step 1. Start

step 2. Declare and initialize the variables.

step 3. Accept the value for $n \leq 50$ from user.

step 4. Begin outer loop i.

step 5. Begin inner loop j.

step 6. Compute $C[i][j] = i * i + j * j$

step 7. End inner loop.

step 8. print C[i][j] value.

step 9. End outer loop.

step 10. Begin outer loop i.

step 11. Begin first inner loop j.

step 12. Begin second inner loop p.

step 13. Begin third inner loop q.

step 14. Compute $C[i][j] = C[p][q]$ if $(i! = p)$ and $(j! = q)$ and $(i! = j)$ and $(j! = p)$

step 15. Print (i, j), (p, q) and C[i][j] else goto step 10.

Step 16. End third inner loop.

step 17. End second inner loop.

step 18. End first inner loop.

Step 19. End outer loop.

step 20. Stop.

The square sum labelling of the largest square sum subgraphs of K_6 is shown in Figure 1 below. A programme in C^{++} is developed based on Algorithm 2.2.1. The critical number of K_n and size of the maximal square sum subgraph of K_n, $6 \leq n \leq 50$ are given in tables [1], [2], [3], [4], [5] and [6] respectively.

Notation: In the table MSG denote the maximal square sum subgraph of G and Cr.No denote the critical number.

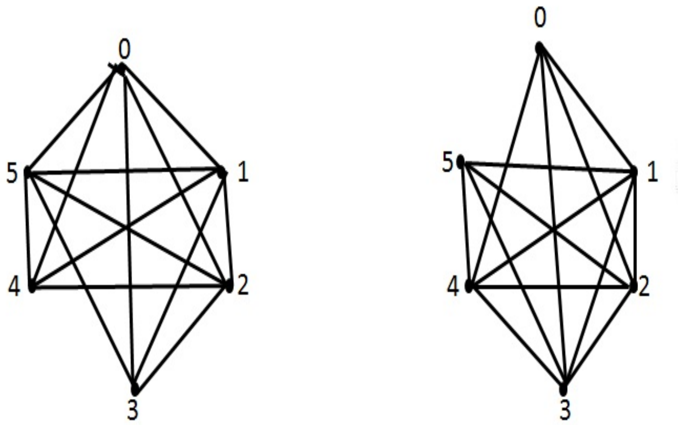

Figure 1. *The Square sum labelings of maximal square sum subgraph of K_6*

Table 1:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
6	K6	{(0, 5)(3, 4)}	14	1
7	K7	{(0, 5)(3, 4)}	20	1
8	K8	{(0, 5)(3, 4)}	27	1
9	K9	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}	34	2
10	K10	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)}	42	3
11	K11	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)} {(0, 10)(6, 8)}	51	4
12	K12	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)},	60	6
13	K13	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)}, {(0, 10)(6, 8)}, {(2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}	71	7
14	K14	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)}, {(0, 10)(6, 8)}, {(2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}, {(0, 13)(5, 12)}{(1, 13)(7, 11)}, {(4, 13)(11, 8)}	81	10
15	K15	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)}, {(0, 10)(6, 8)}, {2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}, {(0, 13) (5, 12)}, {(1, 13)(7, 11)}, {(4, 13)(11, 8)}, {(5, 14)(10, 11)}, {(3, 14)(6, 13)}	93	12
16	K16	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)},{(0, 10)(6, 8)}, {2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}, {(0, 13)(5, 12)}, {(1, 13)(7, 11)}, {(4, 13)(11, 8)},	106	14
17	K17	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)}, {(0, 10)(6, 8)}, {(2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}, {(0, 13)(5, 12)}, {(1, 13)(7, 11)}, {(4, 13)(11, 8)}, {(5, 14) (10, 11)}, {(3, 14)(6, 13)}, {(0, 15)(12, 9)}, {(5, 15)(9, 13)}, {(2, 16)(8, 14)}, {(3, 16)(11, 12)}	120	16
18	K18	{(0, 5)(3, 4)}, {(1, 8)(4, 7)}, {(2, 9)(6, 7)} {(0, 10)(6, 8)}, {(2, 11)(10, 5)}, {(3, 11)(7, 9)}, {(1, 12)(8, 9)}, {(0, 13)(5, 12)}, {(1, 13)(7, 11)}{(4, 13)(11, 8)}, {(5, 14)(10,11)}, {(3, 14)(6, 13)}, {(0, 15)(12, 9)}, {(5, 15)(9, 13)}, {(2, 16)(8, 14)},{(3, 16)(11, 12)},{(0, 17)(8, 15)}, {(1, 17) (11, 3)}, {(4, 17)(7, 16)}, {(6, 17)(10, 15)}	133	20

