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Numerical Solution of Run-Up Flow through a Rectangular Pipe

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Abstract: *The aim of the present investigation is to examine the run-up flow of a viscous incompressible fluid through a pipe whose cross-section is rectangular. The governing equation of the motion under the boundary conditions is solved numerically, by using ADI method. An interesting observation of this investigation is that at large Reynold numbers the velocity distribution along the symmetrical line y = 0 takes the form of a damping wave. At the initial stages the velocity at any nodal point decreases with time is an uneven fashion. The run-up flow also introduces local maxima and minima in the region. At small Reynold numbers the velocity distribution in the region x = 0.2 to x = 0.8 is almost constant for large 't'.*

Key Words: *Run-up flow. ADI method, Rectangular pipe, Reynolds number, Viscous fluid.*

1. Introduction

The problem of Run-up flow of viscous incompressible fluid between rigid boundaries is attracting the attention of researchers due to its importance in several branches of technology. Kazakia and Rivilin [I] and Rivlin [2] have investigated to Run-up flow of viscoelastic fluids between parallel plates and circular geometries. N.CH. Pattabhi Rmacharyulu K.Appala Raju [3] studied Run-up flow in a generalized porous medium.

The aim of the present investigation is to examine the Run-up flow of a viscous incompressible fluid through a pipe whose cross-section is rectangular. Initially the flow is due to constant pressure gradient, when steady state is reached the pressure is suddenly with drawn resulting in Run-up flow, the effect of the Reynold number on the flow field is studied. Unlike closed form solutions in infinite series, this investigation brings out graphically velocity distributions at different times for different Reynold numbers.

2. Mathematical Formulation

Consider the flow of a Newtonian viscous incompressible fluid through a pipe whose cross-section. The centre of pipe is taken as origin and a line parallel to length of pipe through origin is taken as Z-axis. X and Y axis are parallel to sides. The flow is assumed to be symmetrical about X and Y axis. Since the flow is symmetrical the velocity distribution in the first quadrant is investigated. The governing equations are non-dimensionlised and 100 gird points are generated by taking the grid length of 0.1 on either direction. The initial condition which corresponds to fully developed flow under constant pressure gradient is given by

$$W = \frac{pa^2}{2\mu}(1-\bar{y}^2) - \frac{32}{\pi^3}\sum \frac{(-1)^n}{(2n+1)^3} \frac{\cosh(2n+1)\frac{\pi\bar{x}}{2}}{\cosh(2n+1)\frac{\pi}{2}} X \cos(2n+1)\pi\frac{\bar{y}}{2} \qquad (1)$$

Where $\bar{y} = y\frac{a}{b}$ *and* $\bar{x} = \frac{x}{a}$

The velocity at the centre is

$$\overline{w} = \frac{pa^2}{2\mu}[0.41063] \quad (2)$$

Non-dimensionlising (1) using (2), the dimensionless initial condition for velocity up to six digits approximation is given by

$$\overline{W} = \frac{1}{0.41063}[(1-\overline{y}^2) - \frac{32}{\pi^3} X \frac{\cosh\frac{\pi\overline{x}}{2}}{\cosh\frac{\pi}{2}} \cosh\frac{\pi\overline{y}}{2} + \frac{1}{27}\frac{\cosh\frac{3\pi\overline{x}}{2}}{\cosh\frac{3\pi}{2}} \cosh 3\frac{\pi\overline{y}}{2} - \frac{1}{125}\frac{\cosh\frac{5\pi\overline{x}}{2}}{\cosh\frac{5\pi}{2}} \cosh 5\frac{\pi\overline{y}}{2} \quad (3)$$

The governing equations of Run-up flow are

$$\frac{\partial w}{\partial t} = \frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] \quad (4)$$

Non -dimensionlising the above equation by

$$\frac{x}{a} = \overline{x}, \frac{y}{a} = \overline{y}, \ \frac{\overline{w}}{a} t = t', \ and\, w = \overline{w}.\overline{w}$$

it reduces to

$$\frac{\partial \overline{w}}{\partial t'} = \frac{1}{RE}\left[\frac{\partial^2 \overline{w}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{w}}{\partial \overline{y}^2}\right] \quad (5)$$

The equation (5) has to be solved subject to initial conditions, at t = 0, $w = \overline{w}$ and the boundary condition w and the boundary condition w (σ) = 0 for all t, where σ is a boundary of the pipe. ADI method is applied to the equation (5) giving the following finite difference equations

