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A Comparative Study between Einstein's Theory and Rosen's Bimetric Theory through Perfect Fluid Cosmological Model

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Abstract: *Spatially-homogeneous and anisotropic Bianchi type-1 space time is investigated in Einstein's theory of general relativity and its alternative theory Rosen's [Gen. Rel. Grav., vol.**4** (1973)435] bimetric theory of relativity, when source of the gravitational field is perfect fluid. Considering "gamma law equation of state", false vacuum models of the universe are determined in general relativity which are isotropic, inflationary and de-Sitter universes. However, the same false vacuum model of the universe does not survive in Rosen's bimetric theory of relativity but only vacuum model of the universe exists.*

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1. Introduction

1.1. Einstein's Theory of General Relativity

General theory of relativity developed by Einstein is the only coordinate invariant theory which laid foundation for constructing mathematical models of the universe. The field equations in general theory of relativity given by Einstein are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\, T_{ij} \tag{1}$$

where the units are chosen such that G= 1 = C and R_{ij} is the Ricci tensor, R is the Ricci scalar, g_{ij} is the metric tensor and T_{ij} is the Energy momentum tensor of the matter.

1.2. Rosen's Bimetric Theory of Relativity

It is known that most of the cosmological models based on general theory of relativity developed by Einstein contain initial singularities (the big-bang) from which the universe expands. But this theory has some controversies and lapses for which authors have proposed various alternative and modified theories of it to unify gravitation and matter fields in various forms. Thus to get rid of singularities in the said cosmological models, Rosen [2] proposed a new theory of relativity known as bimetric theory of relativity .This theory consists of two metric tensors at each point of the space time whose role is to determine the physical situations. The first metric tensor g_{ij} determines the Riemannian geometry of the curved space time which plays the same role as in general relativity and it interacts with matter. The back ground metric tensor γ_{ij} refers to the geometry of the empty universe and describes the inertial forces. Also it has no direct physical significance but appears in the field equations. Moreover, it interacts with g_{ij} but not directly with matter. One can regards γ_{ij} as giving the geometry that would exist if there were no matter. This theory satisfies the covariant and equivalence principles and agrees with the theory of general relativity up to the accuracy of observations made till the date.

The spatially homogeneous and anisotropic cosmological models have significant role in the description of the Universe in the early stages of its evolution. Also a perfect fluid satisfactorily describes the distribution of matter due to large-scale distribution of galaxies in our universe. Therefore, we consider to investigate the Bianchi type-1 homogeneous model of the Universe with anisotropic background in presence of perfect fluid corresponding to Einstein's theory and Rosen's bimetric theory. The basic aim of comparative study between both the theories of gravitation is to determine the percentage of resemblances of the models constructed in each theory with the physical universe.

The field equations of bimetric theory of gravitation proposed by Rosen [2] are

$$N_j^i - \frac{1}{2} N \delta_j^i = 8\pi k T_j^i \tag{4}$$

where

$$N_j^i = \frac{1}{2} \gamma^{ab} (g^{hi} g_{hj\,|a})_{|b}$$

and

$$N = N_j^i, \quad (i, j=1, 2, 3, 4)\ ; \qquad k = \sqrt{\frac{g}{\gamma}}$$

together with g = determinant of g_{ij} and γ = determinant of γ_{ij}.

Here the vertical bar (|) denotes the covariant differentiation with respect to γ_{ij} and T_j^i is the energy momentum tensor of the matter.

2. Space –Time and Perfect Fluid Distribution

The Bianchi type-1 space time described by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{5}$$

with A, B, C as functions of cosmic time'‘t’. This ensures that the space-time is anisotropic and spatially homogenous.

The energy momentum tensor for perfect fluid distribution for the space-time (5) is given by

$$T_{ij} = (\rho+p)u_iu_j + pg_{ij} \tag{6}$$

where ρ, p and u^i are respectively the energy density, pressure and unit flow vector of the fluid satisfying

$$u_iu^i = -1\ . \tag{7}$$

For the sake of simplification of field equation and to get the viable solution we consider the ‘gamma law’ equation of state as

$$p = (\gamma-1)\rho\ ,\ 0 \le \gamma \le 1. \tag{8}$$

3. Einstein Field Equations and Solutions

Using co-moving coordinate system, Einstein field equations (1) corresponding to eqn. (6) and (7) for the metric (5) take the following explicit forms:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi p\ , \tag{9}$$

$$\frac{C_{44}}{C} + \frac{A_{44}}{A} + \frac{C_4A_4}{CA} = -8\pi p\ , \tag{10}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi p \tag{11}$$

and $$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} = 8\pi \rho . \tag{12}$$

Here and afterwards the subscript '4' after a field variable represents ordinary differentiation with respect to time t.

