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Fourier-Conjugate Models in the Corpuscular-Wave Dualism Concept

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Abstract: *Description of the wave-particle duality and the uncertainty relation based on the Fourier conjugate mathematical models of the particle in the coordinate space and in the frequency space is discussed. The signals recorded by an observer in the coordinate space and the impulse space are satisfied to the fundamental principle of uncertainty. It follows from the uncertainty relation for the extent of the signal in the coordinate space and for the width of its Fourier spectrum, a special case of which are the relations of Heisenberg uncertainty.*

Keywords: *corpuscular-wave dualism; de Broglie wave, correlation ambiguities, Fourier transformations, uncertainty principle.*

1. Introduction

Corpuscular-wave dualism is one of the concepts used to construct the physical pattern of the world, which are developed on the basis of synthesis of physical images and analogies described in the mathematical language. As was mentioned by de Broglie, Poincare believed that there exist an infinitely large number of logically equivalent points and patterns of reality, while the researcher chooses only one of them, based exclusively on the viewpoint of convenience [1]. The traditional description of corpuscular-wave dualism is based on quantization (sampling) of the energy of a moving corpuscle (particle) by analogy with photons. The corpuscular-wave dualism description is usually based on modeling of moving particles as wave packets and on the optical-mechanical analogy. However, the development of a simple and obvious model that explains how a moving particle acquires wave properties is still an urgent task. This work describes a motivated attempt to construct such a model, based on combining physical and information analogies and contributing to understanding of the wave nature of moving matter.

The brilliant de Broglie's guess about wave properties of moving particles was based on the Hamilton-Jacobi optical-mechanical analogy, quantization and presentation of light fields by fluxes of light particles (photons) possessing wave properties (Pauli, Einstein), and theoretical model implying the equivalence of mass and energy (Poincare, Einstein). The idea of quantization is illustrated by a simple example of a harmonic oscillator. The motion of a harmonic oscillator in the phase plane (in the momentum-displacement coordinates) is described by an elliptical phase trajectory. The area embraced by this trajectory is equal to the ratio of the energy E and frequency ν of oscillations, $I = E/\nu$, which was called the adiabatic invariant because it is conserved at small adiabatic changed in frequency. This means that the ratio of the energy of oscillations of the harmonic oscillator to frequency is equal to the energy derivative with respect to frequency or, if differentials are replaced by small increments, to the ratio of the corresponding increments of energy and frequency:

$$I = \frac{E}{\nu} = \frac{dE}{d\nu} = \frac{\Delta E}{\Delta \nu} = \text{const}\,.$$

This equation suggests a possibility of quantization of the adiabatic invariant, which was called the "action" in analytical mechanics. It is the action that is sampled rather than the energy, which is a continuous function of frequency.

The harmonic oscillator whose adiabatic invariant is sampled and whose quantum of the action is equal to Planck's constant h is a quantum oscillator. A specific feature of the quantum oscillator is the fact that its phase trajectory cannot be a closed curve in accordance with Heisenberg's correlation ambiguities. The uncertainty of the phase trajectory position on the phase plane is $1/2\,h$, and the adiabatic invariant of the quantum oscillator is determined by the formula $I=(n+1/2)h$. As the adiabatic invariant for the harmonic oscillator is much greater than Planck's constant, its changing with frequency may be considered as continuous and its phase trajectory may be considered as closed. The existence of the quantum of the action h is a necessary, but not a sufficient condition for Heisenberg's correlation ambiguities. It follows from corpuscular-wave dualism, which, in turn, is based on combining physical images, analogies, and mathematical transformations, including the Fourier transformation.

