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Examining the Effects of Number Sense Instruction on Mathematics Competence of Kindergarten Students

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Abstract: *This study examined the impact of number sense instruction and general classroom instruction on the development and maintenance of mathematics skills of kindergarten students. Participants included kindergarten students from nine classrooms in two elementary schools located in suburban school districts in northeastern United States. The nine classrooms (N = 135) were randomly assigned to either the intervention (Number Sense, NS) group or the comparison (General Classroom Instruction, GCI) group. Overall, results indicated that both groups made significant improvements and maintained them overtime.*

Keywords: *Number Sense, Kindergarten.*

1. Introduction

A strong conception of number and the quantity it represents is a critical part of all areas of daily life. Despite the importance of mathematics competence in daily life, mathematical difficulties are widespread (Doughtery, 2003; Gross-Tsur, Manor, & Shalev, 1996; Murnane, Willett, & Levy, 1995; Ostad, 1998). In addition, there is insufficient research to inform us about instructional approaches that best address the needs of students who are at risk for mathematics failure (Baroody, 1991; Francis, Rivera, Lascaux, Kieffer, & Rivera, 2006; Gersten & Chard, 1999). Despite broad-based concern and a clear direction for reform, progress has been relatively slow, particularly in relation to objectives such as enhancing number sense and lowering the performance gap between advantaged and disadvantaged children (Griffin & Case, 1997). Thus, it is important to design effective interventions for children who are developing or likely to develop problems in mathematics (Dowker, 2005).

Competence in mathematics depends heavily on appropriate and effective instruction, and on opportunities to learn. There are skills that must be developed in the early years (as early as kindergarten) for success in mathematics. Similarly, there is evidence that early intervention can prevent significant difficulties for many learners (Berch, & Mazzocco, 2007; Gersten, Jordan, & Flojo, 2005), and number sense is one of the most important skills necessary for success with basic mathematical computations in the early grades (Berch, 2005; Gersten & Chard, 1999). The term number sense is often used as an umbrella term to describe a well-developed understanding of the concept of number. It refers to "a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (Mcintosh, Reys, & Reys, 1992, p. 3). It also reflects students' early experiences, as well as their cognitive facility (Dowker 2005; Lipton & Spelke, 2003). Researchers believe that a firm understanding of numbers and number systems is central to math learning and that instruction including number sense activities leads to significant reductions in early mathematics failure (e.g., Griffin, Case, & Siegler, 1994).

Given that number sense serves as the foundation for learning formal math concepts and skills, it seems essential to identify interventions, specifically acquisition and/or development of number sense, that support students early in order to prevent later failure in mathematics. Research clearly indicates that lack of number sense can causally affect students' mathematics performance.

Traditionally, mathematics instruction has stressed mastery of algorithms, rote memorization, and repeated practice (Gersten & Chard, 1999). As a result children develop a superficial and procedurally based understanding of mathematics, instead of in-depth knowledge of numbers and the value it represents (Ginsburg & Russell, 1981; Hiebert, 1986; Resnick, 1987; Schoenfeld, 1987).

Procedurally based interventions lie at on one end of a continuum of learning and instruction and focus on executing procedures to solve problems whereas conceptual interventions lie on the other end of the continuum and emphasize understanding of mathematics principles (Bisanz & LeFevre, 1992). Current research findings have suggested that both types of interventions are equally important to promote competence in a domain, because conceptual and procedural knowledge can develop simultaneously (Rittle-Johnson, Siegler, & Alibali, 2001; Sood & Jitendra, 2007, 2013).

The theoretical framework for number sense instruction draws on cognitively guided instruction. Cognitive learning theorists stress the importance of abstraction and concept formation. They view learning from the perspective of the individual and emphasize the interactions between the internal representations of the learner and the external representations in the environment.

Cognitive theorists believe that knowledge is constructed and resides in the mind of the learner as he/she interacts with the environment and reflects upon prior knowledge. Further, learning is dependent upon the context in which it is situated. The social environment becomes the primary unit of analysis, with the individual learner as a participant interacting with others in the social environment. The purpose of instruction is not merely to acquire skills, but to help students become participants in a mathematical community. The development of conceptual understanding involves the development of the ability to construct and reason with mental models. In this theory, modeling takes precedence over abstraction.

Many students fail to spontaneously transfer learned strategies to tasks or situations different from those in the training setting (Chan, 1991; Chan, Cole, & Morris, 1990; Day & Zajakowski, 1991). The advantage of explicit instruction over guided discovery is especially critical for students who fail to make the transition (Moreno, 2004; Tuovinen & Sweller, 1999). According to Gersten, Jordan, and Flojo (2005), explicit instruction for children with mathematics difficulties can be integrated with inquiry based instruction and these approaches do not need to be mutually exclusive. In fact, the National Research Council (2001) suggested that a combination of explicit instruction with open-ended problem solving and teacher-facilitated instruction should be implemented as part of the National Council of Teachers of Mathematics' (NCTM, 2000) reform-based mathematics instruction. In the past decade, a few researchers have developed interventions for students who struggle with early numeracy skills (e.g., quantity discrimination, counting, and number identification) that are most predictive of mathematics failure (Funkhouser, 1996; Griffin, Case, & Siegler, 1994; Griffin, Case, & Capodilupo, 1997; Markovitz & Sowder, 1994; Sood & Jitendra, 2013; Yang, 2003).

Based on a review of the empirical research on number sense, it is evident that number sense is an important prerequisite skill necessary for later mathematics achievement. Number sense instruction was found to be effective not only for older students, but also for kindergarten and preschool students. Studies conducted in the past have focused on a variety of components of number sense. While some studies focused on the development of computational skills (Funkhouser, 1996; Markovits & Sowder,1988; Markovits & Sowder, 1994), others emphasized conceptual understanding of the number sequence, number concepts, one-to-one correspondence, cardinal values of numbers, magnitude of numbers, and benchmarks (Arnold, Fisher, Doctoroff, & Dobbs, 2002; Aunio, Hautamaki, & Van Luit, 2005; Griffin, Case, & Siegler, 1994; Griffin, Case, & Cupodilupo, 1997; Yang, 2002; 2003; Young-Loveridge, 2004). Sood and Jitendra (2013) focused on all aspects of big ideas of number sense.

