International Journal of Advanced Research in Physical Science (IJARPS)
Volume 3, Issue 7, 2016, PP 18-33
ISSN 2349-7874 (Print) & ISSN 2349-7882 (Online)
www.arcjournals.org

Evaluation of the Cosmological Constant from de Broglie Pilot-Wave Dynamics: Inflation with Conformal Coupling

Jung-Jeng Huang

Center for General Education, Division of Natural Science, Ming Chi University of Technology, Taishan, New Taipei City, Taiwan

Abstract: *In lattice Schrödinger picture, we study the possible effects of trans-Planckian physics on the de Broglie-Bohm quantum trajectory of massless conformally coupled scalar field in de Sitter space. Through de Broglie's dynamics, we find that for the Corley-Jacobson type dispersion relations with quartic or sextic correction, there exists a transition in the evolution of the quantum trajectory from well before horizon exit to near horizon exit, thus providing a mechanism for generating a small cosmological constant. Comparing the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory, we also find that for the usual dispersion parameter choice, the latter is smaller than the former. Further, we calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field, and use the backreaction to constraint the magnitude of dispersion parameters. Finally, we show that during the slow-roll inflation at the grand unification phase transition, the reduction of the cosmological constant depends on the choice of dispersion parameters.*

Keywords: *De Broglie-Bohm quantum trajectory, conformal coupling, inflation, cosmological constant, trans-Planckian physics.*

1. Introduction

In the standard inflationary scenario, usual realization of inflation is associated with a slow rolling inflaton minimally coupled to gravity [1]. Nevertheless, it is well known that the extension to the non-minimal coupling with the Ricci scalar curvature can soften the problem related to the small value of the self-coupling in the quartic potential of chaotic inflation [2]. Further, non-minimal coupling terms also can lead to corrections on power spectrum of primordial perturbations [3], a tiny tensor-to-scalar ratio [4] and non-Gaussianities [5]. Recently, it was pointed out that inflation with a conformally coupled inflaton can be realized as the rapid roll inflation [6, 7]. A broad class of models of chaotic inflation in supergravity with an arbitrary inflaton potential was also proposed. In these models the inflaton field is non-minimally coupled to gravity [8, 9].

Moreover, standard inflationary predictions can have two extensions. The first extension is associated with the ambiguity of initial quantum vacuum state, and the choice of initial vacuum state affects the predictions of inflation [10, 11]. For example, a deterministic hidden-variables theory such as the de Broglie-Bohm pilot-wave theory [12, 13] allows the existence of vacuum states with non-standard or nonequilibrium field fluctuations [14, 15], which result in statistical predictions that deviate from those of quantum theory in the context of inflationary cosmology [16, 17]. Recent study also shows that the quantum-to-classical transition of primordial cosmological perturbations can be obtained in the context of the de Broglie-Bohm theory [18].

The second extension concerns the so-called trans-Planckian problem [19, 20] of whether the predictions of standard cosmology are insensitive to the effects of trans-Planckian physics. In fact, nonlinear dispersion relations such as the Corley-Jacobson (CJ) type were used to mimic the trans-Planckian effects on cosmological perturbations [19-21]. These CJ type dispersion relations can be obtained naturally from quantum gravity models such as Horava gravity [22, 23]. Moreover, in several approaches to quantum gravity, the phenomenon of running spectral dimension of spacetime from the standard value of 4 in the infrared to a smaller value in the ultraviolet is associated with modified dispersion relations, which also include the CJ type dispersion relations [24, 25]. These recent results suggest that spacetime becomes effectively two-dimensional at super-Planckian energies, and all particles are conformally coupled to gravity [26].

In the previous work [27-32] we used the lattice Schrödinger picture to study the free scalar field theory in de Sitter space, derived the wave functionals for the Bunch-Davies (BD) vacuum state and its excited states, found the trans-Planckian effects on the de Broglie-Bohm quantum trajectory of massless minimally coupled scalar field for the CJ type dispersion relations, and evaluated the cosmological constant in minimal inflation. In this paper we extend the study to the case of massless conformally coupled scalar field.

The paper is organized as follows. In Section 2, the de Broglie-Bohm pilot-wave theory of a generically coupled scalar field in de Sitter space is briefly reviewed in the lattice Schrödinger picture, and the de Broglie-Bohm quantum trajectories for scalar field are given. In Section 3, we consider the massless conformally coupled scalar field during the slow-roll inflation, and use the CJ type dispersion relation with quartic or sextic correction to obtain the time evolution of the vacuum state wave functional and the corresponding de Broglie-Bohm quantum trajectories. In Section 4, we calculate the finite vacuum energy density, use the backreaction constraint to constraint the magnitude of parameters, and evaluate the cosmological constant in conformal inflation. Finally, conclusions are presented in Section 5. Throughout this paper we will set $\hbar$ =c=1.

2. DE BROGLIE-BOHM PILOT-WAVE THEORY OF SCALAR FIELD IN SCHRÖDINGER PICTURE

In this section, we begin by briefly reviewing how to define the de Broglie-Bohm pilot-wave theory of scalar field in de Sitter space in the lattice Schrödinger picture (for the details see [31]). The Lagrangian density for the scalar field we consider is

$$L = |g|^{\frac{1}{2}} \left\{ \frac{1}{2}\left[g^{\mu\nu}(x)\phi(x)_{,\mu}\phi(x)_{,\nu} \right] - V(\phi) \right\},$$

$$V(\phi) = m^2\phi^2/2 + \xi R\phi^2/2\,, \tag{1}$$

where ϕ is a real scalar field, $V(\phi)$ is the potential , m is the mass of the scalar quanta, R is the Ricci scalar curvature, ξ is the coupling parameter, and g = det $g_{\mu\nu}$, μ,ν =0,1,…,d. For a spatially flat (1+d)-dimensional Robertson-Walker spacetime with scale factor $a(t)$, we have

$$ds^2 = dt^2 - a^2(t)d^2x^i,\ i = 1,2,...,d\,,$$

$$L = a^d\left\{ \frac{1}{2}\left[(\partial_0\phi)^2 - a^{-2}(\partial_i\phi)^2 \right] - V(\phi) \right\}. \tag{2}$$

In the (1+d)-dimensional de Sitter space we have $a(t) = \exp(ht)$, where $h \equiv \dot{a}/a$ is the Hubble parameter which is a constant.

For d=1, in the lattice Schrödinger picture, we obtain from (2) the time-dependent functional Schrödinger equation in momentum space

$$H\psi = i\frac{\partial}{\partial t}\psi\,, \tag{3}$$

$$H = 2\sum_{l=1}^{N/2}\sum_{r=1}^{2} H_{rl} \tag{4}$$

where

$$H_{rl} = \frac{1}{2}p_{rl}^2 + \frac{1}{2}hp_{rl}\phi_{rl} + \frac{1}{2}a^{-2}\omega_l^2\phi_{rl}^2 + \frac{1}{2}\left(m^2 + \xi R\right)\phi_{rl}^2\,, \tag{5}$$

$$\psi[\phi_{rl},t] = \prod_{l=1}^{N/2}\prod_{r=1}^{2}\psi_{rl}(\phi_{rl},t) \equiv \prod_{rl}\psi_{rl}(\phi_{rl},t)\,. \tag{6}$$

Here $\omega_l \equiv (2/\varepsilon)\sin(l\pi/N)$, $\varepsilon = W/N$, *i.e.* W is the overall comoving spatial size of lattice, $\phi_l = \phi_{1l} + i\phi_{2l}$, $p_l = p_{1l} + ip_{2l}$, p_l is the conjugate momentum for ϕ_l, and the subscripts 1and 2 denote the real and imaginary parts respectively.