For the metric (5), the energy conservation equation of general relativity

$$T^{ij}_{;j} = 0 \tag{13}$$

takes the form

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 . \tag{14}$$

Taking $\gamma = 0$, equation (8) reduces to the form

$$p + \rho = 0 \tag{15}$$

which is known as 'false vacuum' or 'de-generate vacuum (Blome and Prister[3]).

Using eqn.(15) and adding eqns.(9),(10)and(11)with three times of eqn.(12),we get

$$\frac{(ABC)_{44}}{ABC} = 24\pi\rho . \tag{16}$$

Applying relation (15) in equation (14) and then integrating, we obtain

$$\rho = K = -p, \tag{17}$$

where $K(\neq 0)$ is the constant of integration.

On substitution (17), equation (16) reduces to

$$(ABC)_{44} = K_1^2 ABC , \tag{18}$$

where $K_1^2 = 24\pi K$.

On integration, (18) yields

$$[(ABC)_4]^2 = K_1^2 (ABC)^2 + K_2 \tag{19}$$

where K_2 is a constant of integration. For exact integration of (19), put $K_2 = 0$ and then integrating we find

$$ABC = e^{K_1 t + K_3} , \tag{20}$$

which can be expressed in the form

$$A = (e^{K_1 t + K_3})^{n_1} ,\ B = (e^{K_1 t + K_3})^{n_2} ,\ C = (e^{K_1 t + K_3})^{n_3} \tag{21}$$

where n_i ,i=1,2,3 are real constants satisfying the condition

$$\sum_{i=1}^{3} n_i = 1 . \tag{22}$$

Here the over determinacy for determining the field variables A,B and C from the field equations (9) to (12) can be settled by actual substitution of the solutions(21) in the field equations .Thus we obtain

$$\sum_{\substack{i,j=1 \\ i<j}}^{3} n_i n_j = \frac{1}{3}. \tag{23}$$

Now equations (22) and (23) yield an explicit relation

$$\sum_{i=1}^{3} n_i^{2} = \frac{1}{3} \text{ and } n_1 = n_2 = n_3 = \frac{1}{3}. \tag{24}$$

Now subject to restriction (24), eqn. (22) yields the admissible solution

$$A = B = C = (e^{K_1 t + K_3})^{\frac{1}{3}}. \tag{25}$$

Therefore the metric (5) corresponding to the solution (25) can be written as

$$dS^2 = -dT^2 + e^{\frac{2}{3}(K_1 t + K_3)}(dX^2 + dY^2 + dZ^2). \tag{26}$$

Hence the spatially-homogeneous anisotropic Bianchi-type-1 model reduces to spatially-homogeneous, isotropic and false vacuum model in Einstein's theory. It is interesting to note that the cosmological model (26) is a de-Sitter universe and hence an Einstein space.

3.1. Some Physical and Geometrical Aspects of the Model

The physical and kinematical parameters involved in the models (26) are as follows:

The energy density and pressure are given by

$\rho = (-p) = 0$.

Since both energy density and pressure are independent of time, so the model has no singularity at t=0 and the space time reduces to a flat space time.

The spatial volume is found to be

$V = (-g)^{\frac{1}{2}} = ABC = e^{K_1 t + K_3}$.

Now V→ a constant as t→0 and V→∞ as t→∞. Thus it is inferred that the model starts with a finite volume and expands continuously with time. As the model has exponential expansion, the expansion in the universe never ends, which was earlier suggested by Willem deSitter of Leydon. Again the space time is flat space time and has exponential expansion, so the model of the universe obtained is inflationary-universe. Also the universe undergoes strongly first-order phase transition. As the universe super cools into a false vacuum phase, the false-vacuum energy density acts as an effective cosmological constant which triggers an epoch of de-Sitter (exponential) expansion.

The magnitude of scalar expansion θ is given by

$\theta = u^{i}_{;i} = \frac{V_4}{V} = K_1$ (a constant).

Hence the model has fixed expansion throughout the evolution of the universe.

The shear scalar σ is calculated as

$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = 0$ and hence $\sigma = 0$.

This indicates that the model remain isotropic and non-shearing throughout the evolution. $\mathrm{Lim}_{t\to\infty}\left(\frac{\sigma}{\theta}\right) = 0$ Confirms that the model of the universe is point wise isotropic (Szafron, 1977).

The generalized mean Hubble parameter

$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{1}{3}K_1$ (a constant).

