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Effect of Magnetic Field on Marangoni Convection in Relatively Hotter or Cooler Liquid Layer

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Abstract: *The effect of uniform vertical magnetic field on the onset of surface tension driven convection in a relatively hotter or cooler layer of electrically conducting liquid is considered. A Fourier series method is used to obtain the characteristic value equation for the Marangoni number M. When instability sets in as stationary convection it is established numerically that both the critical Marangoni number and wave number increase with intensity of the magnetic field, irrespective of whether the liquid layer is relatively hotter or cooler. The asymptotic behavior of the critical Marangoni number for large values of the Chandrasekhar number is also obtained.*

Keywords: *Conducting, Convection, Linearstability, Stationary, Surface-tension.*

1. Introduction

The phenomenon of the onset of surface tension induced convection in a thin horizontal liquid layer heated from below with free upper surface was first established theoretically by Pearson [1] who demonstrated that if the surface tension of the free surface is linearly dependent on temperature then convective instability can occur analogous to those described by Rayleigh [2] in terms of buoyancy (discussed in detail by Chandrasekhar [3]). Nield [4] considered the combined effects of surface tension and buoyancy on the onset of convection in a fluid layer heated from below with free upper surface and found that the two effects causing instability reinforce one another and that as the thickness of the fluid layer decreases, the surface tension effects become more dominant. Further contributions made by many researchers namely, Scriven and Sternling [5], Smith [6], Davis [7], Takashima [8, 9] and Gupta and Surya [10] have refined Pearson's model by incorporating more realistic conditions. For a detailed study of Marangoni convection one may be referred to the work of Normand et al. [11], Koschmieder [12] and Schatz et al. [13].

In view of the stabilizing nature of magnetic field, a fact that has already been established by Chandrasekhar [3] for the buoyancy driven convection, by Nield [14] for the convective instability induced by both surface tension and buoyancy. The influence of magnetic field on the pure Marangoni convection in an electrically conducting liquid layer heated from below have been discussed by Rudraiah et al [15], Maekawa and Tanasawa [16], and by Wilson [17].

Recently, the onset of Marangoni convection in a relatively hotter or cooler liquid layer has been analyzed by Gupta and Shandil [18] and established that irrespective of the thermal nature (conducting or insulating) of the lower boundary, the critical Marangoni number significantly depends on whether the liquid layer is relatively hotter or cooler, and hotter the liquid layer more the postponement of the onset of convection. The problem considered here is a generalization of Gupta and Shandil [18] work to include the effect of vertical magnetic field. A Fourier series method is used to obtain the characteristic value equation analytically. The numerical results are obtained for a wide range of values of the Chandrasekhar number Q. It is shown numerically that both the critical Marangoni number M_c and the critical wave number a_c increase monotonically with Q for a fixed value of the parameter $\alpha_2 T_0$ where T_0 and α_2 being the appropriately chosen mean temperature and coefficient of specific heat (at constant volume) variation due to temperature variation respectively of the fluid layer. The asymptotic behavior of the critical Marangoni number for large values of the Chandrasekhar number is also obtained.

2. Formulation of the Problem

We wish to examine the stability of an infinite horizontal electrically conducting liquid layer of uniform thickness *d* subject to strength externally imposed vertical magnetic field of *H*, which is heated from below with the upper free surface open to the ambient air, where surface tension gradients arise due to temperature perturbations. We choose a Cartesian coordinate system of axes with the *x* and *y* axis in the plane of the lower surface and the *z*- axis along the vertically upward direction so that the fluid is confined between the planes at $z = 0$ and $z = d$. A temperature gradient is maintained across the layer by maintaining the lower boundary at a constant temperature T_0 and the upper boundary at T_1 ($< T_0$). It is assumed that surface tension is given by the simple linear law $\tau = \tau_1 - \sigma(T - T_1)$ where the constant τ_1 is the unperturbed value of τ at the unperturbed surface temperature $T = T_1$ and $-\sigma = (\partial\tau / \partial T)_{T=T_1}$ represents the rate of change of surface tension with temperature, evaluated at temperature T_1, and surface tension being a monotonically decreasing function of temperature, σ is positive.