$U_{i, j, n+1} - U_{i, j, n} = k\ [Ui_{-1, j, n+1} - 2\ U_{i, j, n} + U_{i, j, n+1}, U_{i, j-1, n} - 2U_{i, j, n} + U_{i, j+1, n]}$ (6)

$kU_{i-1, j, n+1} - (1+2k)U_{i, j, n+1} + kU_{i+1, j, n+1} = -k\ U_{i, j-1, n}\ (2k-1)U_{i, j, n} - kU_{i, j+1, n}$ (7)

Substituting I= 0 to 9 in equation (7) we get the following linear equations $AU_{(I, j, n+1)} = B$

Where A=

$$\begin{bmatrix} -(1+2k) & 2k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) \end{bmatrix} \begin{bmatrix} U_{(0,j,n+1)} \\ U_{(1,j,n+1)} \\ U_{(2,j,n+1)} \\ U_{(3,j,n+1)} \\ U_{(4,j,n+1)} \\ U_{(5,j,n+1)} \\ U_{(6,j,n+1)} \\ U_{(7,j,n+1)} \\ U_{(8,j,n+1)} \\ U_{(9,j,n+1)} \end{bmatrix}$$

I= I to 9

$B[i] = -K\ U_{(i-1, j-1)} + (2K-1)\ U_{(i-1, j)} - KU_{(i-1, j+1)}$

$B[10] = -K\ U_{(9, j-1)} + (2K-1)\ U_{(9, j)} - KU_{(9, j+1)} - K\ U_{(10, j)}$

$K\ U_{(i, j-1, n+2)} - (1 + 2K)\ U_{(i, j, n+2)} + KU_{(i, j+1, n+2)}$

$= -K\ U_{(i-1, j, n+1)} + (2K-1)\ U_{(i, j, n+1)} - KU_{(i+1, j, n+1)}$ (8)

Substituting = 0 to 9 in equation (8) we get the following linear equations.

$$AI\, U_{(I,j,n+2)} = B'$$

Where A I=

$$\begin{bmatrix} -(1+2k) & 2k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) & k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k & -(1+2k) \end{bmatrix} \begin{bmatrix} U_{(0,i,n+2)} \\ U_{(1,i,n+2)} \\ U_{(2,i,n+2)} \\ U_{(3,i,n+2)} \\ U_{(4,i,n+2)} \\ U_{(5,i,n+2)} \\ U_{(6,i,n+2)} \\ U_{(7,i,n+2)} \\ U_{(8,i,n+2)} \\ U_{(9,i,n+2)} \end{bmatrix}$$

$B'[i] = -K\, U_{(i-1,\, j-1)} + (2K-1)\, U_{(i,\, j-1)} - KU_{(i+1,\, j-1)}$

$B'[9] = -K\, U_{(i-1,\, 8)} + (2K-1)\, U_{(i,\, 8)} - KU_{(i+1,\, 8)} - K\, U_{(i-1,\, 9)}$

3. Results and Discussion

Equations (7) and (8) are solved numerically by giving different values for K which is influenced by Reynold number at different time steps.

Fig (1) shows distribution of velocity of a fully developed flow (K = 0) along lines x = 0, x = $0.3h^2$, x = 0.6h and x = 0.8h. It is evident from the graph that the velocity decreases as we move away from the symmetrical line towards the boundary. Hence the maximum velocity is obtained at the centre of the cross-section. Fig (2) shows variation of velocity at the centre for different Reynold numbers and time steps. The variation of velocity is observed to take a damping wave form, for large Reynold numbers. The variations are almost smooth for small Reynold numbers. Fig (3) shows the variations of velocity at time N = I on the line y = 0 for different Reynold numbers. There are large variations in the region x = 0, to x = 0.1, where the fluid particle attains local minimum velocity at x = 0.1 and a maximum at x = 0.2. The flow is almost smooth in the region x = 0.3 to x = 0.7. The velocity is found to decrease with the increase of Reynold number.