For each real mode ϕ_{rl}, we have

$$H_{rl}\psi_{rl} = i\frac{\partial}{\partial t}\psi_{rl}, \; r=1,2. \tag{7}$$

$$-\frac{1}{2}\frac{\partial^2\psi_{rl}}{\partial\phi_{rl}^2} + \frac{1}{2}\left[a^{-2}\omega_l^2 + \left(m^2 + \xi R\right) - \frac{1}{4}h^2\right]\phi_{rl}^2\psi_{rl} = i\frac{\partial\psi_{rl}}{\partial t}. \tag{8}$$

Note that (8) arises from the field quantization of the Hamiltonian (5) through the functional Schrödinger representation $\bar{\phi}_{rl} \to \phi_{rl}$, $\bar{p}_{rl} \to -i\partial/\partial\phi_{rl}$, where operators $\bar{\phi}_{rl}$ and $\bar{p}_{rl}$ satisfy the equal time commutation relations $[\bar{\phi}_{rl}, \bar{p}_{rl}] = i$. Thus (8) governs the time evolution of the state wave functional $\left|\psi_{rl}\right\rangle$ of the Hamiltonian operator H_{rl} in the $\{\left|\phi_{rl}\right\rangle\}$ representation. In terms of the conformal time τ defined by

$$d\tau = dt/a, \; \tau = -h^{-1}\exp(-ht) = -h^{-1}a^{-1}, \; -\infty < \tau < 0, \tag{9}$$

the normalized vacuum and its excited states are

$$\psi_{rl(n_{rl})}(\phi_{rl},\tau) = R_{(n_{rl})}(\phi_{rl},\tau)\exp\left(i\Theta_{(n_{rl})}(\phi_{rl},\tau)\right), \; n_{rl} = 0,1,2,... \tag{10}$$

with the amplitude $R_{(n_{rl})}(\phi_{rl},\tau)$ and phase $\Theta_{(n_{rl})}(\phi_{rl},\tau)$

$$R_{(n_{rl})}(\phi_{rl},\tau) = \left[\frac{\sqrt{2h/\pi}}{\sqrt{\pi}\,2^n n_{rl}!\left|H_\nu^{(1)}\right|}\right]^{1/2} H_{(n_{rl})}(\eta_{rl})\exp(-\frac{1}{2}\eta_{rl}^2), \tag{11}$$

$$\Theta_{(n_{rl})}(\phi_{rl},\tau) = -\frac{h\omega_l|\tau|}{2}\frac{\left(\left|H_\nu^{(1)}\right|\right)'}{\left|H_\nu^{(1)}\right|}\phi_{rl}^2 - (\frac{1}{2} + n_{rl})\int\frac{\frac{2}{\pi|\tau|}}{\left|H_\nu^{(1)}\right|^2}d\tau. \tag{12}$$

Here η_{rl} is defined by $\eta_{rl} \equiv \left(\sqrt{2h/\pi}/\left|H_\nu^{(1)}\right|\right)\phi_{rl}$, $H_{(n_{rl})}(\eta_{rl})$ is the nth-order Hermite polynomial, $H_\nu^{(1)}(\omega_l|\tau|)$ is the Hankel function of the first kind of order ν, $\nu^2 = 1/4 - \left(m^2 + \xi R\right)/h^2$, and the prime in (12) denotes the derivative with respect to $\omega_l|\tau|$. The complete wave functionals can be written as $\psi_{[n]}[\phi_{rl},t] = \prod_{rl}\psi_{(n_{rl})}(\phi_{rl},t)$, where $[n] \equiv (n_i, n_j, \cdots)$ means that mode i is in the n_i excited state, mode j is in the n_j excited state, *etc*. For $n_{rl} = 0$, the ground state wave functional corresponds to the BD vacuum. For d=3, we have $\nu^2 = 9/4 - \left(m^2 + \xi R\right)/h^2$, $R = 12h^2$ and the mode index l in ω_l carries labels $(l_i, i = 1,2,3)$ which will be suppressed below.

For d=3, we get from equations (3)-(8) in the continuum limit ($\omega_l \to k$)

$$i\frac{\partial\psi}{\partial t}=\sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial\phi_{rk}^2}+\frac{1}{2}\left[a^{-2}k^2+\left(m^2+\xi R\right)-\frac{9}{4}h^2\right]\phi_{rk}^{\ 2}\}\psi\ , \qquad (13)$$

which implies the continuity equation

$$\frac{\partial|\psi|^2}{\partial t}+\sum_{rk}\{\frac{\partial}{\partial\phi_{rk}}\left[|\psi|^2\frac{\partial\Theta}{\partial\phi_{rk}}\right]\}=0 \qquad (14)$$

and the de Broglie-Bohm velocity field

$$\frac{d\phi_{rk}}{dt}=\frac{\partial\Theta}{\partial\phi_{rk}}, \qquad (15)$$

where $\psi=|\psi|\exp[i\Theta]$. For a single mode ϕ_{rk}, we have $\psi_{rk}=|\psi_{rk}|\exp[i\Theta_{rk}]$ with $\Theta=\sum_{rk}\Theta_{rk}$, the continuity equation

$$\frac{\partial|\psi_{rk}|^2}{\partial t}+\frac{\partial}{\partial\phi_{rk}}\left(|\psi_{rk}|^2\frac{\partial\Theta_{rk}}{\partial\phi_{rk}}\right)=0\ , \qquad (16)$$

and the de Broglie-Bohm velocity field

$$\frac{d\phi_{rk}}{dt}=\frac{\partial\Theta_{rk}}{\partial\phi_{rk}}\ . \qquad (17)$$

Here, ψ is interpreted as a physical field in field configuration space, guiding the evolution of ϕ_{rk} through (13) and (17). Substituting (12) into (17) and using τ gives

$$\frac{d\phi_{rk}}{d\tau}=-k\frac{\left(|H_\nu^{(1)}(k|\tau|)|\right)'}{|H_\nu^{(1)}(k|\tau|)|}\phi_{rk}\ , \qquad (18)$$

which yields the quantum trajectory

$$\phi_{rk}(z)=C\left|H_\nu^{(1)}(z)\right|, \qquad (19)$$

where $z\equiv k|\tau|=(k/a)/h$ is the ratio of physical wave number $k_{phys}\equiv k/a$ to the inverse of Hubble radius, and C is an integration constant.