Let us consider a harmonic wave propagating in a coordinate space and described by a periodic function of the form $\exp[i(\omega t-\mathbf{kr})]$. The phase of this wave, $\varphi=\omega t-\mathbf{kr}$, is determined by the algebraic sum of the time-dependent $\varphi(t)$ and space-dependent $\varphi(\mathbf{r})$ components. The time-dependent component of the phase is found as the product of the circular frequency ω and the time t. The circular frequency of the wave is determined as $\omega=2\pi\nu=2\pi/T$, where T is the wave period. The space-dependent component of the phase is the product of the wave vector $\mathbf{k}$ by the radius-vector $\mathbf{r}$, which describes the spatial position of the point considered. The absolute value of the wave vector $\mathbf{k}$ is determined by the spatial period of the wave Λ, $|\mathbf{k}|=k=2\pi/\Lambda$. By comparing $\omega=2\pi/T$ and $k=2\pi/\Lambda$, we can see that the frequency ω and the absolute value of the wave vector $|\mathbf{k}|=k$ have similar structures. Therefore, the wave vector $\mathbf{k}$ has the meaning of the spatial frequency. In contrast to the frequency ω, the spatial frequency $\mathbf{k}$ is a vector, which is determined by the projections k_x, k_y, and k_z in the Cartesian coordinate system. The wave is described in the coordinate system (t, x, y, z) in the coordinate space and in the coordinate system (ω, k_x, k_y, k_z) in the frequency space. Correspondingly, if a moving physical object in the coordinate space is defined by the mathematical model $s(t,\mathbf{r})$, the Fourier spectrum $s(\omega,\mathbf{k})$ corresponds to this model in the frequency space.

The frequency space is equivalent to the momentum space because their coordinates differ by the factor $\hbar=h/2\pi$: $\mathbf{p}=\mathbf{k}\hbar$ and $E=\omega\hbar$. The motion of physical objects (corpuscle, macroscopic body, or wave) can be described in the coordinate or frequency (momentum) space. These mappings are related via the Fourier transformation and are consistent with the physical reality. For instance, a flying ball can be described by its position, velocity, and momentum; in the case of its rotation, the call is also described by the angular momentum. Certainly, here we mean mathematical models and signals detected by the observer rather than presentations and transformations of objects themselves. In accordance with the classical definition of matter as the objective reality given to us in sensation, signals may be considered as this sensation. The notions of the spatial frequency and of the frequency and momentum spaces are widely used in science and engineering. Examples are optical information technologies and spectroscopy [2, 3].

Let a moving material particle in the coordinate space be described by the function $s(\mathbf{r}+\mathbf{v}t)$, where $\mathbf{r}$ is the radius-vector of the particle location, $\mathbf{v}$ is the velocity vector of the particle, and t is the time. In the frequency space, it is described by the Fourier spectrum $s(\omega,\mathbf{k})$, which is a function of the spatial frequency $\mathbf{k}=k(\mathbf{v}/\mathrm{v})$ (wave vector) and of the temporal frequency ω. The functions $s(\mathbf{r}-\mathbf{v}t)$ and $s(\omega,\mathbf{k})$ are related via the Fourier transform

$$s(\mathbf{r}+\mathbf{v}t)\leftrightarrow\int\int_{-\infty}^{\infty}s(\mathbf{r}+\mathbf{v}t)\exp(-i\mathbf{kr})\exp(-i\omega t)d\mathbf{r}dt=$$

$$= s(\mathbf{k})\int_{-\infty}^{\infty}\exp[-i(\omega-\mathbf{kv})t]dt = 2\pi s(\mathbf{k})\delta(\omega-\Omega), \qquad (1)$$