Following Gupta and Shandil [18], we can write the linearized perturbation equations for an electrically conducting liquid in the presence of magnetic field are

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w = \frac{\mu H}{4\pi\rho}\frac{\partial}{\partial z}\nabla^2 h_z \tag{1}$$

$$(1 - \alpha_2 \mathrm{T}_0)\left(\frac{\partial\theta}{\partial t} - \beta w\right) = \kappa\nabla^2\theta \tag{2}$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H.\frac{\partial w}{\partial z} \tag{3}$$

Where w, θ and h_z denote respectively the z-component of velocity perturbation and temperature perturbation from uniform vertical temperature gradient, and the z-component of the perturbation from the basic vertical magnetic field *H*, ν is the kinematic viscosity; κ is the thermal diffusivity, permeability μ and resistivity η are each assumed constant. β is the temperature gradient which is maintained, T_0 is the unperturbed temperature of the lower surface and each is assumed constant. Further, that coefficient α_2 (due to variation in the temperature) is a constant that ranges from 0 to 10^3 for the liquid with which we are concerned. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, and *t* denotes time.

In seeking solutions of the equations (1), (2) and (3), we must satisfy certain boundary conditions, The boundary conditions at the lower rigid and conducting surface $z = 0$ are straightforward and given by

$$w = 0 \tag{4}$$

$$\frac{\partial w}{\partial z} = 0 \tag{5}$$

$$\theta = 0 \tag{6}$$

The boundary conditions at the upper free surface $z = d$ are more complicated.

Because of the non-deflecting surface, the normal component of the velocity must vanish, that is,

$$w = 0 \tag{7}$$

The stress-balance condition satisfy the equation

$$\rho\nu\frac{\partial^2 w}{\partial z^2} = \sigma\nabla_1^2\theta \tag{8}$$

Here ρ is the density and $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The boundary condition (8) is usually referred to as the Marangoni boundary condition (Pearson [6]). Finally, if we consider conservation of heat transport across the upper free surface, then we have

$$-k\frac{\partial \theta}{\partial z} = q\theta \tag{9}$$

where k is the thermal conductivity of the fluid and q is the heat transfer coefficient.

We now suppose that the perturbations w, θ and h_z are of the form

$$[w(x,y,z,t),\ \theta(x,y,z,t),\ h_z(x,y,z,t)] = [w(z),\ \theta(z),\ h_z(z)]\exp\{i(a_x x + a_y y) + pt\} \tag{10}$$

where $a = \sqrt{a_x^2 + a_y^2}$ is the wave number of the disturbance and p is a time constant (which can be complex). Then express the equations obtained by substituting expression (10) in equations (1)-(3) in dimensionless form by taking $z_* = \frac{z}{d}$, $a_* = ad$, $K_* = \frac{h_z \eta}{\nu H}$, $p_* = \frac{pd^2}{\nu}$, $W_* = \frac{wd}{\nu}$, $\Theta_* = \frac{\theta\kappa}{\beta d \nu}$ and $D = d\frac{d}{dz}$.

Restricting to the case when instability sets in as stationary convection, that is, when the marginal state is characterized by setting $p = 0$ in the resulting equations. Eliminating K from these equations, and omitting asterisk for simplicity, we obtain

$$[(D^2 - a^2)^2 - QD2]W = 0 \tag{11}$$

$$(D^2 - a^2)\theta = -(1 - \alpha_2 T_0)W \tag{12}$$

and the corresponding boundary conditions are

$$\left.\begin{aligned} W(0) &= 0 \\ DW(0) &= 0 \\ \Theta(0) &= 0 \end{aligned}\right\} \tag{13a, b, c}$$

evaluated on the lower rigid boundary $z = 0$, and

$$\left.\begin{aligned} W(1) &= 0 \\ D^2W(1) &= -a^2 M\Theta(1) \\ D\Theta(1) &= -L\Theta(1) \end{aligned}\right\} \tag{14a, b, c}$$

evaluated on the upper free surface $z = 1$, where $Q = \frac{\mu H^2 d^2}{4\pi\rho\nu\eta}$ is the Chandrasekhar number, $M = \frac{\sigma\beta d^2}{\rho\kappa\nu}$ is the Marangoni number and $L = \frac{qd}{k}$ is the Biot number.