Table 2:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
19	K_{19}	Edges having same labels in K_{18}, {(1, 18)(6, 17)(10, 15)}, {(4, 18)(14, 12)}	149	22
20	K_{20}	Edges having same labels in K19 , {(8, 19)(16, 13)}, {(2, 19)(13, 14)}, {(3, 19)(17, 9)}, {(4, 19)(11, 16)}, {(7, 19)(11, 17)}	163	27
21	K_{21}	Edges having same labels in K20 , {(0, 20)(12, 16)}, {(5, 20)(8, 19)(13, 16)}, {(9, 20)(15, 16)}	180	30
22	K_{22}	Edges having same labels in K21 , {(1, 21)(9, 19)}, {(2, 21)(11, 18)}, {(8, 21)(12, 19)}	198	33
23	K_{23}	Edges having same labels in K22 , {(1, 22)(14, 17)},{(3, 22)(13, 18)}, {(4, 22)(10, 20)}, {(6, 22)(14, 18)}	216	37
24	K_{24}	Edges having same labels in K23 , {(1, 23)(13, 19)}, {(2, 23)(7, 22)}, {(4, 23)(16, 17)}, {(6, 23)(9, 22)}, {(9, 23)(13, 21)}, {(23, 11)(17, 19)}	233	43
25	K_{25}	Edges having same labels in K24 , {(2, 24)(16, 18)}, {(3, 24)(21, 12)}, {(7, 24)(15, 20)}, {(11, 24)(16, 21)}	253	47
26	K_{26}	Edges having same labels in K25 , {(0, 25)(7, 24), (15, 20)}, {(2, 25)(10, 23)}, {(5, 25)((11, 23)(17, 19)}, {(8, 25)(17, 20)}, {(10, 25) (14, 23)}	273	52
27	K_{27}	Edges having same labels in K_{26}, {(0, 26)(10, 24)}, {(2, 26)(14, 22)}, {(3,26)(18, 19)}, {(8, 26)(18, 19)},{(7, 26)(10, 25)(14, 23)},{(13, 26) (19, 22)}	293	58

Table 3:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
28	K_{28}	Edges having same labels in K27 , {(1, 27)(17, 21)}, {(11, 27)(15, 25)}, {(4, 27)(13, 24)}, {(5, 27)(15, 23)}, {(6,27)(18, 21)}, {(14, 27)(21, 22)}	314	64
29	K_{29}	Edges having same labels in K28 , {(1, 28)(16, 23)},{(3, 28)(8, 27)}, {(6,28) (12, 26)}, {(9, 28)(17, 24)},{(10, 28)(22, 20)}	337	69
30	K_{30}	Edges having same labels in K29 , {(0, 29)(20, 21)}, {(12, 29)(16, 27)},{(2, 29) (13, 26)(19, 22)}, {(3, 29)(11, 27)(15, 25)}, {(7,29)(19, 23)}, {(8, 29)(28, 11)}, {(14, 29)(19, 26)}, {(15, 29)(21, 15)}	358	77
31	K_{31}	Edges having same labels in K30 , {(0, 30)(18, 24)}, {(1, 30)(15, 26)}, {(5, 30) (14, 27)(21, 22)}, {(7, 30)(18, 25)}, {(10, 30)(18, 26)}	383	82
32	K_{32}	Edges having same labels in K31 , {(1, 31)(11, 29)}, {(2, 31)(26, 17)}, {(3, 31) (21, 23)}, {(5, 31)(19, 25)}, {(7, 31)(29, 13)}, {(13, 31) (17, 29)},{(12, 31)(24, 23)}, {(8, 31)(25, 20)}	406	90
33	K_{33}	Edges having same label in K32 , {(1, 32)(8, 31)(25, 20)}, {(4, 32)(16, 28), {(6, 32)(22, 24)}, {(7, 32)(28, 17)}, {(17, 32)(23, 28)}, {(11, 32)(19, 23}, {(9, 32)(24, 23)(12, 31)}	431	97
34	K_{34}	Edges having same label in K33 , {(1, 33)(19, 27)}, {(4, 33)(9, 32)(12, 31)(24, 23)}, {(6, 33)(15, 30)}, {(9, 33)(27, 21)}, {(10, 33)(17, 30)}, {(13, 33)(23, 27)}, {(14, 33)(18, 31)}	457	104
35	K_{35}	Edges having same label in K34 , {(0, 34)(16, 30)}, {(1, 34)(14, 31)}, {(2, 34)(22, 26)}, {(12, 34)(20, 30)}, {(3, 34)(29, 18)}, {(7, 34)(23, 26)}, {(8, 34)(14, 32)}, {(19, 34)(26, 29)},{(13, 34) (29, 22)}, {(17, 34)(22, 31)}	481	114

