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An Integral involving Gauss's Hypergeometric Function of the Series ${}_1F_1$

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Abstract: *In this present work our main aim is to obtain integral involving Hyper geometric function of the series ${}_1F_1$ by employing one of the integral obtained by MacRobert. Main interesting result of this research paper is that it comes out in the products of the ratio of the Gamma function with Special Cases.For the application point of view integral comes in terms of Gamma function is very usefulin engineering Applications. On specializing the parameters, we can easily obtain some new integrals by rathie and others which are given in Book of Mathai and Saxena.*

Key Words: *Generalized Hypergeometeric functions, Gamma function and integrals.*

1. INTRODUCTION

The definition of the Gauss's Hypergeometric Series [6] and

denoted by ${}_2F_1\left[\begin{matrix}a,b\\c\end{matrix}\middle|\, z\right]$ *which can be further written as*

$$1+\frac{a.b}{c}\frac{z}{1!}+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!}+......=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!} \quad ....(1.1)$$

For a = 1 & b = c or b = 1 and a = c, the series (1.1) reduced to the well known geometric series.and for a = 0 and b = 0 or both zero, the series becomes unity.If a or b or both are negative integers , the series becomes polynomial.Also, if we take p = q =1, the generalized Hypergeometric function reduces to confluentHypergeometeric function [5], given as

$${}_1F_1\left[a;b\middle|z\right]=\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{z^n}{n!} \quad ....(1.2)$$

The result will be defined with the help of known and interesting result by Macrobert [1].

The aim of this paper is Providing an integral invovingHypergeometric function .few interested well known results have been obtained as a limiting cases of main result.

2. RESULT REQUIRED

In our present investigation we use the following intrestring result by Macrobert [1]

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta} dx=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)},......(2.1)$$

Re(α)>0,Re(β)>0.Provided Re(α)>0,Re(β)>0 and a,b are non zero constants and expression[ax+b(1-x)]; where 0 ≤ x ≤ 1 is not zero.

$$\text{and} \quad (a)_{2n} = 2^{2n}\left(\frac{1}{2}a\right)_n\left(\frac{1}{2}a+\frac{1}{2}\right)_n \quad ....(2.2)$$

3. MAIN RESULT

In this section we evaluate integral involving confluent hypergeometricfunction

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta}\ {}_1F_1\left[\begin{matrix}a\\ b\end{matrix}\middle|\frac{4abx(1-x)}{\left[ax+b(1-x)\right]^2}\right]dx$$

$$=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ {}_3F_3\left[\begin{matrix}a, & \alpha, & \beta \\ b, & \frac{\alpha+\beta}{2}, & \frac{\alpha+\beta+1}{2};\end{matrix}\middle| 1\right] ....(3.1)$$

Provided Re(α) > 0, Re(β) > 0 and Re(2b - 2a) > -1.Also a,b are non zero constants and expression[ax + b(1 - x)]; where 0 ≤ x ≤ 1 is not zero

4. DERIVATION

Above result (2.1) can be express as follows.We will express the R.H.S.of (2.1) is I.after that we expressing ${}_1F_1$ as a series with the help of (1.2),we get

$$I=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta}\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{4^n a^n b^n x^n(1-x)^n}{\left[ax+b(1-x)\right]^{2n} n!}dx$$

By changing the Summation and order of integration which is uniformaly convergence in the interval (0,1),we have

$$=\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{2^{2n}a^n b^n}{n!}\int_0^1 x^{n+\alpha-1}(1-x)^{n+\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta-2n}dx$$

using MacRobert's [1] result and evauating we get

$$=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{(\alpha)_n(\beta)_n}{(\alpha+\beta)_n}\frac{2^{2n}}{n!}$$

Now using the result (2.2), we have

$$=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{(\alpha)_n(\beta)_n}{\left(\frac{\alpha+\beta}{2}\right)_n\left(\frac{\alpha+\beta}{2}+\frac{1}{2}\right)_n n!}$$

Hence we obtain main result

$$I=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ {}_3F_3\left[\begin{matrix}a, & \alpha, & \beta \\ b, & \frac{\alpha+\beta}{2}, & \frac{\alpha+\beta+1}{2}\end{matrix}\middle| 1\right]$$

5. Special Case

Here we will discuss one intrestringnew result as a special case

If we put $a=\varphi+\frac{1}{2}$ *and* $b=\varphi$ *in* (3.1), *we get*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta}\ {}_1F_1\left[\begin{matrix}\varphi+\frac{1}{2}\\ \varphi\end{matrix}\middle|\frac{4abx(1-x)}{\left[ax+b(1-x)\right]^2}\right]dx$$

$$=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ {}_3F_3\left[\begin{matrix}\varphi+\frac{1}{2}, & \alpha, & \beta\\ \varphi, & \frac{\alpha+\beta}{2}, & \frac{\alpha+\beta+1}{2}\end{matrix}\middle| 1\right] \quad .....(5.1)$$

Now in (5.1), *if we put* $\phi=\frac{1}{2}(\alpha+\beta)$, *and* $\phi=\beta$, *we obtain*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta}\ {}_1F_1\left[\begin{matrix}\frac{1}{2}(\alpha+\beta+1)\\ \beta\end{matrix}\middle|\frac{4abx(1-x)}{\left[ax+b(1-x)\right]^2}\right]dx$$

$$=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\ {}_1F_1\left[\begin{matrix}\alpha\\ \frac{1}{2}(\alpha+\beta)\end{matrix}\middle| 1\right] \quad ....(5.2)$$

Using Classical Gauss's theorem, we get

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\left[ax+b(1-x)\right]^{-\alpha-\beta}\ {}_1F_1\left[\begin{matrix}\frac{1}{2}(\alpha+\beta+1)\\ \beta\end{matrix}\middle|\frac{4abx(1-x)}{\left[ax+b(1-x)\right]^2}\right]dx$$

$$=\frac{1}{a^{\alpha}b^{\beta}}\frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+n)\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma(\alpha+\beta)\Gamma\left(\frac{\alpha+\beta}{2}+n\right)} \quad ....(5.3)$$

6. Overall Conclusions

By using the Hyper geometric Function of ${}_pF_q$ in known definite integrals obtained by MacRobert[1] we are getting more interested results in terms of ${}_{p+2}F_{q+2}$ and again some specializing the parameters, we gets further more interesting results. If we employing the integral presented in paper, we may evaluate large no. of integrals involving I,G and H-functions,recently introduced by Rathie[14].

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