Research clearly indicates the importance of number sense instruction. It is important to emphasize explicit strategy instruction with open-ended problem solving and teacher mediated instruction to help them understand the big ideas of number sense (National Research Council, 2001). Although there are several early mathematics programs that emphasize explicit knowledge and others that take a more child-centered approach, there are few programs that combine both of these approaches (Griffin & Case, 1997). Sood and Jitendra (2013) is the only study that has examined the impact of instruction on big ideas of number sense on the mathematics performance of kindergarten students.

Given that problems with number sense may have a causal influence on students' math learning difficulties and there is a lack of empirical studies that have examined the effectiveness of number sense interventions for kindergarten students with math difficulties, the purpose of the present study was to evaluate the effectiveness of number sense instruction on the acquisition and maintenance of mathematics competence by kindergarten students.

2. METHODS

2.1. Participants and Materials

Participants were kindergarten students from nine classrooms in two elementary schools, located in suburban school districts in the northeastern Unites States. The sample consisted of 135 students. Seventy-seven (39 female; 38 male) students from five classrooms comprised the number sense (NS) group and 58 (27 female; 21 male) students from four classrooms were in the general classroom instruction (GCI) group. The mean age of the students was 66.36 months.

Table1. *Demographic Description and Pretreatment Group Equivalency Chi Square Results of Participants in the NS + GCI and GCI Group*

	NS		GCI		χ^2	*df*	*t*
Gender Male Female Total	 38 39 77		 31 27 40		.102	1	-
Mean Age in months (SD)	65.22 (4.32)		67.49 (3.90)	-	-	-	2.68
Ethnicity Caucasian Others	 27 50		 25 33		3.745	1	-

Note. NS + GCI = Number Sense, GCI = General Classroom Instruction; df = degrees of freedom

*p<.05

Table 1 provides demographic data for the NS + GCI and GCI groups. A one-way analysis of variance (ANOVA) indicated no statistically significant differences between the two groups, on SESAT, F (1, 99) = 2. In addition, there were no statistically significant differences between groups on oral counting fluency, F (1, 99) = 0.00, counting from, F (1, 99) = 0.10, number identification, F (1, 99) = 0.01, spatial relationships, F (1, 99) = 0.01, number relationships, F (1, 99) = 1.09, five and ten frame identification and representation, F (1, 99) = .32, five and ten frame calculation, F (1, 99) = .27, and nonverbal calculation, F(1, 99) = .98. Chi square analysis also revealed no statistically significant between group differences on gender, x^2 (1, N = 101) =.10, ethnicity, x^2 (1, N = 101) = 3.75, and SES, x^2 (1, N = 101) =1.93. There was a significant difference in the NS + GCI and GCI group on disability category and IEP. Given that from the overall sample only 3 students, all in the GCI group, had a documented disability and an accompanying IEP, these results are expected. In addition, results of an independent t-test conducted on age scores indicated a statistically significant difference between groups (t (99) = 2.68, p = .009) between the two groups. The mean age in months of students in the GCI group (M= 67.49, SD = 3.90) was higher than the mean age of students in the NS + GCI group (M= 65.22, SD = 4.32).

Nine female teachers provided all instruction in the study. Seven teachers were Caucasian and two were African-American with teaching experience ranging from 7 to 29 years. All teachers were certified in elementary education and two teachers were also certified in special education. Teachers in the study were randomly assigned to the two conditions. Teachers assigned to implement the number sense program, received a 2 hour in-school workshop on implementing the program. The workshop provided a rationale for, and content of, the number sense program. Teachers reviewed the teaching scripts as they were modeled and discussed by the researchers. Further, teachers were encouraged to study the scripts and implement them rather than read them verbatim. All questions and concerns regarding the lessons were clarified during the training. To avoid potential problems of diffusion of treatment effects, teachers in the treatment group were requested not to talk about number sense curriculum with their colleagues until after the intervention was completed. The four teachers assigned to the comparison groups continued to implement the standard district assigned curriculum.

2.2. Treatment Conditions

2.2.1. Both Conditions

In all classrooms, mathematics was taught five times a week using the district adopted curricula. Students in both conditions received a total of 60 min of math instruction, five days a week. Both conditions received 40 min of math instruction based on the district assigned curriculum. For the remaining 20 min students in the NS + GCI group received instruction based on the number sense program, whereas students in the GCI group continued to receive math instruction based on the district assigned program. To ensure comparability across conditions, the math instruction was identical in length.

2.2.2. NS Condition

Students received instruction based on the number sense program. The program based on the big ideas of number sense which included number relationships (spatial relationships, one more, one less, two more, and two less, benchmarks of five and ten, and part-part whole relationships) (Van de Walle, 2007). In addition, this program adopted a combination of explicit and cognitive instruction and followed the model, lead, guided practice, and independent practice instructional paradigm that allowed for the development of both procedural and conceptual knowledge (Gersten & Chard, 1999; Kame'enui & Carnine, 2002; Robison, Menchetti, & Torgesen, 2002; NCTM, 2000). Activities for this program were researcher designed or adapted from a variety of resources (published mathematics curricula and books) (Bell, Bell, Bretzlauf, Dellard, Hartfeild, Isaacs, et al., 2004; Columba, Kim, & Moe, 2005; Van de Walle, 2007) and included various formats (whole class, small group, partner work, and individual assignments) and materials (concrete, semi-concrete, and abstract).

