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An Inductive Attempt to Prove Mean Value Theorem for n- Real Valued Functions

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Abstract: *The paper intends to establish a mean value theorem for n- real valued functions. It is proved with the help of Mathematical induction. In addition to this, the mean value theorem for two functions which contain n components each is also instituted with the support of standard mean value theorems. For showing the theorems, the nature of continuity and differentiability of the functions have been adopted conditionally.*

Keywords: *Continuity, Differentiability, Standard mean Value Theorems and Mathematical Induction.*

1. Introduction

Mean value theorems are pillars of modern analysis which help us to gain some new inventions with analytical approach. Many Mathematicians like Michel Rolle, Joseph Louis Lagrange, Augustin-Louis Cauchy etc. contributed their might of valuable results to the field of real analysis. New eras and constructive ideas were opened for the next generation in the continuation of their work. Several generalizations and exclusive extensions have been established by Hardy,G.H[2],Buck,R.C[1],Simmons,G.F[4],Ruddin,W[3] and Smith,K.T[5].Mean value theorems can apply in real life situation for analyzing in a better way to obtain the fruitful solution. In this paper, the authors utilized the concept of mathematical induction. Mathematical induction is an effective and efficient tool in differential calculus to achieve a set of necessary goals.

An inductive attempt is made to prove mean value theorem for n real valued functions. The main essence of the principle of mathematical induction is the noticeable deterioration in performance of a step which leads inevitably to next step. Let K be any inter(may be positive, negative, zero) and $S_{K_0}, S_{K_0+1} \ldots S_K \ldots$ be the propositions where each integer $K \geq K_0$ which satisfy (i) S_{K_0} is true(ii) S_K implies S_{K+1} for every integer K then S_K is valid for every integer $K \geq K_0$.On other words, a statement is valid for K=1,K=2,…, also assume that the statement is valid for K and if it is also to be proved to valid for K+1,then the statement can be generalized for any integer K. This phenomenon is being utilized in many applications and complex situations to obtain the validity of the results.

In this paper, the authors aimed to establish a mean value theorem for n- real valued functions. It is proved with the help of Mathematical induction. In addition to this, the mean value theorem for two functions which contain n components each is also instituted with the back ground of standard mean value theorems. The continuity and differentiability of the functions have been considered for establishing lemmas and the theorems.

2. MEAN VALUE THEOREM FOR N-REAL VALUED FUNCTIONS

The following lemma will be used to prove the mean value theorem for n-real valued functions.

2.1 Lemma:

Prove that $\sum_{i=0}^{n} f'_{2i+1}(c)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))=\sum_{i=1}^{n} f'_{2i}(c)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$ (1)

where f_k (K=1,2,3….n) is continuous on [a,b] and derivable on (a,b).

<u>Proof</u>: It can be shown with the aid of mathematical induction.

In the case of having one and only fuction,it is trivially true.

If the system involves two real valued functions (i=2)

$$f'_1(c)(f_2(b)-f_2(a))=f'_2(c)(f_1(b)-f_1(a))$$

By above result is true by Cauchy's mean value theorem.

If the system necessitates three real valued functions (i=3) ,then

$$(f'_1(c)+f'_3(c))(f_2(b)-f_2(a))=f'_2(c)(f_1(b)-f_1(a)+f_3(b)-f_3(a))$$

It can be verified as below

Define a function h(x) as

$$h(x)=(f_1(x)+f_3(x))(f_2(b)-f_2(a))-f_2(x)(f_1(b)-f_1(a)+f_3(b)-f_3(a))$$

$$h(a)=(f_1(a)+f_3(a))(f_2(b)-f_2(a))-f_2(a)(f_1(b)-f_1(a)+f_3(b)-f_3(a))$$

$$=f_1(a)f_2(b)+f_2(b)f_3(a)-f_2(a)f_1(b)-f_2(a)f_3(b)$$

$$h(b)=(f_1(a)+f_3(a))(f_2(b)-f_2(a))-f_2(a)(f_1(b)-f_1(a)+f_3(b)-f_3(a))$$

$$=f_1(a)f_2(b)+f_2(b)f_3(a)-f_2(a)f_1(b)-f_2(a)f_3(b)$$

Therefore h (a) =h (b) is true. Thus h satisfies the three conditions of Rolle's Theorem.