Table 4:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
36	K_{36}	Edges having same label in K35 , {(0, 35)(21, 28)}, {(4, 35)(20, 29)}, {(5, 35)(17, 31)}, {(6, 35)(19, 30)}, {(10, 35)(13, 34) (29, 22)}, {(15, 35)(33, 19)}, {(19, 35)(31, 25)}, {(28, 29)(20, 35)}	508	122
37	K_{37}	Edges having same label in K36 , {(2, 36)(12, 34)(20, 30)}, {(3, 36)(24, 27)}, {(7, 36)(16, 33)}, {(8, 36)(24, 28)}, {(11, 36)(24, 29)}, {(13, 36)(21, 32)}	538	128
38	K_{38}	Edges having same label in K37 , {(0, 37)(35, 12)}, {(1, 37)(29, 23)}, {(3, 37)(17,33)}, {(4, 37)(32, 19)}, {(5, 37)(35, 13)}, {(6, 37)(26, 27)}, {(9, 37)(15, 35)(19, 33)}, {(11, 37)(23, 21)}, {(12, 37)(27, 28)}, {(16, 37) (20,35)(28, 29)}	565	138
39	K_{39}	Edges having same label in K38 , {(1, 38)(17, 34)(22, 31)}, {(4, 38)(28, 26)}, {(5, 38)(10, 37)}, {(6, 38)(18, 34)}, {(8, 38)(22, 32)}, {(9, 38)(25, 30)}, {(11, 38)(14, 37)}, {(14, 38)(22, 34)}, {(21, 38)(27, 34)}, {(16, 38)(32, 26)}	593	148,
40	K_{40}	Edges having same label in K39 , {(0, 39)(36, 15)}, {(2, 39)(9, 38)(25, 30)}, {(3,39)(21, 33)}, {(4, 39)(24, 31)}, {(7, 39)(27, 29)}, {(8, 39)(17, 36)}, {(12, 39)(24, 33)}, {(17, 39)(21, 37)}, {(20, 39)(25, 36)}, {(23, 29)(31, 33)}, {(13, 39)(27, 31)}	621	159
41	K_{41}	Edges having same label in K40 , {(0, 40)(24, 32)}, {(5, 40)(16, 37) (20, 35)(28,29)},{(7, 40)(25, 32)}, {(10, 40)(16, 38) (32,26)},{(13, 40)(20, 37)}, {(15, 40)(23, 36)}, {(18,40)(30, 32)}	654	166
42	K_{42}	Edges having same label in K41 , {(0, 41)(9, 40)}, {(2, 41)(23, 34)}{(6, 41) (14,39)},{(3, 41)(13, 39)(27, 31)}, {(7, 41)(19, 37)}, {(8,41)(28, 31)}, {(10, 41)(25, 34)}, {(11, 41)(29, 31)}, {(12, 41)(15, 40)(23, 36)}, {(18, 41)(22, 39)}, {(24, 41)(31, 36)}, {(13, 41)(35, 25)},{(23, 41)(37, 29)}	682	179