3. TRANS-PLANCKIAN EFFECTS

In this section we consider the massless conformally coupled ($\nu=1/2$) scalar field in the slow-roll inflation. To study further the effects of trans-Planckian physics, we use the CJ type dispersion relations

$$\omega^2(k/a)=k^2\left[1+b_s(\frac{k}{aM})^{2s}\right], \qquad (20)$$

where M is a cutoff scale, s is an integer, and b_s is an arbitrary coefficient [18-20].

3.1. CJ Type Dispersion Relation with Quartic Correction

We first focus on the CJ type dispersion relation (20) with $s = 1$ and $b_1 > 0$. Notice that this CJ type dispersion relation can be obtained from theories based on quantum gravity models [22-25].

3.1.1. Evolution of Vacuum Wave Functional

Then using $z = k|\tau| = k / ah$, (13) becomes

$$i\frac{\partial \psi}{\partial t} = \sum_{rk} \{ -\frac{1}{2}\frac{\partial^2}{\partial \phi_{rk}{}^2} + \frac{1}{2}\left[z^2\left(1+\sigma^2 z^2\right)h^2 - \frac{1}{4}h^2 \right]\phi_{rk}{}^2 \}\psi \ , \tag{21}$$

where $\sigma^2 \equiv b_1 (h / M)^2$, and the ground state wave functional of (21) becomes

$$\psi_{(0)} = \prod_{rk} A_{k(0)}(\tau)\exp(-\frac{1}{2}B_k(\tau)a^{-1}\phi_{rk}{}^2) \ , \tag{22}$$

where $A_{k(0)}(\tau)$ and $B_k(\tau)$ satisfy

$$A_{k(0)}(\tau) = \exp\left[-i\frac{1}{2}\int B_k(\tau)d\tau + const \right], \tag{23}$$

$$B_k{}^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau} \right] - \left[k^2(1+\sigma^2 z^2) - \frac{1}{4\tau^2} \right] = 0 \ . \tag{24}$$

In region I where $k_{phys} \equiv k / a > M$, *i.e.*, $z > M / h$, the dispersion relation can be approximated by $\omega^2(k / a) \approx k^2\sigma^2 z^2$, and the corresponding wave functional for the initial BD vacuum state is [31]

$$\psi_{(0)}{}^{\mathrm{I}} = \prod_{rk} A_{k(0)}{}^{\mathrm{I}}(\tau)\exp(-\frac{1}{2}B_k{}^{\mathrm{I}}(\tau)a^{-1}\phi_{rk}{}^{\mathrm{I}2}) \ ,$$

$$A_{k(0)}{}^{\mathrm{I}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k{}^{\mathrm{I}}(\tau)d\tau + const \right], \tag{25}$$

$$B_k{}^{\mathrm{I}}(\tau) = \frac{\frac{4}{\pi|\tau|}}{\left|H_{1/4}{}^{(1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{1/4}{}^{(1)}\right|^2\right)'}{\left|H_{1/4}{}^{(1)}\right|^2}\sigma z \ , \tag{26}$$

where the prime in (26) denotes the derivative with respect to $\sigma z^2 / 2$.

On the other hand, in region II where $k_{phys} \equiv k / a < M$, *i.e.*, $z < M / h$, linear relation recovers $\omega^2 \cong k^2$, and the corresponding wave functional for the non-BD vacuum state is [31]

$$\psi_{(0)}{}^{\mathrm{II}} = \prod_{rk} A_{k(0)}{}^{\mathrm{II}}(\tau)\exp(-\frac{1}{2}B_k{}^{\mathrm{II}}(\tau)a^{-1}\phi_{rk}{}^{\mathrm{II}2}) \ ,$$

$$A_{k(0)}{}^{\mathrm{II}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k{}^{\mathrm{II}}(\tau)d\tau + const \right], \tag{27}$$

$$B_k^{\text{II}}(\tau) = \frac{\dfrac{2}{\pi|\tau|}}{\left(\left|C_1^{\text{II}}\right|^2 + \left|C_2^{\text{II}}\right|^2\right)\left|H_{1/2}^{(1)}\right|^2 + 2\,\text{Re}\left[C_1^{\text{II}} C_2^{\text{II}*}\left(H_{1/2}^{(1)}\right)^2\right]}$$

$$- i\frac{k}{2}\frac{\left\{\left(\left|C_1^{\text{II}}\right|^2 + \left|C_2^{\text{II}}\right|^2\right)\left|H_{1/2}^{(1)}\right|^2 + 2\,\text{Re}\left[C_1^{\text{II}} C_2^{\text{II}*}\left(H_{1/2}^{(1)}\right)^2\right]\right\}'}{\left(\left|C_1^{\text{II}}\right|^2 + \left|C_2^{\text{II}}\right|^2\right)\left|H_{1/2}^{(1)}\right|^2 + 2\,\text{Re}\left[C_1^{\text{II}} C_2^{\text{II}*}\left(H_{1/2}^{(1)}\right)^2\right]}, \tag{28}$$

where the prime in (28) denotes the derivative with respect to z, and C_1^{II} and C_2^{II} satisfy $\left|C_1^{\text{II}}\right|^2 - \left|C_2^{\text{II}}\right|^2 = 1$. Let τ_c be the time when the modified dispersion relations take the standard linear form. Then $\sigma^2 z_c^2 = 1$ where $z_c = k|\tau_c| = M / b_1^{1/2} h$ >>1 for $b_1 \sim 1$. The constants C_1^{II} and C_2^{II} can be obtained by the following matching conditions at τ_c for the two wave functionals (25) and (27)

$$\psi_{(0)}^{\text{I}}\Big|_{z_c} = \psi_{(0)}^{\text{II}}\Big|_{z_c}, \tag{29}$$

$$\frac{d\psi_{(0)}^{\text{I}}}{dz}\Big|_{z_c} = \frac{d\psi_{(0)}^{\text{II}}}{dz}\Big|_{z_c}, \tag{30}$$

which can also be rewritten respectively as

$$\text{Re}\left(B_k^{\text{I}}\right)\Big|_{z_c} = \text{Re}\left(B_k^{\text{II}}\right)\Big|_{z_c}, \tag{31}$$

$$\frac{d\,\text{Re}\left(B_k^{\text{I}}\right)}{dz}\Big|_{z_c} = \frac{d\,\text{Re}\left(B_k^{\text{II}}\right)}{dz}\Big|_{z_c}, \tag{32}$$

by requiring $B_k^{\text{I}} = B_k^{\text{II}}$, $\phi_{rk}^{\text{I}} = \phi_{rk}^{\text{II}}$, $A_{k(0)}^{\text{I}} = A_{k(0)}^{\text{II}}$ when $z = z_c$. Using $\left|H_{1/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 = \left(4/\pi\sigma z^2\right)\left(1 - 3/8\sigma^2 z^4 + \ldots\right) \approx 4/\pi\sigma z^2$ with $\sigma = z_c^{-1}$, z_c >>1 and $\left|H_{1/2}^{(1)}(z)\right|^2 = 2/\pi z$, we have from (26), (28), and (31)