(i) h is continuous on [a,b]

(ii) h is derivable on (a,b)

(iii) h(a)=h(b)

The three conditions of Rolle's theorem are satisfied, then there exists at least one constant c which belongs to (a,b) such that h'(c) = 0.

$$(f'_1(c)+f'_3(c))(f_2(b)-f_2(a))-f'_2(c)(f_1(b)-f_1(a)+f_3(b)-f_3(a))=0$$

$$(f'_1(c)+f'_3(c))(f_2(b)-f_2(a))=f'_2(c)(f_1(b)-f_1(a)+f_3(b)-f_3(a))$$

It is observed that the statement is valid for the three real valued functions.

Now, it is assumed that the statement (1) is true for i=k.

$$\sum_{i=0}^{k} f'_{2i+1}(c)\sum_{i=1}^{k}(f_{2i}(b)-f_{2i}(a))=\sum_{i=1}^{k} f'_{2i}(c)\sum_{i=0}^{k}(f_{2i+1}(b)-f_{2i+1}(a))$$

Now it requires to verify the validity of the statement(1) for i=k+1.

Consider L.H.S $=\sum_{i=0}^{k+1} f'_{2i+3}(c)\sum_{i=1}^{k+1}(f_{2i+2}(b)-f_{2i+2}(a))$

$$=(\sum_{i=0}^{k} f'_{2i+1}(c)+f'_{2k+3}(c))(\sum_{i=1}^{k}(f_{2i}(b)-f_{2i}(a))+f_{2k+2}(b)-f_{2k+2}(a))$$

$$=(\sum_{i=0}^{k} f'_{2i+1}(c)\sum_{i=1}^{k}(f_{2i}(b)-f_{2i}(a))+\sum_{i=0}^{k} f'_{2i+1}(c)(f_{2k+2}(b)-f_{2k+2}(a))$$

$$+f'_{2k+3}(c)\sum_{i=1}^{k}(f_{2i}(b)-f_{2i}(a))+f'_{2k+3}(c)(f_{2k+2}(b)-f_{2k+2}(a))$$

By the application of cauchy's mean value therem,

$$\sum_{i=1}^{k} f'_{i}(c)(g_{m}(b)-g_{m}(a))=g'_{m}(c)\sum_{i=1}^{k}(f_{i}(b)-f_{i}(a))$$

$$\sum_{i=0}^{k} f'_{2i+1}(c)(f_{2k+2}(b)-f_{2k+2}(a))=f'_{2k+2}(c)\sum_{i=0}^{k}(f_{2i+1}(b)-f_{2i+1}(a)$$

and $\sum_{i=1}^{k} f'_{2i}(c)(f_{2k+3}(b)-f_{2k+3}(a))=f'_{2k+3}(c)\sum_{i=1}^{k}(f_{2i}(b)-f_{2i}(a))$

$$=\sum_{i=1}^{k} f'_{2i}(c)\sum_{i=0}^{k}(f_{2i+1}(b)-f_{2i+1}(a))+\sum_{i=1}^{k} f'_{2i}(c)(f_{2k+3}(b)-f_{2k+3}(a))$$

$$+f'_{2k+2}(c)\sum_{i=0}^{k}(f_{2i+1}(b)-f_{2i+1}(a)+f'_{2k+2}(c)(f_{2k+3}(b)-f_{2k+3}(a))$$

$$=(\sum_{i=1}^{k} f'_{2i}(c)+f'_{2k+2}(c))(\sum_{i=0}^{k}(f_{2i+1}(b)-f_{2i+1}(a))+f_{2k+3}(b)-f_{2k+3}(a))$$

$$=\sum_{i=1}^{k+1} f'_{2i+2}(c)\sum_{i=0}^{k+1}(f_{2i+3}(b)-f_{2i+3}(a))$$

= R.H.S

By mathematical induction, it is also valid and true for K+1.