Fig (4) and (5) show that with lapse of time i.e. N = 6 and N = 8 and K = 0.2 the velocity gradually decreases from the centre and attains minimum at x = 0.1 and increases from there and velocity profile is almost parabolic obtaining maximum round about at x = 0.4. The same phenomena is observed when K = 0.3 (figures 6 and 7). Fig (6) and (7) show that at the beginning stages of Run-up flow the velocity distributions is non-symmetrical. Fig (8) and Fig (9) show that when Reynold number is small the velocity distribution is almost symmetrical about x = 0.1 and x = 0.4. Fig (10) and Fig (11) show the decrease of velocity with time for a given Reynold number. It can be observed that the fall in velocity in the region from x = 0.2 to x = 0.7 is very large while in other region it is gradual. An interesting observation of this investigation is that at large Reynold numbers the velocity distribution along the symmetrical line y = 0 takes form of a damping wave. At the initial stages the velocity at any nodal point decreases with time in an uneven fashion. The Run-up flow also introduces local maxima and minima in the region. At small Reynold numbers the velocity distribution in the region x = 0.2 to x = 0.8 is almost constant for large t.

4. Conclusions

The maximum velocity is obtained at the centre of the cross-section. For large Reynold numbers, the variation of velocity is takes a damping wave form. At the initial stages the velocity at any nodal point decreases with time in an uneven fashion. The Run-up flow introduces local maxima and minima in the region. At small Reynold numbers the velocity distribution in the region x = 0.2 to x = 0.8 is almost constant for large t. Velocity profile is almost parabolic obtaining maximum round about the centre.