$$1 = \left|C_1^{\text{II}}\right|^2 + \left|C_2^{\text{II}}\right|^2 + 2\left|C_1^{\text{II}}\right|\left|C_2^{\text{II}}\right|\cos(2z_c - \theta), \tag{33}$$

where we chose $C_1^{\text{II}} = \left|C_1^{\text{II}}\right|$ and $C_2^{\text{II}} = \left|C_2^{\text{II}}\right|\exp(i\theta)$, and θ is a relative phase parameter. Then from (33) and $\left|C_1^{\text{II}}\right|^2 - \left|C_2^{\text{II}}\right|^2 = 1$ we have

$$\left|C_1^{\text{II}}\right| = -\csc(2z_c - \theta), \quad \left|C_2^{\text{II}}\right| = -\cot(2z_c - \theta), \tag{34}$$

where $\sin(2z_c - \theta) < 0$, $\cos(2z_c - \theta) > 0$. Substituting (26) and (28) into (32) and keeping terms up to order $1/z_c$ on the right-hand side of (32) , we obtain

$$\frac{1}{z_c} = -4\left|C_1^{\text{II}}\right|\left|C_2^{\text{II}}\right|\sin(2z_c - \theta). \tag{35}$$

Using (34) in (35) gives

$$\cot(2z_c - \theta) = -\frac{1}{4z_c}. \tag{36}$$

Then we have

$$\left|C_2^{\mathrm{II}}\right| = \frac{1}{4z_c}, \quad \left|C_1^{\mathrm{II}}\right| = \sqrt{1 + \left|C_2^{\mathrm{II}}\right|^2} \cong 1 + \frac{1}{32 z_c^2} \cong 1, \tag{37}$$

or

$$\sin(2z_c - \theta) \cong -1, \quad \cos(2z_c - \theta) \cong \frac{1}{4z_c}. \tag{38}$$

3.1.2. De Broglie-Bohm Quantum Trajectory

In Section 2, we defined the pilot-wave scalar field theory through de Broglie's first-order dynamics (13) and (17). Using the results about the evolution of vacuum wave functional in previous subsection, we can further define it through Bohm's second-order dynamics (21) and (39):

$$\frac{d^2\phi_{rk}}{dt^2} = -\frac{\partial}{\partial\phi_{rk}}(V + Q). \tag{39}$$

Here, the classical potential V and the so-called 'quantum potential' Q are given by

$$V = \sum_{rk}\frac{1}{2}\left[z^2\left(1+\sigma^2 z^2\right)h^2 - h^2/4\right]\phi_{rk}^2, \tag{40}$$

$$Q = -\sum_{rk}\frac{1}{2\left|\psi_{(0)}\right|}\frac{\partial^2\left|\psi_{(0)}\right|}{\partial\phi_{rk}^2}, \tag{41}$$

where $\psi_{(0)}$ is given by (22)-(24) and $\left|\psi_{(0)}\right|$ is given by the continuum limit of (11) for $n_{rl} = 0$. Note that Bohm's dynamics in general yields more possible quantum trajectories than de Broglie's dynamics does [30], and this distinction between Bohm's and de Broglie's dynamics was also emphasized recently by Valentini [33]. This is what we expect, because Bohm regarded (39) as the law of motion, with the de Broglie guidance equation (17) added as a constraint on the initial momenta.

However, recently it was pointed out that Bohm's dynamics is unstable. Small deviations from initial quantum equilibrium do not relax and instead grow with time [34]. On the other hand, de Broglie's dynamics is a tenable physical theory. Therefore, we will investigate the quantum trajectories of scalar field through de Broglie's dynamics hereafter.

In region I, from (25) and (26) we have

$$\Theta_{(0)}(\phi_{rk}^{\mathrm{I}}, \tau) = -\frac{k}{4}\frac{\left(\left|H_{1/4}^{(1)}\right|^2\right)'}{\left|H_{1/4}^{(1)}\right|^2}\sigma z a^{-1}\phi_{rk}^{\mathrm{I}\,2} - \frac{1}{2}\int\frac{\frac{4}{\pi|\tau|}}{\left|H_{1/4}^{(1)}\right|^2}d\tau, \tag{42}$$

where $\left|H_{1/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 = \left(4/\pi\sigma z^2\right)\left(1 - 3/8\sigma^2 z^4 + \ldots\right) \approx 4/\pi\sigma z^2$, and the prime in (42) denotes the derivative with respect to $\sigma z^2/2$. Substituting (42) into (17) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ gives

$$\frac{d\phi_{rk}^{\mathrm{I}}}{d\tau} = \frac{k}{z}\phi_{rk}^{\mathrm{I}}. \tag{43}$$

The general solution of (43) is

$$\phi_{rk}^{\mathrm{I}}(z) = \overline{C}^{\mathrm{I}} z^{-1}. \tag{44}$$

On the other hand, in region II, from (27) and (28) we have

$$\Theta_{(0)}(\phi_{rk}{}^{\mathrm{II}},\tau) = -\frac{k}{4}\frac{\left(\left|H_{1/2}{}^{(1)}\right|_{md}{}^{2}\right)'}{\left(\left|H_{1/2}{}^{(1)}\right|_{md}{}^{2}\right)} a^{-1}\phi_{rk}{}^{\mathrm{II}\,2} - \frac{1}{2}\int \frac{\frac{2}{\pi|\tau|}}{\left(\left|H_{1/2}{}^{(1)}\right|_{md}{}^{2}\right)} d\tau \ , \tag{45}$$

where $\left|H_{1/2}{}^{(1)}\right|_{md}$ means $\left|H_{1/2}{}^{(1)}\right|$ modified according to

$$\left|H_{1/2}{}^{(1)}\right|_{md} \equiv \left\{\left(|C_1|^2 + |C_2|^2\right)\left|H_{1/2}{}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1 C_2{}^{*}\left(H_{1/2}{}^{(1)}\right)^2\right]\right\}^{1/2} ,$$

$\left|H_{1/2}{}^{(1)}(z)\right|^2 = 2/\pi z$, and the prime in (45) denotes the derivative with respect to z . Note that in region II, $\left|H_{1/2}{}^{(1)}\right|_{md}$ becomes

$$\left|H_{1/2}{}^{(1)}\right|_{md} = \left|H_{1/2}{}^{(1)}\right|\left\{\left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2 + 2\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\left[\cos(2z-\theta)\frac{z^2-1}{z^2+1} - \sin(2z-\theta)\frac{2z}{1+z^2}\right]\right\} , \tag{46}$$

which reduces to $\left|H_{1/2}{}^{(1)}\right|$ for $z \to z_c \gg 1$ (well before horizon exit) by using (33). Substituting (45) into (17) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ gives

$$\frac{d\phi_{rk}{}^{\mathrm{II}}}{d\tau} = \frac{k}{2z}\phi_{rk}{}^{\mathrm{II}} . \tag{47}$$

The general solution of (47) is

$$\phi_{rk}{}^{\mathrm{II}}(z) = \overline{C}^{\,\mathrm{II}} z^{-1/2} . \tag{48}$$