The program consisted of four units. The first two units included five lessons each and the last two units included four lessons each. Each unit focused on the four big ideas of number sense: (a) spatial relationships, (b) one more, one less, and two more, and two less, (c) benchmarks of five and ten, and (d) part-part whole relationships. Table 2 presents the sequence of lessons in the number sense program.

Table2. *Sequence of Lessons in Number Sense Program*

Unit Number	Topic
Unit 1	Spatial relationships
Lesson 1	Dot patterns (1,2, 4, 6, 8,10)
Lesson 2	Dot patterns (3,5, 7,9)
Lesson 3	Combining smaller patterns to make bigger patterns (2, 3, 4, 5, 6)
Lesson 4	Combining smaller patterns to make bigger patterns (7, 8, 9, 10)
Lesson 5	Review
Unit 2	One More, One Less; Two More and Two Less
Lesson 1	One More
Lesson 2	Two More
Lesson 3	One Less
Lesson 4	Two Less
Lesson 5	Review
Unit 3	Benchmarks of Five and Ten
Lesson 1	Five Frame (Identification and Representation)
Lesson 2	Ten Frame (Identification and Representation)
Lesson 3	Five Frame (Calculation)
Lesson 4	Ten Frame (Calculation)
Unit 4	Part-Part Whole Relationships
Lesson 1	Part-Part Whole for numbers 2 to 4
Lesson 2	Part-Part Whole for numbers 5 and 6
Lesson 3	Determining missing part when whole and one part is known.
Lesson 4	Review + Additional Practice

The first unit focused on spatial relationships. The objective of this unit was to teach students to recognize sets of objects in patterned arrangements and tell how many there are without counting. Knowledge of spatial relationships is important because it encourages reflective thinking and allows students to see how numbers are related (Van de Walle, 2007). The first two lessons in this unit focused on introducing dot patterns for numbers 1-10. During these lessons, the teacher displayed dot cards for 3s and had students identify the number and describe the displayed pattern. For example, if the dot pattern displayed represented the number 2, students first stated the number "2" and then described the pattern in terms of the placement of the dots. The description for the pattern could be "I noticed that the 2 dots were placed in a straight line one below the other." After having introduced the patterns, the teacher showed students the dot pattern chart with all the possible representations of the numbers introduced. In Lessons 3 and 4, students were presented with two number patterns/quantities. They learned how to combine the smaller patterns/quantities to form one big number pattern/quantity. Before introducing number combinations, the counting from strategy was reviewed to ensure that all students knew how to count on from a given number. After reviewing the counting from strategy, students were shown two dot cards. Each dot card was displayed for 3s.When students had identified the numbers; the cards were placed where all students could see them. The teacher then modeled how to combine the patterns. Lesson 5, which was the last lesson in this unit, was a review lesson. In this lesson, students reviewed recognizing number patterns/quantities and combining two smaller patterns to form a big number pattern/quantity. After the review, students played a game called "How Many?" Students played this game in pairs and each student was given a set of dot cards (1-5). When the teacher provided a cue, students turned over their top card and placed it on the table. They then combined the two smaller patterns/quantities to form a big pattern/quantity.

The second unit included five lessons that focused on the big idea of one more, one less, two more, and two less. The primary objective of this unit was to teach students how numbers are related to one another. For example, 8 is 1 more than 7 or 2 less than 10. This is important because activities that emphasize the basic relations (i.e., *more, less, same)* between numbers involve more than just the ability to count. Instead, the focus is on understanding that all numbers are related to one another in a variety of ways (Van de Walle, 2007). The first lesson in this unit introduced the concept of "one more" and allowed students to see how numbers were related to one another (e.g., 6 is 1 more than 5). The "One More Activity" that introduced the concept required students to recognize the dot pattern presented and then use counters to show a pattern that was one more than the presented pattern. For example, if the dot pattern presented represented the number 5, students used their counters to make a pattern that represented the number 6, which is one more than 5. This was followed by comparing the two patterns to see how the two numbers were related. For example, 6 is one more than 5. Lesson 2 introduced "two more." This lesson also had students identify a dot pattern/quantity and then use counters to display the number that was two more than the presented number. For example, 5 is 2 more than 3. Students were then required to solve "Two More than Stories." The lesson ended with a predicting game, in which students learned how to make the calculator a Two-More-Than-Machine by pressing "0 + 2 =." Lessons 3 and 4 focused on "One less and Two less" and followed the same sequence of activities as Lessons 1 and 2. In Lesson 3, students read the book *Ten, Nine, Eight* by Molly Bang and in Lesson 4, they made the calculators a Two-Less-Than Machine to play the prediction game. The last lesson reviewed all the four concepts of one more, one less, two more, and two less. In this lesson, students had the opportunity to make up their own number story.

The third unit comprised four lessons that focused on the benchmarks of 5 and 10. Developing an ability to relate a number to a benchmark or criterion (such as 5 or 10) has been proven to improve students' acquisition of mathematical concepts (Sowder & Schappelle, 1994). The number 10 plays a significant role in the numeration system. Since two fives total 10, both 5 and 10 can be valuable benchmarks for students to utilize when understanding relationships of numbers less than 10, as students can consider different combinations of numbers (e.g., 8 is "5 and 3 more" and "2 away from 10"). These anchors also help students develop an ability to mentally compute larger numbers (Van de Walle, 2007). The first lesson in this unit had students identify a number and then represent that number on a five-frame. Students were then asked to identify the number of boxes that contained a dot and the number of boxes that were empty. Lesson 2 introduced the tens-frame and followed the same sequence of activities as Lesson 1. In Lesson 3, students were presented with five frames representing numbers from 1-5 and were required to first identify the number and then state how many more counters were needed to make 5. Lesson 4 followed the same sequence of activities as Lesson 3, but instead of using five frames to make 5, students learned how to use a ten-frame to make 10. They also looked at an array often-frame cards to find two ten-frame cards that made ten, and solved 'make 10' story problems.