where $\delta(\omega-\Omega)$ is the Dirac delta function. Here we use the shift theorem and introduce the notation $\Omega=\mathbf{kv}$. The function $2\pi s(\mathbf{k})\delta(\omega-\Omega)$ is the Fourier spectrum $s(\omega,\mathbf{k})$ of the function $s(\mathbf{r}+\mathbf{v}t)$, which describes the particle moving in the coordinate space. As it follows from Eq. (1), the Fourier spectrum of the signal considered as a moving particle in the frequency space has the form of a δ-function localized at the temporal frequency $\Omega=\mathbf{kv}$ and the spatial frequency $\mathbf{k}=\frac{\mathbf{v}}{\mathrm{v}^2}\Omega$. The directions of the corpuscle velocity $\mathbf{v}$ and the wave vector (spatial frequency) $\mathbf{k}$ coincide, whereas the absolute value of the wave vector $\mathbf{k}$ (wave number *k*) is determined by the ratio of the frequency Ω to the velocity v:

$$|\mathbf{k}| = k = \frac{\Omega}{\mathrm{v}}$$

The frequency Ω equal to the product of the wave vector and the velocity vector of particle motion $\Omega=\mathbf{kv}$, which is identical to the formula for the Doppler frequency shift and testifies to the kinematic nature of the frequency Ω. The velocity $\mathbf{v}$ is determined by the relative velocity of the corpuscle and observer coordinate systems; for the wave induced by corpuscle, it is determined by the group velocity. The dimension of the velocity as the ratio of the dimensions of the temporal and spatial frequencies, $[v]=[\omega]/[k]$, indicating the reality of the wave-induced motion process.

Using the definition of the group velocity as a derivative of the frequency with respect to the wave number $v=\frac{d\Omega}{dk}$, we write simple transformations of the expression for the frequency Ω:

$$\Omega = \mathbf{kv} = \frac{\mathbf{kv}}{\mathrm{v}}\frac{d\Omega}{dk}. \qquad (2)$$

According to the quantum mechanics concept, we have $\Omega=\frac{E}{\hbar}$. As the energy *E* is the kinetic energy of particle motion, $E=\frac{mv^2}{2}=\frac{p^2}{2m}$, where m is the particle mass and $\mathbf{p}=m\mathbf{v}=\frac{\mathbf{v}}{\mathrm{v}}p$ is the particle momentum, Eq. (2) yields

$$\Omega = \frac{\mathbf{kv}}{\mathrm{v}\hbar}\frac{dE}{dk} = \frac{\mathbf{kv}}{\mathrm{v}\hbar}\frac{d}{dk}\left(\frac{p^2}{2m}\right) = \frac{\mathbf{kv}}{\mathrm{v}}\frac{p}{\hbar m}\frac{dp}{dk} = \mathbf{kv}$$

from whence $\frac{1}{\hbar}\frac{dp}{dk}=1$ and

$$p = \hbar\frac{2\pi}{\Lambda_B} = \hbar k_B,$$

where Λ_B is the de Broglie wave length and k_B is the de Broglie wave number:

$$\Lambda_B = \frac{h}{p},\ k_B = \frac{2\pi}{\Lambda_B} = \frac{p}{\hbar}. \qquad (3)$$

According to Eq. (3), the length of the de Broglie wave induced by particle motion is determined by the ratio of Planck's constant to the momentum. The kinetic energy of the moving particle, as well as its momentum, it relative in the observer coordinates system.

Corpuscular-wave dualism is a fundamental property of moving matter; its universality follows from the quantum concept and adequate presentation of corpuscle motion in the Fourier-conjugate coordinate and frequency spaces. It is related to the correlation ambiguities, which have a universal character and follow from the uncertainty principle for the functions $s(t)$ and $s(\Omega)$ describing the moving particle, which are considered as the Fourier-conjugate signals in the coordinate and frequency spaces. These functions bear information about the moving particle and satisfy the uncertainty principle for the signals

$$\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt \int_{-\infty}^{\infty} \Omega^2 |s(\Omega)|^2 d\Omega \geq \frac{\pi}{2} \mathrm{E}^2 ,$$

where E is the signal energy; $\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt$ is the energy of the second derivative of the Fourier spectrum of the signal by frequency; $\frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega^2 |s(\Omega)|^2 d\Omega$ is the energy of the second derivative signal by time. The uncertainty principle yields the uncertainty ratio

$$\Delta t \Delta\Omega \geq \frac{1}{2}, \tag{4}$$

where Δt and $\Delta\Omega$ are the signal length in the coordinate space and the width of its Fourier spectrum, respectively:

$$(\Delta t)^2 = \frac{1}{E_s} \int_{-\infty}^{\infty} t^2 |s(t)|^2 dt ,$$

$$(\Delta\Omega)^2 = \frac{1}{2\pi E_s} \int_{-\infty}^{\infty} \Omega^2 |s(\Omega)|^2 d\Omega .$$