Then, substituting (44) and (48) into the matching condition at z_c for $\phi_{rk}{}^{\mathrm{I}}$ and $\phi_{rk}{}^{\mathrm{II}}$

$$\phi_{rk}{}^{\mathrm{I}}\Big|_{z_C} = \phi_{rk}{}^{\mathrm{II}}\Big|_{z_C} , \tag{49}$$

We obtain

$$\overline{C}^{\,\mathrm{II}} = \overline{C}^{\,\mathrm{I}} z_c{}^{-1/2} . \tag{50}$$

Furthermore, for $z \approx 1$ (near horizon exit), (46) also approximately reduces to $\left|H_{1/2}{}^{(1)}\right|$ by using (37) and $z_c \gg 1$, and the solution of (17) becomes

$$\phi_{rk}{}^{\mathrm{II}}(z) \approx \overline{C}^{\,\mathrm{II}} z^{-1/2} . \tag{51}$$

Since for d=3 ϕ_{rk} contains a factor $a^{3/2}$ which is proportional to $z^{-3/2}$, we use a field redefinition $u_{rk} \equiv a^{-3/2}\phi_{rk}$, $a = (k/h)z^{-1}$, and (50) to rewrite (44) and (51) as

$$u_{rk}{}^{\mathrm{I}} = \left(\frac{k}{h}\right)^{-3/2} \bar{C}^{\mathrm{I}} z^{1/2}, \quad u_{rk}{}^{\mathrm{II}} = \left(\frac{k}{h}\right)^{-3/2} \bar{C}^{\mathrm{I}} z_c{}^{-1/2} z. \tag{52}$$

Thus we see from (44) and (51) that for fixed k and $z_c >> 1$, as z decreases from $z = z_c >> 1$ to $z \approx 1$, the scalar field decreases from one large value to a much smaller value which is a factor $1/z_c$ less than the field value at $z = z_c$, i.e., there exists a transition in the time evolution of the quantum trajectory of scalar field.

3.2. CJ Type Dispersion Relation with Sextic Correction

In this subsection, we consider the CJ type dispersion relation (20) with $s = 2$ and $b_2 > 0$, and repeat the preceding calculations for this type dispersion relation.

3.2.1. Evolution of Vacuum Wave Functional

For this case, only (21), (24), and (26) are changed into

$$i\frac{\partial \psi}{\partial t} = \sum_{rk} \{ -\frac{1}{2}\frac{\partial^2}{\partial \phi_{rk}{}^2} + \frac{1}{2}\left[z^2\left(1+\sigma^2 z^4\right)h^2 - \frac{1}{4}h^2 \right]\phi_{rk}{}^2 \}\psi, \tag{53}$$

$$B_k{}^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau}\right] - \left[k^2(1+\sigma^2 z^4) - \frac{1}{4\tau^2}\right] = 0, \tag{54}$$

$$B_k{}^{\mathrm{I}}(\tau) = \frac{\frac{6}{\pi|\tau|}}{\left|H_{1/6}{}^{(1)}\right|^2} - i\frac{k}{2}\frac{\left(\left|H_{1/6}{}^{(1)}\right|^2\right)'}{\left|H_{1/6}{}^{(1)}\right|^2}\sigma z^2, \tag{55}$$

where $\sigma^2 \equiv b_2 (h/M)^4$, and the prime in (55) denotes the derivative with respect to

$\sigma z^3/3$ .Using $\left|H_{1/6}{}^{(1)}\left(\sigma z^3/3\right)\right|^2 = \left(6/\pi\sigma z^3\right)\left(1 - 1/\sigma^2 z^6 + \ldots\right) \approx 6/\pi\sigma z^3$ with $\sigma = \bar{z}_c{}^{-2}$,

$\bar{z}_c = k|\bar{\tau}_c| = M/b_2{}^{1/4} h$ >>1 for $b_2 \sim 1$, and $\left|H_{1/2}{}^{(1)}(z)\right|^2 = 2/\pi z$, we obtain from (55),(28),and (31)

$$1 = \left|C_1{}^{\mathrm{II}}\right|^2 + \left|C_2{}^{\mathrm{II}}\right|^2 + 2\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\cos(2\bar{z}_c - \theta), \tag{56}$$

$$\left|C_1{}^{\mathrm{II}}\right| = -\csc\left(2\bar{z}_c - \theta\right), \quad \left|C_2{}^{\mathrm{II}}\right| = -\cot(2\bar{z}_c - \theta), \tag{57}$$

where $\sin(2\bar{z}_c - \theta) < 0$ and $\cos(2\bar{z}_c - \theta) > 0$. Substituting (55) and (28) into (32) and keeping terms up to order $1/\bar{z}_c$ on the right-hand side of (32), we find

$$\frac{2}{\bar{z}_c} = -4\left|C_1{}^{\mathrm{II}}\right|\left|C_2{}^{\mathrm{II}}\right|\sin\left(2\bar{z}_c - \theta\right). \tag{58}$$

Using (34) in (58) gives

$$\cot(2\bar{z}_c - \theta) = -\frac{1}{2\bar{z}_c}. \tag{59}$$

Then we have

$$\left|C_2^{\,\mathrm{II}}\right| = \frac{1}{2\bar{z}_c}, \quad \left|C_1^{\,\mathrm{II}}\right| = \sqrt{1+\left|C_2^{\,\mathrm{II}}\right|^2} \cong 1+\frac{1}{8\bar{z}_c^{\,2}} \cong 1, \tag{60}$$

or

$$\sin(2\bar{z}_c-\theta) \cong -1, \quad \cos(2\bar{z}_c-\theta) \cong \frac{1}{2\bar{z}_c}. \tag{61}$$

3.2.2. De Broglie-Bohm Quantum Trajectory

In region I, from (25) and (55) we have

$$\Theta_{(0)}(\phi_{rk}^{\,\mathrm{I}},\tau) = -\frac{k}{4}\frac{\left(\left|H_{1/6}^{(1)}\right|^2\right)'}{\left|H_{1/6}^{(1)}\right|^2}\sigma z^2 a^{-1}\phi_{rk}^{\,\mathrm{I}\,2} - \frac{1}{2}\int\frac{\frac{6}{\pi|\tau|}}{\left|H_{1/6}^{(1)}\right|^2}d\tau \;\;, \tag{62}$$

where $\left|H_{1/6}^{(1)}(\sigma z^3/3)\right|^2 = (6/\pi\sigma z^3)(1-1/\sigma^2 z^6+\ldots) \approx 6/\pi\sigma z^3$, and the prime in (62) denotes the derivative with respect to $\sigma z^3/3$ .Substituting (62) into (17) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ gives

$$\frac{d\phi_{rk}^{\,\mathrm{I}}}{d\tau} = \frac{3}{2}\frac{k}{z}\phi_{rk}^{\,\mathrm{I}}. \tag{63}$$