Hence $\sum_{i=0}^{n} f'_{2i+1}(c)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))=\sum_{i=1}^{n} f'_{2i}(c)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$

2.2 Theorem: Let $f_1, f_{2,\ldots,}f_n$ are n-real valued functions defined on [a,b] which satisfy the following conditions.

(i) $f_1, f_{2,\ldots,}f_n$ continuous on [a,b]

(ii) $f_1, f_{2,\ldots,}f_n$, derivable on (a,b)

then there exists at least one $c\in(a,b)$ such that

$$\sum_{i=0}^{n} f'_{2i+1}(c)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))=\sum_{i=1}^{n} f'_{2i}(c)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a)) \quad (2)$$

Proof: Define a function

$$g(x)=\sum_{i=0}^{n} f_{2i+1}(x)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))-\sum_{i=1}^{n} f_{2i}(x)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$$

Clearly g is continuous on [a,b] and derivable on (a,b).

For employing Rolle's theorem, it is necessary to verify that g(a)=g(b),

$$g(a)=\sum_{i=0}^{n} f_{2i+1}(a)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))-\sum_{i=1}^{n} f_{2i}(a)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$$

$$=\sum_{i=1}^{n} f_{2i}(b)\sum_{i=0}^{n} f_{2i+1}(a)-\sum_{i=0}^{n} f_{2i+1}(b)\sum_{i=1}^{n} f_{2i}(a)$$

$$g(b)=\sum_{i=0}^{n} f_{2i+1}(b)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))-\sum_{i=1}^{n} f_{2i}(b)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$$

$$=\sum_{i=1}^{n} f_{2i}(b)\sum_{i=0}^{n} f_{2i+1}(a)-\sum_{i=0}^{n} f_{2i+1}(b)\sum_{i=1}^{n} f_{2i}(a)$$

Therefore g(a)=g(b) is true.

Thus,g satisfies the following conditions.
(i) g is continuous on [a,b]
(ii) g is derivable on(a,b)
(iii) g(a)=g(b)
The three conditions of Rolle's theorem are satisfied, hence by Rolle's theorem there exists at least one $c\in(a,b)$ such that g'(c) =0

$$\sum_{i=0}^{n} f'_{2i+1}(c)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))-\sum_{i=1}^{n} f'_{2i}(c)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))=0$$

$$\sum_{i=0}^{n} f'_{2i+1}(c)\sum_{i=1}^{n}(f_{2i}(b)-f_{2i}(a))=\sum_{i=1}^{n} f'_{2i}(c)\sum_{i=0}^{n}(f_{2i+1}(b)-f_{2i+1}(a))$$

Hence the proof

2.4 Lemma: Show that $\dfrac{\sum_{i=1}^{n} f'_i(c)}{g'_m(c)}=\dfrac{\sum_{i=1}^{n}(f_i(b)-f_i(a))}{(g_m(b)-g_m(a))}$ (3)

where f_i (i=1,2,3…n) and g_m are continuous on [a,b] and derivable on (a,b).
Proof: The above result is proved by mathematical induction
if n=1 ,then

L.H.S= $\dfrac{f'_1(c)}{g'_m(c)}=\dfrac{f_1(b)-f_1(a)}{(g_m(b)-g_m(a))}$ =R.H.S

By Cauchy's mean value theorem the above result is true
Let us assume to consider that the statement (3) is valid for n=k.

$$\frac{\sum_{i=1}^{k} f'_i(c)}{g'_m(c)}=\frac{\sum_{i=1}^{k}(f_i(b)-f_i(a))}{(g_m(b)-g_m(a))}$$

Now, it is essential to verify that the statement (3) is true for n=k+1

L.H.S $=\dfrac{\sum_{i=1}^{k+1} f'_i(c)}{g'_m(c)}$

$$=\frac{\sum_{i=1}^{k} f'_i(c)}{g'_m(c)}+\frac{f'_{k+1}(c)}{g'_m(c)}$$

$$=\frac{\sum_{i=1}^{k}(f_i(b)-f_i(a))}{(g_m(b)-g_m(a))}+\frac{(f_{k+1}(b)-f_{k+1}(a))}{(g_m(b)-g_m(a))}$$

$$=\frac{\sum_{i=1}^{k+1}(f_i(b)-f_i(a))}{(g_m(b)-g_m(a))}$$

=R.H.S
Therefore the statement (3) is valid for n=K+1.