The functions that describe quantum mechanics objects may be considered as signals in the coordinate and frequency spaces because the frequency space is an analog of the momentum space. Multiplying Eq. (4) by Planck's constant, we obtain

$$\Delta t \Delta E \geq \frac{\hbar}{2}, \tag{5}$$

where $\Delta E = \hbar \Delta\Omega$. This is the ratio of Heisenberg's ambiguities for fluctuations of time intervals and energy at quantum mechanics scales.

The momentum and frequency spaces in quantum mechanics are related as $\mathbf{p} = \hbar \mathbf{k}$. Multiplying and dividing the left-hand side of inequality (4) by the group velocity v, e.g., in the *z* direction, and taking into account that $\mathrm{v}\Delta t = \Delta z$ and $\frac{\Delta\Omega}{\mathrm{v}} = \Delta k_B$, we obtain

$$\Delta z \Delta k_B \geq \frac{1}{2}, \tag{6}$$

where $k_B = \Omega/\mathrm{v}$. Expression (6) relates the uncertainties of the spatial Δz and frequency Δk_B coordinates. Multiplying inequality (6) by Planck's constant and taking into account that $\hbar k_B = p$, we obtain Heisenberg's correlation ambiguities for a quantum mechanics object in the coordinate and momentum spaces:

$$\Delta z \Delta p \geq \frac{\hbar}{2}. \tag{7}$$

This result in the same way follows from the principle of uncertainty, and the uncertainty relation for the spatial signal $\left(\Delta z \Delta k \geq \frac{1}{2} \right)$ when multiplying this inequality on $\hbar$. Where Δk - the uncertainty of the spatial frequency in the chosen direction, Δz - the uncertainty of spatial coordinates. In quantum mechanics, the minimum uncertainty $\Delta z \Delta p = \frac{\hbar}{2}$ occurs when a quantum particle is modeled by a Gaussian wave packet envelope. In this case the analogy traces with a minimum of uncertainty for the time-dependent $\left(\Delta t \Delta \Omega = \frac{1}{2}\right)$ and spatial $\left(\Delta z \Delta k = \frac{1}{2}\right)$ Gaussian signals resulting from the uncertainty principle [3, 4]. There is no contradiction in corpuscular-wave dualism; it just means that an adequate method of description is determined by the chosen method of observation [4]. The spatial-frequency image of a moving object as the Fourier spectrum of a physical signal is feasible. Its wave properties are manifested, for example, in diffraction phenomena and in the Doppler effect.

2. Conclusion

Wave-particle dualism is based on the synthesis of physical images, analogies and displayed on language of mathematics. The concept of wave-particle duality can be described in terms of the Fourier conjugate mathematical models of movement particle. These models are adequately displayed as signals in the coordinate space and in the frequency (pulse) space with regard to the quantum of action. The signals detected by an observer in the coordinate and in the momentum spaces satisfy to the fundamental principle of uncertainty. It follows from the uncertainty relation for the extent of the signal in the coordinate space and the width of its Fourier spectrum, a special case of which the Heisenberg uncertainty relation is.

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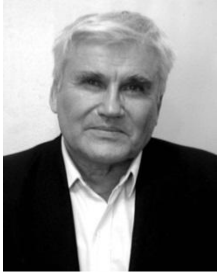
Authors’ Biography



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