Table 5:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
43	K_{43}	Edges having same label in K42 , {(1, 42)(33, 26)}, {(2, 42)(18, 38)}, {(4, 42)(22,36)},{(9, 42)(18, 39)}, {(15, 42)(30, 33)}, {(16, 42)(24, 38)}, {(19, 42)(30, 35)}, {(11, 42)(21, 38)(27, 34)}	716	187
44	K_{44}	Edges having same label in K43 , {(1, 43)(35, 25)(13, 41)}, {(2, 43)(22, 37)}, {(4,43)(32, 29)},{(6, 43)(11, 42)(21, 38)(27, 34)}, {(7, 43)(37, 23)}, {(9, 43)(29, 33)},{(11, 43)(17, 41)}, {(14, 43)(26, 37)}, {(18, 43)(27, 38)},{(20, 43)(32, 35)}, {(23, 41)(19, 43)(37, 29)}	748	198
45	K_{45}	Edges having same label in K44 , {(1, 44)(41, 16)}, {(2, 44)(28, 34)}, {(3, 44)(24,37)}, {(5, 44)(19, 40)}, {(6, 44)(26, 36)}, {(7, 44)(31, 32)}, {(8, 44)(20, 40)}, {(12, 44)(28, 36)}, {(13, 44)(16, 43)}, {(25, 44)(31, 40)}, {(27, 44)(36, 37)}, {(23, 44)(28, 41)}, {(17, 44)(25, 40)}	779	211
46	K_{46}	Edges having same label in K45 , {(0, 45)(27, 36)}, {(4, 45)(21, 40)}, {(5, 45)(23,39)(31, 33)}, {(7, 45)(15, 43)}, {(11, 45)(25, 39), {(10, 45)(19, 42)(30, 35)}, {(15, 45)(27, 39)}, {(17, 45)(33, 35)}, {(20, 45)(24, 43)}	815	220
47	K_{47}	Edges having same label in K46 , {(1, 46)(31, 34)}, {(2, 46)(26, 38)}, {(4, 46)(14,44)}, {(8, 46)(32, 34)}, {(3, 46)(10, 45)(19, 42)(30,35)}, {(9, 46)(26, 39)}, {(12, 46)(18, 44)}, {(13, 46)(29, 38)}, {(17, 46)(31, 38)}, {(18, 46)(42, 46)},{(7, 46)(22, 41)}	849	231
48	K_{48}	Edges having same label in K47 , {(1, 47)(19, 43)(23, 41)(37, 29)}, {(6, 47)(34,33)}, {(4, 47)(17, 44)(25, 40)}, {(9, 47)(21, 43)}, {(11, 47)(31, 37)}, {(13, 47)(23, 43)}, {(16, 47)(23, 44)(28, 41)}, {(18, 47)(33, 38)}, {(21, 47)(25, 45)}, {(27, 47)(43,33)}, {(29, 47)(37, 41)}	886	242
49	K_{49}	Edges having same label in K48 , {(1, 48)(28, 39)}, {(4, 48)(32, 36)}, {(5, 48)(27,40)}, {(6, 48)(24, 42)}, {(7, 48)(12, 47)}, {(9, 48)(33, 36)}, {(11, 48)(20, 45)(24, 43)}, {(19,48)(27, 44)(36, 37)}, {(22, 48)(32, 42)}, {(29, 48)(36, 43)}	924	252

Table 6:

n	k_n	Edges having same labels	Size of MSG	Cr.No.
50	K_{50}	Edges having same label in K49 , {(2, 49)(14, 47)(17, 46)(31, 38)}, {(3, 49)(27,41)}, {(8, 49)(16, 47)(28, 41)(44, 23)}, {(9, 49)(39,31)}, {(11, 49)(29, 41)}, {(12, 49)(32, 39)}, {(12,49)(32, 39)}, {(17, 49)(29, 43)}, {(18, 49)(31, 42)}, {(22, 49)(26, 47)},{(13, 49)(47, 19)}, {(26, 49)(31, 46)}, {(24, 49)(36, 41)}	960	265

The square graphs $W2, n \geq 6$ and $K2, m + n \geq 6$ are complete graphs, the critical number and size of the maximal square sum subgraph of W^2,n≥ 6 and K^2, m+n≥6 are also obtained from the algorithm. We believe that there is great potential for developing practical mathematics with numbered graphs.

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