The last unit in the number sense program focused on part-part whole relationships. The most significant skill of early understanding of mathematics involves the interpretation of numbers in terms of *part-part-whole relationships* (Fischer, 1990). This relationship requires knowledge that a number consists of two or more parts. For example, 7 can be thought of as a set of 5 and a set of 2 or a set of 1 and a set of 6. An activity in this lesson was called "Divide the Counters." This activity required students to divide a given set of counters into two smaller parts. For example, each student was given 3 counters and they were required to first divide the counters into two parts and then describe their arrangement (e.g., 3 is made up of 2+1, 1+2, 3+0, or 0+3). This activity was continued with 2 and 4 counters. Lesson 2 followed the same sequence of activities, but with numbers 5 and 6. Lesson 3 focused on part-part whole relationships and incorporated practice opportunities for students. Lesson 4 allowed for extra practice on part-part-whole relationships. An activity in this lesson not only allowed students to practice "part-part whole" combinations, but also allowed them to use the calculator to check their work. For this activity, students learned how to make a "Calculator Parts of 6 Machine" by pressing the keys "6 - 6 =" keys.

2.3. Measures and Data Collection

All participants completed: (a) the Stanford Early Achievement Test (SESAT) the kindergarten section of the Stanford Achievement Test - 10 (SAT-10) and a set of Early Numeracy-Curriculum Based Measures (EN-CBM) and Number Sense Measures prior to the instruction (pretest), (b) the EN-CBM and Number Sense Measures at posttest immediately following instruction, and (c) the EN-CBM and Number Sense Measures as the maintenance test three weeks after termination of the instruction. Six research assistants were trained to administer and score the EN-CBM, and Number Sense Measures with scripted directions and answer keys. Classroom teachers conducted the SESAT.

2.3.1. SESAT

SESAT is the kindergarten section of the SAT 10 (Stanford Achievement Test -10). The test includes 40 multiple choice questions. Some of the questions require students to compare numbers and sets up to 10, count forward or backward from an initial number, identify ordinal position, identify the number of elements in a set having up to 10 elements, match number names and notation, solve problems using numerical reasoning, translate between visual representations, sentences, and symbolic notation, extend a visual pattern, identify missing elements in a visual pattern, identify possible outcomes, read and interpret tables and graphs, identify coins, compare estimates of capacity, estimate length using non-standard units, and so forth. It has adequate validity (r = .64) and reliability (r = .88) (Clarke, Baker, Smolkowski, & Chard, 2008).

2.3.2. Early Numeracy-CBM and Number Sense Measures (EN-CBM)

The EN-CBM (EN-CBM; Clarke & Shinn, 2004) consisted of: (a) Oral Counting Fluency, (b) Counting From, and (c) Number Identification. These measures were selected from commonly used early numeracy curriculum based measures. They were selected keeping in mind the age of students participating in this study.

Oral counting fluency (Clarke & Shinn, 2004) was a 1 min probe that assessed the ability to orally state numbers in sequence beginning with number 1. For this measure, students were asked to state numbers in sequence until they reached the highest number they could state in 1 min. Each correctly identified number got one point, and no points were assigned if students failed to identify or incorrectly identified a number. The score ranged from a minimum of 0 to a maximum of 100. The criterion measures to determine validity were the SESAT and the Number Knowledge

Test (NKT). Concurrent validity correlations for this measure range from .49 to .70 and predictive validity correlations range from .46 to .72 (Clarke, Baker, Smolkowski, & Chard, 2008).

Counting from (Chard, et al., 2005) is a measure that requires students to count from a given number, with a maximum of 5 correct in a sequence. For example, students were required to count orally starting from 3 for 5 numbers (i.e. to 8). This measure comprised of two items (e.g., counting from 5 and counting from 3). Each correctly identified number was assigned one point and no points were assigned if students failed to identify or incorrectly identified a number. The score ranged from a minimum of 0 to a maximum of 10. Reliability and validity information of this measure was not found in existing research. Hence the Cronbach's alpha reliability for this measure was calculated based on the sample of this study (.97 at pretest, .99 at posttest and .97 at delayed posttest).

Number identification (Clarke & Shinn, 2004) also was used. Based on previous research and keeping in mind that students were just starting kindergarten this measure required students to identify numbers between 0 and 10 (Clarke, Baker, Smolkowski, & Chard, 2008; Gersten, Clarke, & Jordan, 2007). For this measure, students were presented with a sheet of randomly selected and ordered numbers that they had to identify. Each correctly identified number was assigned one point and no points were assigned if a student failed to identify or incorrectly identified a number. The score ranged from a minimum of 0 to a maximum of 50. The criterion measures to determine validity were the SESAT and the NKT. Predictive validity correlations for the modified version ranged from .62 -.67 for kindergarten students.

2.3.3. Number Sense Measures

The number sense measures included a set of four curriculum-based measures designed to assess student's development of number sense. The number sense measures included the following: (a) Spatial relationships, (b) Number relationships, (c) Benchmarks of Five and Ten; and (d) Nonverbal Calculations. These measures were developed by the investigator, keeping in mind the concepts covered in the number sense program. Items in these measures were generated by conducting a thorough search of existing curricula (Scott Foresman, Investigations, Houghton Mifflin, and Everyday Mathematics), other existing measures, research articles, books, and by consulting with experts.

Spatial relationships targeted students' ability to recognize dot patterns (1-10) and state the number within 3 seconds of its presentation. Examiners presented dot cards in a rapid succession displaying each card for 3 seconds and recorded student responses to each pattern. If a student provided an incorrect response, no corrective feedback was provided. If students did not provide a response within 3 seconds, the next dot card was presented. This measure comprised 10 items, and each correctly identified pattern got one point. No points were assigned if students failed to identify or incorrectly identified a pattern. The score ranged from a minimum of 0 to a maximum of 10. The Cronbach's alpha reliability, based on the sample in this study was .74 at pretest, .70 at posttest and .55 at delayed posttest.