The general solution of (63) is

$$\varphi_{rk}^{\,\mathrm{I}}(z) = \hat{C}^{\,\mathrm{I}} z^{-3/2}. \tag{64}$$

On the other hand, in region II, from (27) and (28) we have the same general solution as (48) for $z \to \bar{z}_c \gg 1$

$$\phi_{rk}^{\,\mathrm{II}}(z) = \hat{C}^{\,\mathrm{II}} z^{-1/2}. \tag{65}$$

Substituting (64) and (65) into the matching condition at $\bar{z}_c$ for $\phi_{rk}^{\,\mathrm{I}}$ and $\phi_{rk}^{\,\mathrm{II}}$

$$\phi_{rk}^{\,\mathrm{I}}\Big|_{\bar{z}_C} = \phi_{rk}^{\,\mathrm{II}}\Big|_{\bar{z}_C}\;, \tag{66}$$

we obtain

$$\hat{C}^{\,\mathrm{II}} = \hat{C}^{\,\mathrm{I}}\bar{z}_c^{\,-1}. \tag{67}$$

Furthermore, for $z \approx 1$ (near horizon exit), (46) also approximately reduces to $\left|H_{1/2}^{(1)}\right|$ by using (37) and $\bar{z}_c \gg 1$, and the solution of (17) becomes

$$\phi_{rk}^{\,\mathrm{II}}(z) \approx \hat{C}^{\,\mathrm{II}} z^{-1/2}. \tag{68}$$

Since for d=3 ϕ_{rk} contains a factor $a^{3/2}$ which is proportional to $z^{-3/2}$, we use a field redefinition

$u_{rk} \equiv a^{-3/2}\phi_{rk}$, $a = (k/h)z^{-1}$, and (67) to rewrite (64) and (68) as

$$u_{rk}^{\mathrm{I}} = \left(\frac{k}{h}\right)^{-3/2} \hat{C}^{\mathrm{I}}, \quad u_{rk}^{\mathrm{II}} = \left(\frac{k}{h}\right)^{-3/2} \hat{C}^{\mathrm{I}} \bar{z}_c^{-1} z \ . \tag{69}$$

Thus we see from (64) and (68) that for fixed k and $\bar{z}_c >> 1$, as z decreases from $z = \bar{z}_c >> 1$ to $z \approx 1$, the scalar field decreases from one large constant to a much smaller value which is a factor $1/\bar{z}_c$ less than the field value at $z = \bar{z}_c$, i.e., there exists a transition in the time evolution of the quantum trajectory of scalar field.

Comparing (52) with (69) for scalar field, we find that for $b_1 \approx b_2 \sim 1$, $z_c \approx \bar{z}_c >> 1$,we have $\hat{C}^{\mathrm{I}} = \overline{C}^{\mathrm{I}} z_c^{1/2}$, $\hat{C}^{\mathrm{II}} = \overline{C}^{\mathrm{II}}$ and as z decreases from $z > z_c \approx \bar{z}_c >> 1$ to $z \approx 1$, the former is larger than the latter by a factor $(z/z_c)^{1/2}$ during the early ($z > z_c$) evolution of the quantum trajectory of scalar field. Therefore, if we compare the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory, the latter is smaller than the former.

4. VACUUM ENERGY, BACKREACTION AND COSMOLOGICAL CONSTANT

Using the results of Section 3, we now proceed to calculate the finite vacuum energy density, use the backreaction to constraint the parameters in nonlinear dispersion, and evaluate the cosmological constant. Note that in the slow-roll approximation, the energy density of the scalar field is $\rho_\phi \cong V(\phi)$, where the potential $V(\phi) = h^2\phi^2$. Thus the relation between the expectation value of the vacuum energy density ρ_ϕ and the vacuum wave functional $\psi_{(0)}$ in (22) is

$$\left\langle \rho_\phi \right\rangle = \left\langle \psi_{(0)} \middle| \rho_\phi \middle| \psi_{(0)} \right\rangle = h^2 \varepsilon^{-3} \sum_{rk} \int_{-\infty}^{\infty} du_{rk} \left| \psi_{rk(0)}(u_{rk}, \tau) \right|^2 u_{rk}^2$$

$$= h^2 \frac{1}{8\pi^3} \int d^3k \, \frac{1}{2} \frac{1}{\mathrm{Re}(\ B_k(\tau) a^{-1})} a^{-3}$$

$$= h^2 \frac{1}{2\pi^2} \int k^2 \frac{1}{2} \frac{1}{\mathrm{Re}(\ B_k(\tau) a^{-1})} a^{-3} dk \ , \tag{70}$$

where we use a field redefinition $u_{rk} \equiv a^{-3/2}\phi_{rk}$,

$$\left| \psi_{rk(0)}(u_{rk}, \tau) \right|^2 = a^{3/2} \frac{\sqrt{\mathrm{Re}(B_k(\tau) a^{-1})}}{\sqrt{\pi}} \exp\left(-\mathrm{Re}(B_k(\tau) a^{-1}) \frac{u_{rk}^2}{a^{-3}} \right), \tag{71}$$

$\mathrm{Re}(\ B_k(\tau) a^{-1})$ denotes the real part of $B_k(\tau) a^{-1}$, and the factor $a^{3/2}$ in (71) appears through the normalization condition

$$\int_{-\infty}^{\infty} du_{rk} \left| \psi_{rk(0)}(u_{rk}, \tau) \right|^2 = 1 \ . \tag{72}$$

For $s=1$ and $b_1>0$, in region I, we have $\left|H_{1/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 \approx 4/\pi\sigma z^2$ with $\sigma = z_c^{-1}$. Then, using $a = 1/h|\tau| = k/hz$ and (20) in (70), we obtain

$$\left\langle \rho_\phi \right\rangle_{s=1}^{\mathrm{I}} = \frac{1}{4\pi^2} h^4 z_c \int_{z_c}^{\alpha z_c} dz = \frac{1}{4\pi^2} h^4 z_c^{\,2} (\alpha - 1), \tag{73}$$

where $z_c = M / b_1^{1/2} h$ and $\alpha z_c = M_{Pl}/h$ (here $M_{Pl} = G^{-1/2} = 1.22\times 10^{19}$ GeV is the Planck mass) are the boundaries of the interval of integration. On the other hand, in region II, (28) can be expressed as

$$B_k^{\mathrm{II}}(\tau) = \frac{\dfrac{2}{\pi|\tau|}}{\left|H_{1/2}^{(1)}\right|_{md}^{2}} - i\frac{k}{2}\frac{\left(\left|H_{1/2}^{(1)}\right|_{md}^{2}\right)'}{\left|H_{1/2}^{(1)}\right|_{md}^{2}}, \tag{74}$$

where $\left|H_{1/2}^{(1)}\right|_{md}$ is defined as

$$\left|H_{1/2}^{(1)}\right|_{md} \equiv \left\{\left(\left|C_1^{\mathrm{II}}\right|^2 + \left|C_2^{\mathrm{II}}\right|^2\right)\left|H_{1/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1^{\mathrm{II}} C_2^{\mathrm{II}*}\left(H_{1/2}^{(1)}\right)^2\right]\right\}^{1/2}, \tag{75}$$

with $\left|H_{1/2}^{(1)}(z)\right|^2 = \dfrac{2}{\pi z}$. From (33), (37), and (38), we notice that $\left|H_{1/2}^{(1)}\right|_{md}$ can be approximated by $\left|H_{1/2}^{(1)}\right|$ as z decreases from $z = z_c \gg 1$ to $z=1$ (horizon exit). Then, using $a = 1/h|\tau| = k/hz$ and (28) in (70), we obtain

$$\left\langle \rho_\phi \right\rangle_{s=1}^{\mathrm{II}} = \frac{1}{4\pi^2} h^4 \int_1^{z_c} z dz = \frac{1}{4\pi^2} h^4 \left(\frac{1}{2} z_c^{\,2} - \frac{1}{2}\right). \tag{76}$$