2.5 Lemma:

Prove that $\sum_{i=1}^{n} f'_i(c)\sum_{i=1}^{n}(g_i(b)-g_i(a))=\sum_{i=1}^{n} g'_i(c)\sum_{i=1}^{n}(f_i(b)-f_i(a))$ (4)

where f_k and g_k (k=1,2,3..n) are continuous on [a,b] and derivable on (a,b).

Proof:-

By Cauchy mean value theorem, it can be stated as

$f_s'(c)\,(g_t(b)-g_t(a)) = g_t'(c)\;(f_s(b)-f_s(a))$ where s,t are any natural numbers

Case (i): if n=1

then L.H.S= $f_1'(c)\,(g_1(b)-g_1(a))$= $g_1'(c)\,(f_1(b)-f_1(a))$=R.H.S

Hence by Cauchy's mean value theorem the state is true for n=1

Case (ii): if n=2

then, L.H.S= $f_1'(c)+f_2'(c)\;(g_1(b)-g_1(a)+g_2(b)-g_2(a))$

$$= f_1'(c)\;(g_1(b)-g_1(a)+g_2(b)-g_2(a)) + f_2'(c)\;(g_1(b)-g_1(a)+g_2(b)-g_2(a))$$

$$= f_1'(c)\,(g_1(b)-g_1(a)) + f_2'(c)\;(g_1(b)-g_1(a)) + f_1'(c)\;(g_2(b)-g_2(a)) + f_2'(c)\;(g_2(b)-g_2(a))$$

$$= g_1'(c)\;(f_1(b)-f_1(a)) + g_1'(c)\;(f_2(b)-f_2(a)) + g_2'(c)\;(f_1(b)-f_1(a)) + g_2'(c)\;(f_2(b)-f_2(a))$$

$$= g_1'(c)\;(f_1(b)-f_1(a)+f_2(b)-f_2(a)) + g_2'(c)\;(f_1(b)-f_1(a)+f_2(b)-f_2(a))$$

$$= g_1'(c)+g_2'(c)\;(f_1(b)-f_1(a)+f_2(b)-f_2(a))$$

=R.H.S

Therefore the statement(4) is true for n=2.

Now, it is presumed that the statement (4) is true for n=k

$$\sum_{i=1}^{k} f_i'(c)\sum_{i=1}^{k}(g_i(b)-g_i(a)) = \sum_{i=1}^{k} g_i'(c)\sum_{i=1}^{k}(f_i(b)-f_i(a))$$

and $f_{k+1}'(c)(g_{k+1}(b)-g_{k+1}(a)) = g_{k+1}'(c)(f_{k+1}(b)-f_{k+1}(a))$

Since $\sum_{i=1}^{k} f_i'(c)(g_{k+1}(b)-g_{k+1}(a)) = g_{k+1}'(c)\sum_{i=1}^{k}(f_i(b)-f_i(a))$

Similarly it can be proved $\sum_{i=1}^{k} g_i'(c)(f_{k+1}(b)-f_{k+1}(a)) = f_{k+1}'(c)\sum_{i=1}^{k}(g_i(b)-g_i(a))$

Consider L.H.S,

$$\sum_{i=1}^{k+1} f_i'(c)\sum_{i=1}^{k+1}(g_i(b)-g_i(a))$$

$$= \sum_{i=1}^{k} f_i'(c)\sum_{i=1}^{k}(g_i(b)-g_i(a)) + \sum_{i=1}^{k} f_i'(c)(g_{k+1}(b)-g_{k+1}(a))$$

$$+ f_{k+1}'(c)\sum_{i=1}^{k}(g_i(b)-g_i(a)) + f_{k+1}'(c)(g_{k+1}(b)-g_{k+1}(a))$$

