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The Way for Obtaining Newton's Second Law without using the Variational Principles of Mechanics

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Abstract: *The equation of motion of the body undergone by different type of forces (the Second Newton's Law) is derived without the implementation of the variational principles of mechanics. It is, however, mathematically equivalent to the familiar formulation. The formulation is based on the introduction of the scalar quantity E=E(**r**(t), **v**(t), t), where **r**(t) is the position of the body, **v**(t) is the velocity of the body and t is the independent time variable. The quantity E introduced is considered to be as a mechanical energy of the body including the potential and kinetic terms. From the condition dE/dt=0 the equation of motion in the form of Newton's law can be derived. Some results are discussed.*

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1. Introduction

The equation of motion of a mechanical system traditionally begins with Newton's laws of motion which relate the force, momentum, and acceleration vectors. Analytical mechanics in the form of the Lagrange equations or Hamiltonian formalism provides an alternative and very powerful tool for obtaining the equations of motion.[1,2] The derivation of Lagrange's equations in advanced mechanics texts typically applies the calculus of variations to the principle of least action. The aim of this communication is to provide a derivation of the second Newton's law straightforwardly from the principle of the energy conservation of the moving body relative to independent time variable.

Let E=E(**r**(t), **v**(t), t) be a scalar function, which inherently links with the material body moving in three dimension space, where **r**(t) is the position of the body and **v**(t)=d**r**/dt is the velocity of the body relative to chosen system of coordinate and t is the independent time variable. The time is considered to be absolute and the motion of the body is taking place relative to rigid orthogonal system of coordinate (for example) in the space. We also assume that E=E(**r**(t), **v**(t), t) is to be invariant relative to the time variable. Therefore

$$\frac{dE}{dt} = 0 \tag{1}$$

Taking into account that E is dependent on **r**(t), **v**(t) and t, we have

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} + \nabla_{\boldsymbol{r}} E \cdot \boldsymbol{v} + \nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a} = 0 \tag{2}$$

Where $\nabla_{\boldsymbol{r}} E$ is the gradient of E over **r** and $\nabla_{\boldsymbol{v}} E$ is the gradient of E over the velocity **v** while **a** = d**v**/dt is the acceleration of the body relative to chosen system of coordinate.

In (2) $\nabla_{\boldsymbol{r}} E \cdot \boldsymbol{v}$ and $\nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a}$ are the dot products. If we multiply the right hand side of (2) by **v,** we get

$$\boldsymbol{v} \cdot \frac{\partial E}{\partial t} + \boldsymbol{v}^2 \cdot \nabla_{\boldsymbol{r}} E + \boldsymbol{v} \cdot \nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a} = 0 \tag{3}$$

Equation (3) now can be divided by $\mathbf{v}^2$ (as a scalar quantity), which leads to the next equation

$$\frac{\boldsymbol{v}}{\boldsymbol{v}^2} \cdot \frac{\partial E}{\partial t} + \nabla_{\boldsymbol{r}} E + \frac{\boldsymbol{v}}{\boldsymbol{v}^2} \cdot \nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a} = 0\ . \tag{4}$$

Suppose for the moment that E does not dependent explicitly on t (or equivalently ∂E/∂t=0). In this case we have

$$\nabla_{\boldsymbol{r}} E + \frac{\boldsymbol{v}}{\boldsymbol{v}^2} \cdot \nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a} = 0. \tag{5}$$

We will redefine operators in (5) as

$$\boldsymbol{F} = -\nabla_{\boldsymbol{r}} E \quad (6)$$

and

$$m = \frac{\boldsymbol{v}}{\boldsymbol{v}^2} \cdot \nabla_{\boldsymbol{v}} E \quad (7)$$

Which are considered to be the force acting upon the body (vector quantity) and the mass of the body (the scalar quantity as a dot product).

Combining (5), (6) and (7) we arrive at

$$\boldsymbol{F} = m\boldsymbol{a} \quad (8)$$

On the other hand, from (6) we can determine E as the path integral over the line **r**(t):

$$E = E^{(p)} = -\int \boldsymbol{F}\, d\boldsymbol{r} + C_1 , \quad (9)$$

where C_1 is an arbitrary constant.

The quantity $E^{(p)}$ may be considered as a special part of the energy of the body, which is known as the potential energy.