As H is found to be constant and not a function of time,so the model is a steady state model.

The scale factor in the model is given by

$S^3 = ABC = e^{K_1 t + K_3}$.

Hence S increases as time increases.

An important observational quantity is the deceleration parameter 'q' which can be defined as

$q = -\frac{VV_{44}}{V_4^2} = -1.$

The sign of q indicates whether the model inflates or not. The +ve sign of q corresponds to the standard decelerating model, whereas the negative sign of q indicates inflation. As the result found here is q=-1, so the model corresponds to an inflationary model of the universe.

The rotation 'ω' given by the vorticity tensor ω_{ij} as

$\omega^2 = \frac{1}{2}\omega_{ij}\,\omega^{ij}$.

Here $\omega^2 = 0$ implies $\omega = 0$ .Since the rotation of the model found to be zero, the model is non-rotating in nature.

The Kretshmann curvature invariant L in model is found to be

$L = \frac{5}{27}\ (K_1)^4 \,.\, e^{-4(K_1\, t + K_3)}$.

So when t→0, L→ a constant and when t→ ∞, L→ 0 .The above results confirms that the models posses no geometrical singularities at t=0.

We know that when the Ricci tensor R_{ij} is proportional to the metric tensor g_{ij} the space time is called Einstein space(Petrov,1969).

Here $R_{ij} = ¼R\, g_{ij}$ is true so the space time is an Einstein space.

It is found that $\frac{\rho}{\theta^2}$ =0.This indicates that the model approaches homogeneity in a point wise (Szafron, 1977).

4. ROSEN'S FIELD EQUATIONS AND SOLUTIONS

The background flat space -time corresponding to the metric (5) is

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \ . \qquad (27)$$

By use of co-moving coordinates and equations (6) and (7), Rosen's field equations (4) for the metrics (5) and (27) can be written as

$$(\frac{A_4}{A})_4 - (\frac{B_4}{B})_4 - (\frac{C_4}{C})_4 = 16\pi k\mathrm{p} \ , \qquad (28)$$

$$(\frac{A_4}{A})_4 - (\frac{B_4}{B})_4 + (\frac{C_4}{C})_4 = -16\pi k\mathrm{p} \ , \qquad (29)$$

$$(\frac{A_4}{A})_4 + (\frac{B_4}{B})_4 - (\frac{C_4}{C})_4 = -16\pi kp \ , \qquad (30)$$

$$(\frac{A_4}{A})_4 + (\frac{B_4}{B})_4 + (\frac{C_4}{C})_4 = 16\pi k\rho \ . \qquad (31)$$

From field equations (28) to (30), we have

$$(\frac{A_4}{A})_4 = (\frac{B_4}{B})_4 = (\frac{C_4}{C})_4 \ . \qquad (32)$$

Adding eqns.(29) and (30), we get

$$(\frac{A_4}{A})_4 = -16\pi kp \ , \qquad (33)$$

Now use of eqn.(32) and (33)in eqn.(31),we can find

$$\rho + 3p = 0 \, . \qquad (34)$$

On substitution $\rho = -p$ from (15) in eqn.(34), we get p = 0 and $\rho = 0$. (35)

As p=0 and $\rho = 0$, so the Bianchi type-1 cosmological model in presence of perfect fluid do not exist in bimetric theory.

Using eqn.(35) in eqn.(33)and then putting the value in eqn.(32),the metric potential are found as

A=B=C= $e^{K_8 t}$, (36)

where K_8 is the constant of integration.

Thus in view of eqn.(36) ,the metric (5) takes the form

$$ds^2 = -dt^2 + e^{2K_8 t}\,[dx^2 + dy^2 + dz^2] \quad . \tag{37}$$

Thus it is observed that Bianchi type-1 cosmological perfect fluid model does not survive in bimetric theory but only the vacuum model of the universe exists. It is interesting that the model (37) is spatially homogeneous, isotropic and has no singularity at t=0.

5. Conclusion

It is observed that cosmological false vacuum model exist in Einstein's theory whereas vacuum model only survives in Rosen's bimetric theory. The model which found in Einstein's theory is uniformly expanding, non-rotating, non-shearing, isotropic and has no geometrical singularity. As Edwin Hubble and M.L.Humason (American astronomers) who after studying the red shift (see Doppler effect) in the spectral lines of the distant galaxies have concluded that the universe is expanding, thus our investigation and result found here reveals that Einstein theory is more viable than Rosen's bimetric theory. It is also interesting to conclude that as the model in Rosen's bimetric theory does not admit singularity which is of physical nature, so Rosen's aim in developing his own theory has been fulfilled in this article.

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