Number relationships required students to state the number that is one more, one less, two more, or two less than the given number ranging from 1-10. This measure comprised 20 items, 5 items each for the four targeted skills. Numbers that were skipped or not correctly identified were marked as incorrect. Each correctly identified number was given one point and no points were assigned if students failed to state the correct answer. The score ranged from a minimum of 0 to a maximum of 20. The Cronbach's alpha reliability, based on the sample in this study, was .93 at pretest, .97 at posttest and .96 at delayed posttest.

Benchmarks of five and ten was divided into two parts. Students were first required to identify the number represented on the five- and ten-frame, and then use counters to represent the same number on a five- or ten-frame. Next, students were required to determine how many more counters would be needed to make five or ten when presented with a five- or ten-frame. This measure comprised 6 items, three items each for the five and ten frames. Numbers that were skipped or not correctly identified were marked as incorrect. Each correct item received two points for identification and representation, and one point for calculation. No points were assigned if students failed to respond or state the incorrect answer. The scores ranged from a minimum of 0 to a maximum to 12 points for identification and representation and from a minimum of 0 to a maximum of 6 points for calculation. The Cronbach's alpha reliability for five and ten frame identification and representation, based on the sample in this study, was .64 at pretest, .80 at posttest and .70 at delayed posttest. For five and ten frame calculation, the Cronbach's alpha was .80 at pretest, .89 at posttest, and .90 at delayed posttest.

Nonverbal calculation required students to solve four addition and four subtraction problems using counters. The measure began with three warm-up trials that were administered prior to the assessment. During these trials, students were asked to identify the specific number of counters on the mat. The warm-up trials helped students practice what they would be required to do on the test and also to familiarize students on how the materials would be used. After the warm-up session, students were presented with four addition and four subtraction problems (e.g., 3 + 2 ; 5 + 2; 3 + 5;4 + 2 ; 2 - l ;5 - 3; 5 - 2; 6 - 5). Addition problems were presented before subtraction problems. For the addition problems, the examiner first placed a specific number of counters on the mat and asked students to identify the number of counters. Next, the examiner added some counters to the same set, and students were required to identify the total number of counters. For subtraction problems, the examiner placed counters on a mat and had students identify the number. Next, the examiner took away counters from the set and asked students to state how many counters were left. For addition and subtraction problems, students could state the answer by saying the number word or by showing the number of counters. Items were scored as correct if students displayed the appropriate number of counters and/or gave the appropriate number word. For each correct answer, students received a score of 2, one point for identifying the original amount and one point for the correct answer. No points were assigned if students failed to respond or state the incorrect answer. The score ranged from a minimum of 0 to a maximum of 16. The Cronbach's alpha reliability based on the sample of this study was .86 at pretest, .85 at posttest and .66 at delayed posttest.

2.4. Data Analysis

Pretreatment group equivalency was tested by conducting a one-way analysis of variance (ANOVA) on the two groups' scores on the SAT 10, EN-CBM, and the Number Sense Measures at pretest. T-tests and Chi-square tests were also conducted to determine treatment group equivalency on key demographic variables (e.g., age, ethnicity, SES, disability, and gender). In addition, means, standard deviations, and effect sizes were calculated.

To determine if there was a differential effect between the two groups a Repeated Measures ANOVA -2 Group (NS + GCI and GCI) x 3 Time (pretest, posttest, and delayed posttest) was conducted. In addition, simple effects analyses with Bonferroni correction were conducted to determine if there were group differences at each time of testing. Further, paired samples tests were conducted to analyze the extent to which EN-CBM and Number Sense Measures scores were maintained between pretest to posttest, and posttest to delayed posttest.

3. RESULTS

3.1. Pretreatment Equivalency

The ANOVA applied to the pretest scores indicated no statistically significant differences between conditions on SESAT, F (1, 99) = 2.3, p = .13. In addition, students' pretreatment performance on the EN-CBM and number sense measures also indicated no statistically significant differences between groups on oral counting fluency, F (1, 99) = 0.00 = .97, counting from, F (1, 99) = 0.10, = .75, number identification, F (1, 99) = 0.01, = .92, spatial relationships, F (1, 99) = 0.01, = .91, number relationships, F (1, 99) = 1.09, = .30, five and ten frame identification and representation, F (1, 99) = .32, p = .57, five and ten frame calculation, F (1, 99) = .27, p =.61, and nonverbal calculation, F(1, 99) = *.98*,p = .33.

3.2. Differential Effects of Number Sense Instruction

3.2.1. EN-CBM

Results for repeated measures ANOVA indicated no statistically significant main effects for group for all three EN-CBM measures (Oral counting fluency F (1, 99) = .138, p = .71, counting from F (1, 99) = .2.38, p = .126 and number identification F (1, 99) = .132, p = .717). However, results indicated a statistically significant effect for time of testing on oral counting fluency, F (2, 198) = 138.96, p = .000, counting from testing F (2, 171) = 22.16, p = .00), and number identification F (2, 176) = 55.30, p = .00). The simple effect for time also was examined post hoc, using a Bonferonni adjustment. Results indicated statistically significant differences from pretest to posttest (p < .001), but no significant differences from posttest to delayed posttest (p = .073) for both oral counting fluency and counting from. For number identification results indicated statistically significant differences from pretest to posttest (p < .001), and from posttest to delayed posttest (p = .008).