If we multiply (7) by **v** we can write

$$\nabla_{\boldsymbol{v}} E = m\boldsymbol{v} = \boldsymbol{p} \, . \quad (10)$$

This quantity may be considered as the impulse of the body.

We will further consider m as a constant quantity which constitutes only the physical essence of the body and does not dependent on t, **v**(t) or **a**(t). The integration of (10) over **v** gives

$$E = E^{(k)} = \int m\boldsymbol{v}\, d\boldsymbol{v} + \ C_{\boldsymbol{2}} = \frac{1}{\boldsymbol{2}} m\boldsymbol{v}^2 + C_2 \quad (11)$$

where C_2 is the constant.

This part of energy E is well known as the kinetic energy of the body. Combining (9) and (11) we have

$$E = \frac{1}{2}\left(E^{(k)} + E^{(p)}\right) + C_3 \quad (12)$$

where C_3 is the constant.

If we subtract (9) from (11), we arrive at the next equation

$$\frac{1}{2} m\boldsymbol{v}^2 + \int \boldsymbol{F}\, d\boldsymbol{r} = const \quad (14)$$

which constitutes the energy conservation law of the moving body.

Therefore, the quantity E may be interpreted as a half of the sum of the potential and the kinetic energy provided that C_3=0. Moreover, the mass and the energy are linked to each other by the expression of (7). It is actually another formulation of the mass and the energy equivalence.

Now let us return to (4) and take into account the term

$$\frac{\boldsymbol{v}}{\boldsymbol{v}^2} \cdot \frac{\partial E}{\partial t} = \boldsymbol{F}' \quad (15)$$

This term represents the additional force acting upon the body in the case when the energy of the body is explicitly dependent on the time variable. If we consider that E is also the function of **v**(t), we may represent ∂E/∂t as

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial v_x}\frac{\partial v_x}{\partial t} + \frac{\partial E}{\partial v_y}\frac{\partial v_y}{\partial t} + \frac{\partial E}{\partial v_z}\frac{\partial v_z}{\partial t} \quad (16)$$

Equation (15) may also be represented as a scalar product

$$\frac{\partial E}{\partial t} = \left(\frac{\partial E}{\partial v_x}\boldsymbol{i} + \frac{\partial E}{\partial v_y}\boldsymbol{j} + \frac{\partial E}{\partial v_z}\boldsymbol{k}\right)\left(\frac{\partial v_x}{\partial t}\boldsymbol{i} + \frac{\partial v_y}{\partial t}\boldsymbol{j} + \frac{\partial v_z}{\partial t}\boldsymbol{k}\right) = \nabla_{\boldsymbol{v}} E \cdot \boldsymbol{a}' \quad (17)$$

where

$$\boldsymbol{a}' = \frac{\partial v}{\partial t} \quad (18)$$

This acceleration of the body may be interpreted as the space acceleration relative to rigid system of coordinates or the local reference-frame acceleration relative to unmoving space.

Combining (4) and (18) we get

$$\boldsymbol{F}' = \frac{\boldsymbol{v}}{\boldsymbol{v}^2}\nabla_{\boldsymbol{v}}E\frac{\partial \boldsymbol{v}}{\partial t} = m\boldsymbol{a}' \quad (19)$$

By means of (19) we can rewrite (8) as

$$\boldsymbol{F} = m(\boldsymbol{a} + \boldsymbol{a}') = m\boldsymbol{a} + m\boldsymbol{a}' = m\boldsymbol{a} + \boldsymbol{F}' \quad (17)$$

Where $\boldsymbol{F}'$ is the additional force, which is usually interpreted as the force of inertia in the case when the reference frame is accelerated.

2. Conclusion

In summary, this work reports the way for analytical obtaining of the second Newton's law from the energy conservation principle. The physical worth for the application of this principle is the fact that additional force acting upon the body must be including into the equation of motion when the energy of the body explicitly dependent on the time variable. For example, the force of inertia is directly included into the equation of motion by means of the principle described.

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Author's Biography

Alexander F. Yaremchuk, Prof. Dr. Ph., received Ph.D. degree in physics and Ph.D. degree in electronic technology from National Research University of electronic technology (MIET) in 1986 and 2011. In 1980 he joined the technology department of the University. In 2012 he was appointed as Full Professor of Nanoelectronics and Functional Electronics. His current research interests involve the numerical simulation of semiconductor devices for electronics and optoelectronic applications, the theoretical study of transport processes in nanostructures. He is also author of more than 50 scientific papers.