$$=\sum_{i=1}^{k} g_i'(c)\sum_{i=1}^{k}(f_i(b)-f_i(a))+g_{k+1}'(c)(\sum_{i=1}^{k}(f_i(b)-f_i(a)))$$
$$+\sum_{i=1}^{k} g_i'(c)(f_{k+1}(b)-f_{k+1}(a))+g_{k+1}'(c)(f_{k+1}(b)-f_{k+1}(a))$$

$$=\sum_{i=1}^{k+1} g_i'(c)\sum_{i=1}^{k+1}(f_i(b)-f_i(a))$$

= R.H.S

$$\therefore \sum_{i=1}^{n} f_i'(c)\sum_{i=1}^{n}(g_i(b)-g_i(a))=\sum_{i=1}^{n} g_i'(c)\sum_{i=1}^{n}(f_i(b)-f_i(a))$$

2.6 Corollary:

Let f and g contain n components $f_1, f_2, f_3 \ldots f_n$ and $g_1, g_2 \ldots g_n$ respectively .All components are real valued functions defined on [a, b] which satisfy the following conditions.

(I) if $f_1, f_2 \ldots f_n$ and $g_1, g_2 \ldots g_n$ are continuous on [a, b]
(II) if $f_1, f_2 \ldots f_n$ and $g_1, g_2 \ldots g_n$ are derivable on (a, b)

then there exists at least one $c \in (a,b)$ Such that

$$\sum_{i=1}^{n} f_i'(c)\sum_{i=1}^{n}(g_i(b)-g_i(a))=\sum_{i=1}^{n} g_i'(c)\sum_{i=1}^{n}(f_i(b)-f_i(a))$$

Proof: Define a function

h(x) = $\sum_{i=1}^{n} f_i(x)\sum_{i=1}^{n}(g_i(b)-g_i(a))-\sum_{i=1}^{n} g_i(x)\sum_{i=1}^{n}(f_i(b)-f_i(a))$

Clearly it is continuous on [a,b] and derivable (a,b).
Now, the third condition of Rolle 's theorem is to be verified as below

$$\sum_{i=1}^{n} f_i(a)\sum_{i=1}^{n}(g_i(b)-g_i(a))-\sum_{i=1}^{n} g_i(a)\sum_{i=1}^{n}(f_i(b)-f_i(a))$$
$$=\sum_{i=1}^{n} f_i(b)\sum_{i=1}^{n}(g_i(b)-g_i(a))-\sum_{i=1}^{n} g_i(b)\sum_{i=1}^{n}(f_i(b)-f_i(a)) \qquad (5)$$