3.2.2. Number Sense Measures

Results for all Number Sense Measures indicated statistically significant main effects for group (Spatial relationships, F (1, 99) = 5.40, p = .0; number relationships, F (1, 99) = 7.54, p = .007; five and ten frame identification and representation, F (1, 99) = 13.75, p < .001; five and ten frame calculation F (1, 99) = 7.59, p = .007; and nonverbal calculation F (1, 99) = 8.99, p = .007) and time of testing (Spatial relationships, F (2, 184) = 45.05, p < .001; number relationships, F (2, 187) = 74.17, p < .001; five and ten frame identification and representation, F (2, 166) = 42.03, p < .001; five and ten frame calculation F (2, 192) = 9.32, p < .01; and nonverbal calculation F (1, 147) = 50.34, p < .001). The simple effect for time was also examined post hoc, using a Bonferonni adjustment. Results indicated statistically significant differences from pretest to posttest for all measures and students in the NS+ GCI group outperformed students in the GCI group on all Number Sense Measures at posttest and delayed posttest.

4. DISCUSSION

This study examined the effectiveness of number sense instruction on mathematics competence of kindergarten students. The primary goal was to determine if number sense instruction combined with explicit instruction would improve students' number sense performance. Overall results indicated that both groups made significant improvements; however, students in the NS group significantly outperformed students in the GCI group on both Early Numeracy-CBM (oral counting fluency, counting from, number identification) and Number Sense Measures (spatial relationships, number relationships, benchmarks of five and ten, non-verbal calculations) at posttest. In addition, the NS + GCI group significantly outperformed students on all but one EN-CBM measure (Number Identification) and all number sense measures at delayed posttest, conducted three weeks after the intervention.

Overall, results of the repeated measures ANOVA indicated that even though students in both groups improved performance, students in the NS + GCI group significantly outperformed students in the GCI group on both EN-CBM and number sense measures at posttest. Application of the Bonferroni correction that adjusts the level of significance based on multiple comparisons revealed that there were no significant differences between the NS + GCI and GCI group on EN-CBM between pretest and posttest. However, the NS + GCI group outperformed the GCI group on all number sense measures, even after the Bonferroni correction. In addition, the NS + GCI group outperformed students on all but one early numeracy measure (Number Identification) and all number sense measures at delayed posttest conducted three weeks after the intervention. Further, the differences between the NS + GCI group and GCI group on all but the Number Identification (NI) measure were significant at the delayed posttest. These results indicate that students in the NS + GCI group maintained the new knowledge and skills.

Results of this study support previous research in that carefully sequenced activities focusing on the big ideas of number sense (e.g., spatial relationships; one more, one less, two more, and two less; benchmarks of five and ten; part-part whole relationship) (Van de Walle, 2007) as well as the use of explicit instruction (e.g., following the model, lead, guided practice, and independent practice paradigm) (Gersten & Chard, 1999; Kame'enui et al., 2002) when combined with opportunities for practice, lead to significant improvements in number sense skills of kindergarten students (Carnine, 1997, Van de Walle, 2007).

4.1. Effect of NS + GCI vs. GCI on Students' Mathematical Competence

4.1.1. EN-CBM

Even though there were no significant differences between groups on the pretest, results for all three EN-CBM indicated an increase from pretest to posttest for both the NS + GCI and GCI groups, however, students in the NS + GCI group significantly outperformed students in the GCI group. The increase from pretest to posttest observed on EN-CBM is notable because results clearly indicated that students who received more explicit number sense instruction performed better than students who did not receive explicit instruction. It should be noted however that these results ceased to be significant with the application of Bonferroni correction, a more stringent criteria for significance testing.

Results from posttest to delayed posttest for two out of three EN-CBMs (Oral Counting and Counting From) showed similar trends. Students maintained their new learning over the three-week time lag between posttest and delayed posttest. However, results of Number Identification were contradictory because at delayed posttest the GCI group outperformed the NS + GCI group. The reason for this contradictory trend for the Number Idenitification measure is unknown. Some possible explanations relate to the amount of time that was spent on teaching number identification skills. It is possible that the students in the GCI group who received 60 min of instruction based on the general classroom instruction had more time to learn and practice the skill than students in the NS + GCI group who received only 40 min of instruction on the district assigned curriculum. Number identification was not a core skill of the number sense curriculum and therefore students might not have spent the same amount of time learning and practicing the skill as students in the GCI group. In addition, this measure did not involve the use of manipulatives or interaction between examiner and student. Therefore, it is possible that students were more easily distracted during data collection for this measure as opposed to other measures that allowed for more interaction between the examiner and student. This could be an important reason for the group differences because students in the NS + GCI group were used to working of manipulatives and facilitated communication during the course of the number sense intervention.

4.1.2. Number Sense Measures

Results of all number sense measures indicated increase in the scores from pretest to posttest for both the NS + GCI and GCI group. Students in the NS + GCI group significantly outperformed students in the GCI group on all number sense measures. These results continued to be significant even after correcting for multiple comparisons. The results pertaining to the research question on the maintenance of number knowledge conducted via comparison of the scores of the NS and GCI group on the delayed posttest indicated that the NS group maintained their significantly higher scores on the number sense measures at least three weeks after the intervention. In sum, gains from pretest to posttest and the ability to maintain skills learned indicate the effectiveness of both curricula. At the same time, results clearly indicate that the number sense curriculum was more effective.

There are a number of possible explanations for the positive performance of the NS group. One plausible explanation is that the number sense program focused on big ideas of number sense (spatial relationships, one more, one less, two more, and two less, benchmarks of five and ten, and part-part whole relationships). As previous research has illustrated, big ideas are critical, because organizing information around key concepts maximizes student learning (Carnine, 1997; Prawat, 1989) and enhances attainment of skills and knowledge (Kame'enui et al., 2002; Ritchhart, 1999; Van de Walle, 2007). Further, a sound knowledge of number sense allows students to work more flexibly with numbers and is critical to all aspects of mathematics (Sowder, 1994). Number sense is the foundation for the development of student learning and understanding of complex problems (Case, 1998; Griffin, 2003, 04a, & 04b; Sowder & Schapelle, 1994; Yang, 2002).