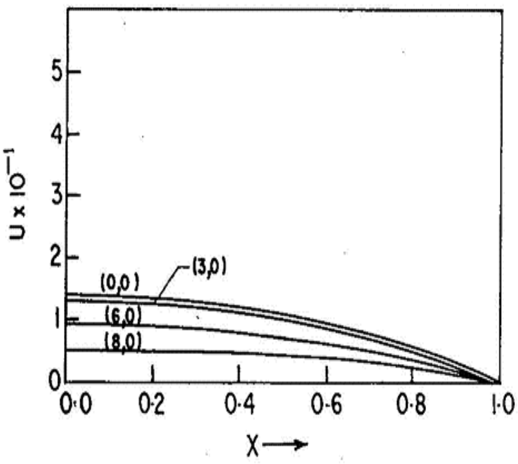

Fig.1

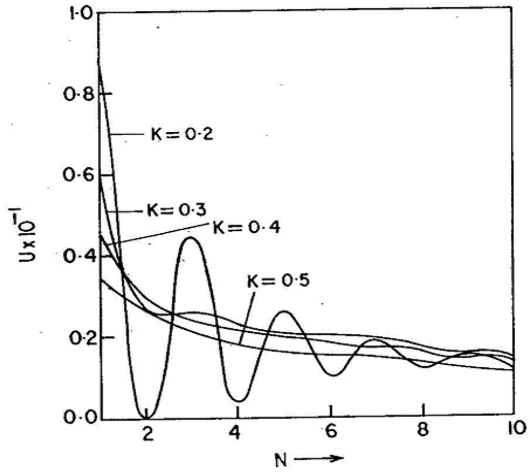

Fig.2

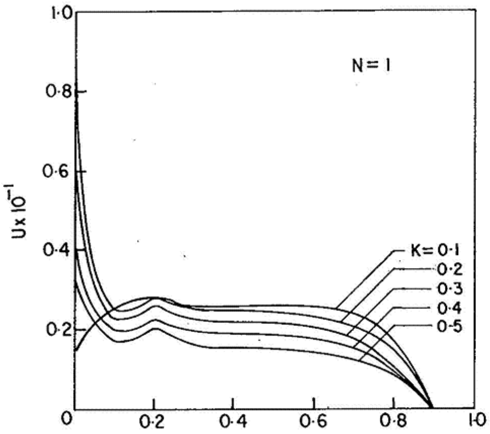

Fig.3

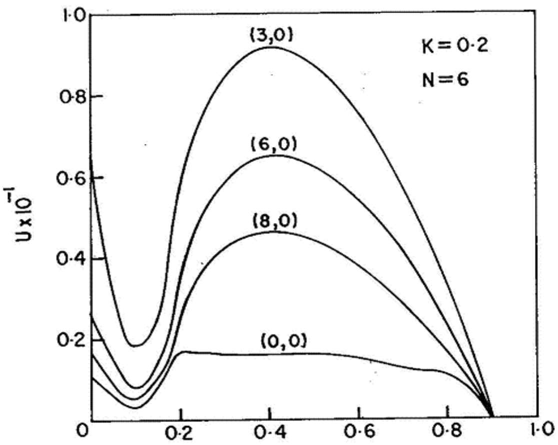

Fig.4

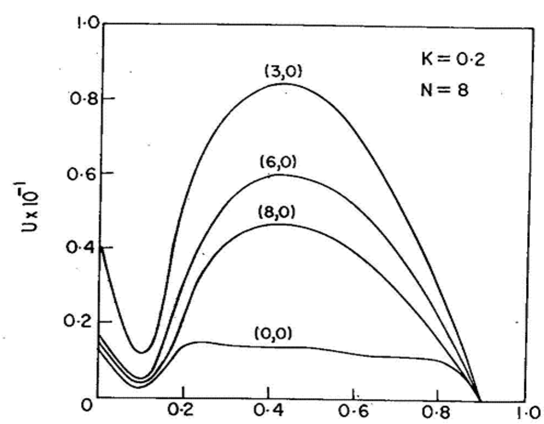

Fig.5

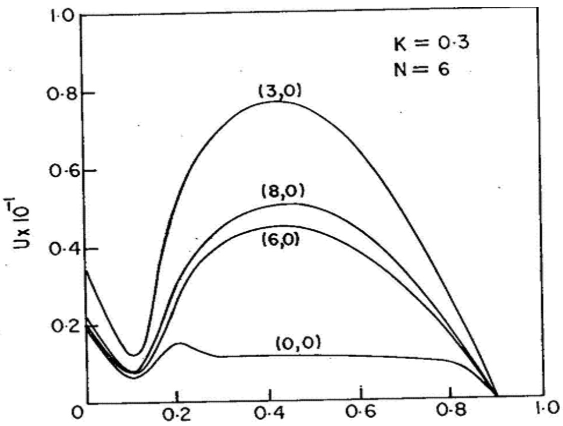

Fig.6

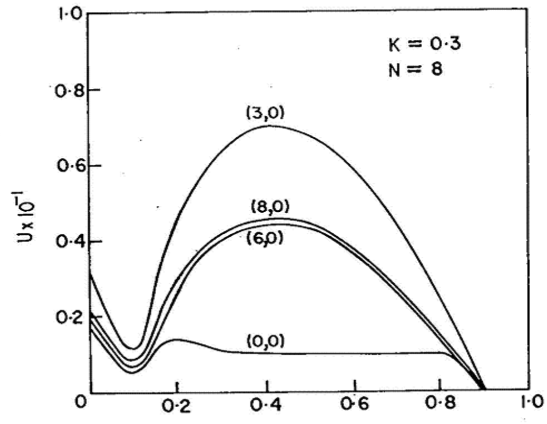

Fig.7

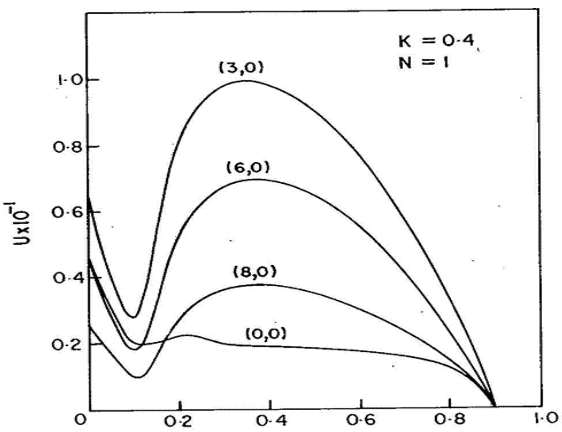

Fig.8

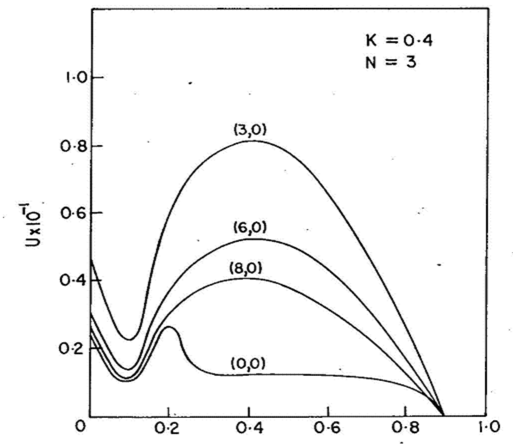

Fig.9

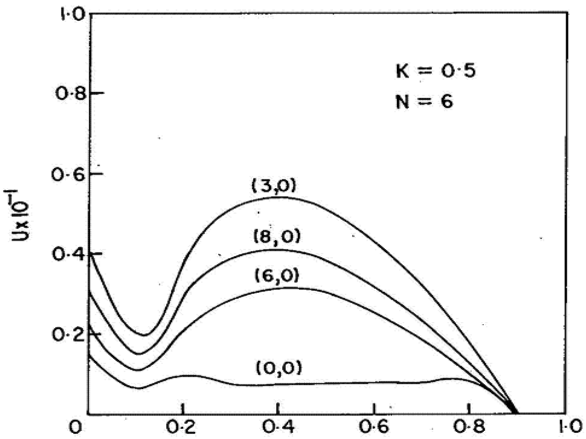

Fig.10

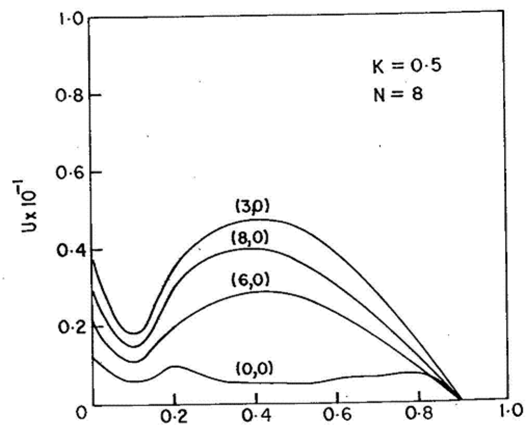

Fig.11

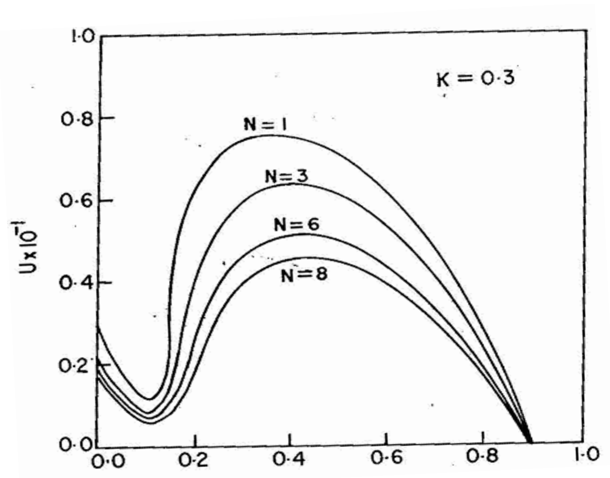

Fig.12

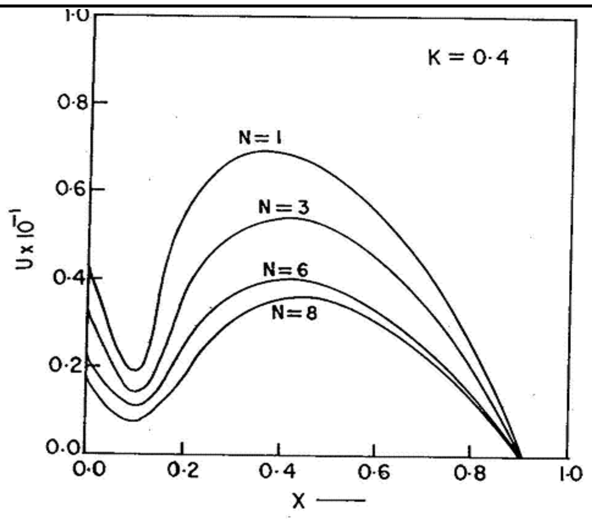

Fig.13

References

[1] Kazakia, J.Y., and Rivlin, R.S., Run-up and spin-up in a viscoelastic fluid-1, Rheologia Acta. 20 (1981).
[2] Rivlin, Run-up and spin-up in viscoelastic fluids - III, Rhelogica Acta. 21, 213 (1982).
[3] Pattabhi Ramacharyulu N.Ch., and Appala Raju K., Run-up flow in a generalized porous medium, Indian J. Pure and Appied Maths. 15(6), 665 (1984).
[4] M. K. Jain, Numerical solution of Differential Equations, Willey Eastern Limited, New Delhi.
[5] G. D. Smith, Numerical Solutions of Partial Differential Equations, Oxford University Press, London. 1965.
[6] D. Samuel, I. Conte, Carl de Boor, Elementary numerical analysis.
[7] F. Ames William, Numerical methods for partial differential equations.
[8] S. Dorn William and Daniel D Me Cracken, Numerical Methods with Fortran IV cast studies.