From (73) and (76) we have

$$\left\langle \rho_\phi \right\rangle_{s=1} = \left\langle \rho_\phi \right\rangle_{s=1}^{\mathrm{I}} + \left\langle \rho_\phi \right\rangle_{s=1}^{\mathrm{II}} = \frac{1}{4\pi^2} h^4 \left[z_c^{\,2}(\alpha - \frac{1}{2}) - \frac{1}{2}\right]. \tag{77}$$

For $z_c \gg 1$ and $\alpha = b_1^{1/2}(M_{Pl}/M) > 1$, (77) becomes

$$\left\langle \rho_\phi \right\rangle_{s=1} \cong \frac{1}{4\pi^2} h^4 z_c^{\,2} (\alpha - \frac{1}{2}). \tag{78}$$

From (78) we see that there is no back reaction problem if the energy density due to the quantum fluctuations of the inflaton field is smaller than that due to the inflaton potential

$$\left\langle \rho_\phi \right\rangle_{s=1} < V(\phi). \tag{79}$$

In the slow-roll approximation, using $V(\phi) \cong 3M_{Pl}^{2}h^{2}/8\pi$ and (78) in (79) gives the constraint on the parameter b_1 as $b_1 > \frac{4}{9\pi^2}\left(\frac{M}{M_{Pl}}\right)^2$. For $M \sim 10^{16}$ GeV (the energy scale during inflation), we have $b_1 > 3.0\times 10^{-8}$.

For $s = 2$ and $b_2 > 0$, in region I, we have $\left|H_{1/6}^{(1)}\left(\sigma z^3/3\right)\right|^2 \approx 6/\pi\sigma z^3$ with $\sigma = \bar{z}_c^{-2}$. Then, using $a = 1/h|\tau| = k/hz$ and (55) in (70) we obtain

$$\left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{I}} = \frac{1}{4\pi^2}h^4\bar{z}_c^{\,2}\int_{\bar{z}_c}^{\beta\bar{z}_c}\frac{1}{z}dz = \frac{1}{4\pi^2}h^4\bar{z}_c^{\,2}\ln\beta\ , \tag{80}$$

where $\bar{z}_c = M/b_2^{1/4}h$ and $\beta\bar{z}_c = M_{Pl}/h$ are the boundaries of the interval of integration. On the other hand, in region II, (28) can be again expressed as (74) with $\left|H_{1/2}^{(1)}\right|_{md}$ defined by (75). From (58), (62), and (63), we also notice that $\left|H_{1/2}^{(1)}\right|_{md}$ can be approximated by $\left|H_{1/2}^{(1)}\right|$ as z decreases from $z = \bar{z}_c \gg 1$ to $z = 1$.

Then, using $a = 1/h|\tau| = k/hz$ and (55) in (70), we obtain

$$\left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{II}} = \frac{1}{4\pi^2}h^4\int_{1}^{\bar{z}_c} zdz = \frac{1}{4\pi^2}h^4\left(\frac{1}{2}\bar{z}_c^{\,2} - \frac{1}{2}\right). \tag{81}$$

From (80) and (81) we have

$$\left\langle \rho_\phi \right\rangle_{s=2} = \left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{I}} + \left\langle \rho_\phi \right\rangle_{s=2}^{\mathrm{II}} = \frac{1}{4\pi^2}h^4\left[\bar{z}_c^{\,2}(\ln\beta + \frac{1}{2}) - \frac{1}{2}\right]. \tag{82}$$

For $\bar{z}_c \gg 1$ and $\beta = b_2^{1/4}(M_{Pl}/M) > 1$, (82) becomes

$$\left\langle \rho_\phi \right\rangle_{s=2} \cong \frac{1}{4\pi^2}h^4\bar{z}_c^{\,2}(\ln\beta + \frac{1}{2}). \tag{83}$$

Moreover, we see that there is no back reaction problem if

$$\left\langle \rho \right\rangle_{s=2} < V(\phi). \tag{84}$$

Using $V(\phi) \cong 3M_{Pl}^{2}h^{2}/8\pi$ and (83) in (84) gives the constraint on the parameter b_2 as $b_2 > \frac{4}{9\pi^2}(\frac{M}{M_{Pl}})^4(\ln\beta + \frac{1}{2})^2$. For $M \sim 10^{16}$ GeV, we have $b_2 > 8.5\times 10^{-3}$.

Comparing (78) with (83), we find that $\langle \rho_\phi \rangle_{s=2} < \langle \rho_\phi \rangle_{s=1}$ if the inequality $\bar{z}_c^2(\ln\beta + 1/2) < z_c^2\alpha$, or $[(\ln\beta + 1/2)/(M_{Pl}/M)]^2 < b_2/b_1$ is satisfied. For example, the usual parameter choice $b_1 \sim b_2 \sim 1$ satisfies the inequality. On the other hand, we have $\langle \rho_\phi \rangle_{s=2} > \langle \rho_\phi \rangle_{s=1}$ if the inequality $\bar{z}_c^2(\ln\beta + 1/2) > z_c^2\alpha$, or $[(\ln\beta + 1/2)/(M_{Pl}/M)]^2 > b_2/b_1$ is satisfied. For example, the parameter choice $b_1 \sim 10^4$ and $b_2 \sim 10^{-2}$ satisfies the inequality.

In the case that $\langle \rho_\phi \rangle_{s=1}$ is larger than $\langle \rho_\phi \rangle_{s=2}$, using (78) in the cosmological constant $\Lambda = 8\pi\rho_{vac}/M_{Pl}^2$ gives $\Lambda = \frac{2h^2}{\pi b_1^{1/2}}\left(\frac{M}{M_{Pl}}\right)$, which is 5.2×10^{24} GeV ^{2} for $b_1 \sim 1$, $M \sim 10^{16}$ GeV, $h \sim 10^{14}$ GeV and 1.7×10^{28} GeV ^{2} for $b_1 \sim 10^{-7}$, $M \sim 10^{16}$ GeV, $h \sim 10^{14}$ GeV.

In the case that $\langle \rho_\phi \rangle_{s=2}$ is larger than $\langle \rho_\phi \rangle_{s=1}$, using (83) in $\Lambda = 8\pi\rho_{vac}/M_{Pl}^2$ gives $\Lambda = \frac{2h^2}{\pi b_2^{1/2}}\left(\frac{M}{M_{Pl}}\right)^2\left(\ln\beta + \frac{1}{2}\right)$, which is 3.3×10^{22} GeV ^{2} for $b_2 \sim 1$, $M \sim 10^{16}$ GeV, $h \sim 10^{14}$ GeV and 2.8×10^{23} GeV ^{2} for $b_2 \sim 10^{-2}$, $M \sim 10^{16}$ GeV, $h \sim 10^{14}$ GeV.