Another potential reason for these effects could be the use of a combination of cognitive and explicit instruction (Moreno, 2004; NRC 2001; Rittle-Johnson, Siegler, & Alibali, 2001; Tuovinen & Sweller, 1999). Students in the NS + GCI group were not only encouraged to learn through experience, but also were taught using the model, lead, guided, and independent practice instructional paradigm. When introducing a new activity, the teacher first explained the activity and then modeled the activity for the students. This allowed students to not only explore different ways of approaching the problems but also allowed teachers to assist students to solve the problems correctly and efficiently. Following this instructional procedure is critical for those students who have trouble understanding keys facts. Number sense instruction should be explicit and not left to natural development or incidental learning (Aunio, Hautamaki, & Van Luit, 2005; Gersten & Chard, 1999). Further, a strong understanding of concepts and skills is facilitated by reliable, sound, and complete explanations (Leinhardt, 1989; Leinhardt & Putnam, 1987).

Third, the number sense curriculum included activities that involved real world issues that students could relate with. For example, during the review activity for unit One More, One Less, Two More, and Two Less, with the help of the teacher, students made up a story about a rabbit going into the donut forest and picking up one more or two more donuts or dropping or eating one or two donuts. Helping students identify and understand the association of numbers to real-world quantities not only helps them develop a flexible knowledge of numbers but also allows them understand the world in a mathematical manner within a meaningful social context (Eggen & Kauchak, 2000; Sowder & Schappelle, 1994; Thompson, 1990; Van de Walle, 2007; Young-Loveridge, 2004).

The fourth potential reason for improvement could be the use of manipulatives and scaffolding of instruction by introducing new concepts using concrete objects and then moving to semi-concrete and abstract materials. During the entire curriculum students were given the opportunity to work with a number of different manipulatives (e.g., unifix cubes, dot cards, counters, five and ten frames, bean counters, toothpicks, attribute blocks, and dice). The use of manipulatives was designed to make the activities fun and engaging for all students. Moreover, activities were designed to allow teachers to scaffold instruction and provide students with opportunities to move from concrete to semi concrete to abstract materials. Although the exact role of manipulatives cannot be determined, the outcome supports previous research, which indicates that manipulatives "support learners as they internalize skills and strategies" (Santoro et al., 2006, p. 125) and encourage learning of key concepts (Griffin, Case, & Capodilupo, 1997; Larkin, 2001; van Garderen, 2006). In addition, the use of manipulatives is valuable in developing deep understanding of novel and complicated concepts (Arcavi, 2003; Griffin, Case, Seigler, 1994; Sowder & Schappelle, 1994).

The fifth potential reason is the use of hands on activities and number games. For example, when learning about dot patterns, students were required to make a dot card train to figure out what pattern comes next. This was engaging for the students as it moved away from the rote procedures and memorization. Similarly, while learning about part-part whole relationships, students were required to use bean counters to figure out different parts of a whole as opposed to simply memorizing addition facts (2+2 = 4; 3+1 = 4; 1+ 3 = 4; 4 + 0 = 4; 0+4 = 4). Researchers have reported on the importance of incorporating of hands-on-activities because they allow students to construct their own knowledge rather than passively replicate the knowledge imparted by others (Burns & Silbey, 2000; Simon & Schifter, 1991). In addition, previous research has indicated that inclusion of games along with other methods of instruction is highly motivating for students (Kamii, 1982; Young-Loveridge, 2004).

Another feature of the curriculum that students appeared to enjoy was the incorporation of children's literature. Though limited, the curriculum allowed for opportunities for students to make predictions while the teacher read books. For example, one of the lessons had the teacher read the book Ten Black Dots by Donald Crew and after each page students had to predict which number would come next or what number was one more than the number of dots that they saw on the page that was being read. Previous research on early childhood learning indicates that integration of children's literature into the mathematics curriculum not only teaches mathematical concepts in the context of a story, but also develops mathematical thinking by allowing students to provide a variety of responses and make historical, cultural, and real world connections. Incorporation of literature also prevents math anxiety, thereby creating an environment that is conducive to promoting math literacy. In addition, it provides the ability to assess student understanding by questioning (Columba, Kim, & Moe, 2005; Furner, Yahya, & Duffy, 2005; Griffiths & Clyne, 1991; Young-Loveridge, 2004).

Sixth, students were provided with numerous opportunities to practice learned skills. Each lesson began with a review of what was taught the previous day with a number of opportunities for students to practice the skills learned throughout the curriculum. Earlier research indicates that review must be sufficient and provide students with several opportunities to apply previously learned knowledge (e.g., a concept) until they demonstrate mastery of the concept or skill (King-Sears, 2001). In sum, one can presume that a combination of these six factors created an environment conducive to learning where students were able to practice, maintain, and internalize number sense concepts.

Despite the above positive characteristics of the number sense curriculum, implementation of the curriculum provided information about future improvements. First, the duration of the intervention was too short. Feedback from the teachers indicated that more time should have been spent on certain topics, especially Spatial Relationships and One More, One Less, Two More, Two Less. Even though results show improvement in student performance, teachers indicated that students struggled to grasp the concept in that short span of time. Currently, these topics are taught across two weeks of the curriculum, with five lessons for each topic. A revision of the NS curriculum should include at least eight days of instruction on each topic.

Second, some of the activity sheets were overwhelming for the students and need to be modified. Teacher feedback indicated that some worksheets had too many problems and it was difficult for students to stay on task. At present the worksheets have an average of six items, ranging from four to eight items per worksheet. The revised curriculum should have no more than 5 items per worksheet.