Case (i): If n=1

then L.H.S = $f_1(a)\ (g_1(b)-g_1(a))-g_1(a)\ (f_1(b)-f_1(a))$

$= f_1(a)\ g_1(b)-f_1(a)\ g_1(a)-g_1(a)\ f_1(b)+g_1(a)\ f_1(a)$

$= f_1(a)\ g_1(b)-g_1(a)\ f_1(b)$

R.H.S = $f_1(b)(g_1(b)-g_1(a))-g_1(b)\ (f_1(b)-f_1(a))$

$= f_1(b)\ g_1(b)-f_1(b)\ g_1(a)-g_1(b)\ f_1(b)+g_1(b)\ f_1(a)$

$= f_1(a)\ g_1(b)-f_1(b)\ g_1(a)$

Therefore the statement (5) is valid for n=1

Case (ii): If n=2

then L.H.S = $f_1(a)+f_2(a)\ (g_1(b)+g_2(b)-g_1(a)-g_2(a))$

$-\ g_1(a)-g_2(a)\ (f_1(b)-f_1(a)+f_2(b)-f_2(a))$

$=(f_1(a)+f_2(a))(g_1(b)+g_2(b))+(f_1(a)+f_2(a))(-g_1(a)-g_2(a))$

$$+ (-g_1(a) - g_2(a))(f_1(b) + f_2(b)) + (-g_1(a) - g_2(a))(-f_1(a) - f_2(a))$$

$$= (f_1(a) + f_2(a))(g_1(b) + g_2(b)) - (g_1(a) + g_2(a))(f_1(b) + f_2(b))$$

$$\text{R.H.S} = f_1(b) + f_2(b)\ ((g_1(b) + g_2(b) - g_1(a) - g_2(a))$$

$$- (g_1(b) + g_2(b))(f_1(b) - f_1(a) + f_2(b) - f_2(a))$$

$$= (f_1(b) + f_2(b))(g_1(b) + g_2(b)) + f_1(b) + f_2(b)\ (-g_1(a) - g_2(a))$$

$$+ (-g_1(b) - g_2(b))(f_1(b) + f_2(b)) + (-g_1(b) - g_2(b))(-f_1(a) - f_2(a))$$

$$=((f_1(a) + f_2(a))(g_1(b) + g_2(b)) - (g_1(a) + g_2(a))(f_1(b) + f_2(b))$$

The validity of the statement (5) exist for n=2.

Consider that the statement (5) is true for n=k

$$\sum_{i=1}^{k} f_i(a)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - \sum_{i=1}^{k} g_i(a)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

$$= \sum_{i=1}^{k} f_i(b)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - \sum_{i=1}^{k} g_i(b)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

It is enough to prove that the statement is true for n=k+1.

Consider L.H.S

$$\sum_{i=1}^{k+1} f_i(a)\sum_{i=1}^{k+1}(g_i(b) - g_i(a)) - \sum_{i=1}^{k+1} g_i(a)\sum_{i=1}^{k+1}(f_i(b) - f_i(a))$$

$$= \sum_{i=1}^{k} f_i(a)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - \sum_{i=1}^{k} g_i(a)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

$$+ \sum_{i=1}^{k} f_i(a)(g_{k+1}(b) - g_{k+1}(a)) - \sum_{i=1}^{k} g_i(a)(f_{k+1}(b) - f_{k+1}(a))$$

$$+ f_{k+1}(a)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - g_{k+1}(a)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

$$+ f_{k+1}(a)(g_{k+1}(b) - g_{k+1}(a)) - g_{k+1}(a)(f_{k+1}(b) - f_{k+1}(a))$$

$$= \sum_{i=1}^{k} f_i(a)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - \sum_{i=1}^{k} g_i(a)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

$$+ g_{k+1}(b)\sum_{i=1}^{k} f_i(a) - g_{k+1}(a)\sum_{i=1}^{k} f_i(a)) - f_{k+1}(b)\sum_{i=1}^{k} g_i(a) + f_{k+1}(a)\sum_{i=1}^{k} g_i(a))$$

$$+ f_{k+1}(a)\sum_{i=1}^{k} g_i(b) - f_{k+1}(a)\sum_{i=1}^{k} g_i(a) - g_{k+1}(a)\sum_{i=1}^{k} f_i(b) + g_{k+1}(a)\sum_{i=1}^{k} f_i(a)$$

$$+ f_{k+1}(a)\, g_{k+1}(b) - f_{k+1}(a)\, g_{k+1}(a) - g_{k+1}(a)\, f_{k+1}(b) + g_{k+1}(a)\, f_{k+1}(a)$$

$$= \sum_{i=1}^{k} f_i(a)\sum_{i=1}^{k}(g_i(b) - g_i(a)) - \sum_{i=1}^{k} g_i(a)\sum_{i=1}^{k}(f_i(b) - f_i(a))$$

$$+ g_{k+1}(b)\sum_{i=1}^{k} f_i(a) - f_{k+1}(b)\sum_{i=1}^{k} g_i(a) + f_{k+1}(a)\sum_{i=1}^{k} g_i(b) - g_{k+1}(a)\sum_{i=1}^{k} f_i(b)$$

$$+ f_{k+1}(a)\, g_{k+1}(b) - g_{k+1}(a)\, f_{k+1}(b)$$

$$\text{R.H.S} = \sum_{i=1}^{k+1} f_i(b)\sum_{i=1}^{k+1}(g_i(b) - g_i(a)) - \sum_{i=1}^{k+1} g_i(b)\sum_{i=1}^{k+1}(f_i(b) - f_i(a))$$