5. Conclusions

In the lattice Schrödinger picture, we have considered the de Broglie-Bohm pilot-wave theory of a generically coupled free real scalar field in de Sitter space. To investigate the possible effects of trans-Planckian physics on the quantum trajectory of the vacuum state of scalar field, we focused on the massless conformally coupled scalar field in the slow-roll inflation, and considered the CJ type dispersion relation with quartic or sextic correction.

We find that there exists a transition in the evolution of the quantum trajectory from well before horizon exit to near horizon exit, providing a possible mechanism for generating a small cosmological constant. Moreover, we find that for the usual dispersion parameter choice, if we compare the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectory, the latter is much smaller than the former. Note that these results about the quantum trajectory also appeared in the case of massless minimally coupled scalar field for CJ type dispersion relations with quartic or sextic corrections [32].

Finally, we calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field, use the backreaction to constraint the magnitude of parameters in nonlinear dispersion relation, and show how the corresponding cosmological constant reduces during the slow-roll inflation at the grand unification phase transition.

Acknowledgments

The author thanks M.-J. Wang for valuable discussions on trans-Planckian physics, and his colleagues at Ming Chi University of Technology for helpful suggestions.

References

[1] Linde A. D., *Particle Physics and Inflationary Cosmology*. Harwood, Chur (1990).

[2] Komatsu E. and Futamase T., Complete constraints on a nonminimally coupled chaotic inflationary scenario from the cosmic microwave background, Phys. Rev. D **59**, 064029 (1999).

[3] Kaiser D. I., Primordial spectral indices from generalized Einstein theories, Phys. Rev. D **52**, 4295 (1995).

[4] Hwang J. and Noh N., COBE constraints on inflation models with a massive non-minimal scalar field, Phys. Rev. D **60**, 123001 (1999).

[5] Qiu T. and Yang K. C., Non-Gaussianities of single field inflation with non-minimal coupling, Phys. Rev. D **83**, 084022 (2011).

[6] Kofman L. and Mukohyama S., Rapid roll inflation with conformal coupling, Phys. Rev. D **77**, 043519 (2008).

[7] Chiba T. and Yamaguchi M., Extended slow-roll conditions and rapid-roll conditions, J. Cosmol. Astropart. Phys. **0810,** 021 (2008).

[8] Kallosh R. and Linde A. D., New models of chaotic inflation in supergravity, J. Cosmol. Astropart. Phys. **1011**, 011 (2010).

[9] Linde A. D., Noorbala M. and Westphal A., Observational consequences of chaotic inflation with nonminimal coupling to gravity, J. Cosmol. Astropart. Phys. **1103,** 013 (2011).

[10] Danielsson U. H., Note on inflation and trans-Planckian physics, Phys. Rev. D **66**, 023511 (2002).

[11] Armendariz-Picon C. and Lim E. A., Vacuum choices and the predictions of inflation, J. Cosmol. Astropart. Phys. **0312,** 006 (2003).

[12] Bohm D., A suggested interpretation of the quantum theory in terms of “hidden” variables I, Phys. Rev. **85**, 166 (1952).

[13] Bohm D., A suggested interpretation of the quantum theory in terms of “hidden” variables II, Phys. Rev. **85**, 180 (1952).

[14] Valentini A., Signal-locality, uncertainty, and the subquantum H-theorem I, Phys. Lett. A **156**, 5 (1991).

[15] Valentini A., Signal-locality, uncertainty, and the subquantum H-theorem II, Phys. Lett. A **158**, 1 (1991).

[16] Valentini A., Astrophysical and cosmological tests of quantum theory, J. Phys. A: Math. Theor. **40**, 3285 (2007).

[17] Valentini A., Inflationary cosmology as a probe of primordial quantum mechanics, Phys. Rev. D **82**, 063513 (2010).

[18] Pinto-Neto N., Santos G. and Struyve W., The quantum-to-classical transition of primordial cosmological perturbations, Phys. Rev. D **85**, 083506 (2012).

[19] Martin J. and Brandenberger R., Trans-planckian problem of inflationary cosmology, Phys. Rev. D **63**, 123501 (2001).

[20] Brandenberger R. and Martin J., The robustness of inflation to changes in super-Planck-scale physics, Mod. Phys. Lett. A **16**, 999 (2001).

[21] Martin J. and Brandenberger R., Corley-Jacobson dispersion relation and trans-Planckian inflation, Phys. Rev. D **65**, 103514 (2002).

[22] Horava P., Quantum gravity at a Lifshitz point, Phys. Rev. D **79**, 084008 (2009).

[23] Horava P., Spectral dimension of the universe in quantum gravity at a Lifshitz point, Phys. Rev. Lett. **102**, 161301 (2009).

[24] Amelino-Camelia G., Arzano M., Gubitosi G. and Magueijo J., Dimensional reduction in the sky, Phys. Rev. D **87**, 123532 (2013).

[25] Amelino-Camelia G., Arzano M., Gubitosi G. and Magueijo, J., Rainbow gravity and scale-invariant fluctuations, Phys. Rev. D **88**, 041303 (2013).

[26] Amelino-Camelia G., Arzano M., Gubitosi G. and Magueijo J., Gravity as the breakdown of conformal invariance, Int. J. Mod. Phys. D **24**, 1543002 (2015).

[27] Huang J.-J. and Wang, M.-J., Green’s functions of scalar field in de Sitter space: discrete lattice formalism.-I, Nuovo Cimento A **100**, 723 (1988).

[28] Huang J.-J., Excited states of de Sitter space scalar fields: lattice Schrödinger picture, Mod. Phys. Lett. A **21**, 1717 (2006).

[29] Huang J.-J., Inflation and squeezed states: lattice Schrödinger picture, Mod. Phys. Lett. A **24**, 497 (2009).

[30] Huang J.-J., Pilot-wave scalar field theory in de Sitter space: lattice Schrödinger picture, Mod. Phys. Lett. A **25**, 1 (2010).

[31] Huang J.-J., Bohm quantum trajectories of scalar field in trans-Planckian physics, Advances in High Energy Physics. Vol. 2012, Article ID 312841, 19 pages, doi:10.1155/2012/312841, (2012).

[32] Huang J.-J., Evaluation of the cosmological constant from de Broglie pilot-wave dynamics: inflation with minimal coupling, International Journal of Advanced Research in Physical Science, Vol.1, Issue 8, pp 35-48 (2014).

[33] Valentini A., *Many worlds? Everett, quantum theory, and reality.* eds. Saunders S., et al., Oxford University Press, Oxford (2010).

[34] Colin S. and Valentini, A., Instability of quantum equilibrium in Bohm's dynamics, Proc. R. Soc. A 470, 20140288 (2014).