Third, the teachers were hesitant to use the provided materials. For example, counters were provided for all students in the intervention group, but at least one teacher was not comfortable giving them to all students. Instead she preferred to use one set and work as a group calling upon students to solve the problem. The primary reason for this was her concern about staying on time and keeping students focused on task instead of playing with the manipulatives. The teachers who were comfortable using the materials had at least two aides to help them distribute the materials and monitor student work. On the other hand the teacher who did not use the materials as planned had no help. Considering that the teacher did not have any help her method of using just one set of materials was practical. At the same time, previous research indicates that the use of manipulatives is very important to promote conceptual understanding; therefore, this aspect of the curriculum will be maintained. In addition, ways to help teachers use the materials effectively may be included during professional development. For example, the

Teacher could place materials on student tables prior to the start of class or place a collection of counters in the center of the table for all students to use. Both these strategies were used by the comparison group teachers and students completed the activities responsibly. Another possibility could be to have teachers work by demonstrating the use of counters in the classroom but then have students take the counters home to practice the skill learned. This could be easily implemented by including a packet for parents providing them with details about what their child was learning in school and how to practice at home.

The fourth issue pertains to the use of technology. Some review activities required students to use the calculator. Two of the three intervention teachers were not comfortable using the calculators and did not teach the lesson as designed. Reasons for not using the calculators were similar to reasons for not using manipulatives. The teacher who used the calculator with her students was the one who had one aide per table, making it easier for them to guide and monitor students. The other teachers, however, completely ignored these activities because they felt that their students would not be able to complete the task. Some modifications to the curriculum could be to provide teachers with a large calculator so they could walk around modeling for the students and then call on students to check the answers themselves. Another possibility would be to work in small groups. While one group would work with the teacher on the calculator the others could work on the assigned task independently. The fact that teachers did not use materials appropriately is reflected in the treatment fidelity data collected for observed lessons. The results of treatment fidelity (88.57%) suggested that the intervention was not implemented exactly as intended. Although teachers, on average, completed the majority of the steps outlined for implementation, it may be possible that overall student performance would have been better had teachers implemented the curriculum with higher level of integrity.

Fifth, since the number sense curriculum was comprised of multiple components (explicit instruction, children's literature, technology, hand on activities, use of manipulatives); it is difficult to parcel out the effective components of the curriculum. Thus, future research should parcel out the impact of each component alone, or combinations of components, to determine which are necessary for improved student outcomes.

Despite the aforementioned issues, the results from the study demonstrated that the intervention was effective in teaching the big ideas of number sense. The results from this study extend previous research indicating that explicit instruction helps students understand and master skills and supports cognitive instruction by allowing students to think and explore before coming up with an answer. Further, this study extends the current literature on the topic by being one of the first to teach domain specific concepts and skills to kindergarten students combining cognitive and explicit instruction.

Teachers in both groups were not completely satisfied with either the number sense program or the district assigned curriculum. While teacher perceptions regarding the district assigned varied regarding both skills and the instructional design, teacher perceptions about the number sense program indicated variations only regarding the skills taught. All teachers in the NS+ GCI group were satisfied with the instructional design principle adopted by the number sense program and were willing to recommend the use of the methods to other teachers.

5. LIMITATIONS AND FUTURE DIRECTIONS

Even though overall results of the study are encouraging, caution should be exercised when considering the results. Several limitations of the study are outlined below. First, even though the number of participants in the study was statistically adequate, the total sample size was small. Second, it was not possible to randomize at the individual student level, therefore classrooms were randomly assigned to the NS + GCI or GCI condition. The robustness that could have been gained by individual level random assignment was not logistically feasible within the school setting. In terms of analyses the assignment of classrooms and not students to conditions raises the issue of the appropriate unit of analysis. Not having a sufficient number of classrooms posed a problem in terms of using the classroom as the unit of analysis. Therefore, the study design resulted in using students as the unit of analysis. In order to overcome some of these design limitations multi-level modeling (MLM) can be conducted. In sum, future research should be conducted to validate the methodologies of number sense instruction with a larger group of students, belonging to different schools, and representing diverse populations. If possible randomization at the school level could be attempted to gain greater generalizability.

In addition to the limitations noted above there were also issues specific to the curriculum. First, since the NS + GCI group received instruction on both the district assigned curriculum and the number sense curriculum, it is impossible to isolate the unique contribution of the number sense curriculum. Improvement in performance could be due to a combination of the two curricula. Thus, future investigations should study the effectiveness of the numbers sense curriculum by using the NS curriculum alone, also comparing outcomes with other curricula.

Second, the duration of this study was short. Each big idea was taught only for a period of one week. It is a possibility that students would have shown greater improvements had the instructional time been longer. As noted above, future research should consider teaching some of the big ideas (spatial relationships and one more, one less, two more, and two less) that students struggled with for a longer time.

6. CONCLUSION

In summary, the present study reconfirms that preliminary evidence from the Sood and Jitendra (2013) study and reaffirms that instruction combining big ideas of number sense and explicit instruction can enhance mathematics competence of kindergarten students. Although further replication of these preliminary findings is necessary, the findings of this study can shed some light on current instructional practices for kindergarten students that are foundational for later mathematical learning.

First, since a strong understanding of beginning mathematics skills is the foundation of later mathematics achievement, it is important that the knowledge of number sense is not left to incidental learning. This is also important because not all students enter school with well-developed knowledge of numbers (Case 1985; Hiebert, 1986; Siegler & Robinson, 1982) and they need formal instruction to help them acquire and maintain basic mathematics skills.

This study also stresses on the importance of early intervention. It is critical that students be taught the big ideas of number sense as early as kindergarten. It is important to intervene at kindergarten because it is the first year that most students begin formal education. Not all students who come to kindergarten are fluent in mathematics and it is the appropriate time for them to learn the basics to not only decrease the gap between students who know the concepts, but also be prepared for first grade and beyond (Gersten, Jordan, & Flojo, 2005; Jordan, Kaplan, Olah, & Locuniak, 2006; Mazzocco & Thompson, 2005).

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