$$=\sum_{i=1}^{k} f_i(b)\sum_{i=1}^{k}(g_i(b)-g_i(a))-\sum_{i=1}^{k} g_i(b)\sum_{i=1}^{k}(f_i(b)-f_i(a))$$
$$+\sum_{i=1}^{k} f_i(b)(g_{k+1}(b)-g_{k+1}(a))-\sum_{i=1}^{k} g_i(b)(f_{k+1}(b)-f_{k+1}(a))$$
$$+f_{k+1}(b)\sum_{i=1}^{k}(g_i(b)-g_i(a))-g_{k+1}(b)\sum_{i=1}^{k}(f_i(b)-f_i(a))$$
$$+f_{k+1}(b)(g_{k+1}(b)-g_{k+1}(a))-g_{k+1}(b)(f_{k+1}(b)-f_{k+1}(a))$$
$$=\sum_{i=1}^{k} f_i(b)\sum_{i=1}^{k}(g_i(b)-g_i(a))-\sum_{i=1}^{k} g_i(b)\sum_{i=1}^{k}(f_i(b)-f_i(a))$$
$$+g_{k+1}(b)\sum_{i=1}^{k} f_i(b)-g_{k+1}(a)\sum_{i=1}^{k} f_i(b))-f_{k+1}(b)\sum_{i=1}^{k} g_i(b)+f_{k+1}(a)\sum_{i=1}^{k} g_i(b))$$
$$+f_{k+1}(b)\sum_{i=1}^{k} g_i(b)-f_{k+1}(b)\sum_{i=1}^{k} g_i(a)-g_{k+1}(b)\sum_{i=1}^{k} f_i(b)+g_{k+1}(b)\sum_{i=1}^{k} f_i(a)$$
$$+f_{k+1}(b)g_{k+1}(b)-f_{k+1}(b)g_{k+1}(a)-g_{k+1}(b)f_{k+1}(b)+g_{k+1}(b)f_{k+1}(a)$$
$$=\sum_{i=1}^{k} f_i(a)\sum_{i=1}^{k}(g_i(b)-g_i(a))-\sum_{i=1}^{k} g_i(a)\sum_{i=1}^{k}(f_i(b)-f_i(a))$$
$$+g_{k+1}(b)\sum_{i=1}^{k} f_i(a)-f_{k+1}(b)\sum_{i=1}^{k} g_i(a)+f_{k+1}(a)\sum_{i=1}^{k} g_i(b)-g_{k+1}(a)\sum_{i=1}^{k} f_i(b)$$
$$+f_{k+1}(a)g_{k+1}(b)-g_{k+1}(a)f_{k+1}(b)$$

Therefore L.H.S = R.H.S

Hence by mathematical induction, it is stated that h(a)=h(b) is true for any integer K.

It is identified that h satisfies the following conditions.

(i) h(x) is continuous on [a,b]
(ii) h(x) is derivable on (a,b)
(iii) h(a)=h(b)

The three conditions of Rolle 's theorem are satisfied. Then there exists at least one $c \in (a,b)$ such that h'(c) = 0

$$h'(x)=\sum_{i=1}^{n} f_i'(x)\sum_{i=1}^{n}(g_i(b)-g_i(a))-\sum_{i=1}^{n} g_i'(x)\sum_{i=1}^{n}(f_i(b)-f_i(a))$$

By the Rolle's theorem, h'(c) =0

$$\sum_{i=1}^{n} f_i'(c)\sum_{i=1}^{n}(g_i(b)-g_i(a))-\sum_{i=1}^{n} g_i'(c)\sum_{i=1}^{n}(f_i(b)-f_i(a))=0$$
$$\therefore \sum_{i=1}^{n} f_i'(c)\sum_{i=1}^{n}(g_i(b)-g_i(a))=\sum_{i=1}^{n} g_i'(c)\sum_{i=1}^{n}(f_i(b)-f_i(a))$$

Hence the proof.

3. CONCLUSIONS

The mean value theorem of n-real valued functions is established with an inductive approach. In addition to this, the mean value theorem for two functions which contain n components each is also proved with the back ground assistance of standard mean value theorems. Few necessary lemmas are also situationally substantiated.

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