Solution to equations (11) and (12) is sought subject to boundary conditions (13a, b, c)–(14a, b, c). Thus we have an eigenvalue problem of order six. It is evident that when Q = 0 the system reduces to the case which has been analyzed by Gupta and Shandil [18].

3. Solution of the Problem

The Fourier series method as presented by Nield [4] is convenient for the problem under consideration. The constants to be eliminated are denoted by

$$\lambda_1 = D^2W(0),\ \lambda_2 = D^2W(1),\ \lambda_3 = \Theta(1).$$

We let

$$W = \sum_{n=1}^{\infty}\left[A_n - \frac{2}{n^3\pi^3}\{\lambda_1 - (-1)^n\lambda_2\}\right]\sin n\pi z \tag{15}$$

$$\theta = \sum_{n=1}^{\infty}\left[B_n - \frac{2}{n\pi}(-1)^n \lambda_3 \sin n\pi z\right] \tag{16}$$

where the boundary conditions (13a), (13c) and (14a) have already been used while writing equation (15) and (16)

Then, we have

$$D^2W = \sum_{n=1}^{\infty}[A_n(-n^2\pi^2) + \frac{2}{n\pi}\{\lambda_1 - (-1)^n \lambda_2\}]\sin n\pi z \tag{17}$$

$$D^4W = \sum_{n=1}^{\infty} A_n(n^4\pi^4)\sin n\pi z \tag{18}$$

$$D^2\Theta = \sum_{n=1}^{\infty} B_n(-n^2\pi^2)\sin n\pi z \tag{19}$$

The differential equations (11) and (12) are satisfied by substituting the complete Fourier expansions for W, Θ and their derivatives from equations (15)-(19) and equating the coefficients of $\sin n\pi z$, we obtain

$$[(n^2\pi^2 + a^2)^2 + Qn^2\pi^2]A_n = \frac{2}{n^3\pi^3}[\lambda_1 - (-1)^n \lambda_2]\{a^2(2n^2\pi^2 + a^2) + Qn^2\pi^2\} \tag{20}$$

$$(1-\alpha_2 T_0)A_n - (n^2\pi^2 + a^2)B_n = \frac{2(1-\alpha_2 T_0)}{n^3\pi^3}[\lambda_1 - (-1)^n \lambda_2] - \frac{2a^2}{n\pi}(-1)^n \lambda_3, \tag{21}$$

The remaining boundary conditions require that

$$\sum_{n=1}^{\infty} n\pi A_n - \frac{2}{n^2\pi^2}\lambda_1 + \frac{2}{n^2\pi^2}(-1)^n \lambda_2 = 0, \tag{22}$$

$$\lambda_2 + a^2 M \lambda_3 = 0, \tag{23}$$

$$\sum_{n=1}^{\infty}(-1)^n n\pi B_n + (1+L)\lambda_3 = 0, \tag{24}$$

From equations (20)-(21), A_n and B_n can be expressed in terms of λ_1, λ_2 and λ_3 which are when substituted in equations (22), (24) and on making use of equation (23), then yield two homogeneous equations in λ_1 and λ_2. Elimination of these constants gives the eigenvalue equations as

$$\begin{vmatrix} \sum_{n=1}^{n} E_n & \sum_{n=1}^{n}(-1)^n E_n \\ (1-\alpha_2 T_0)\sum_{n=1}^{n}(-1)^n F_n & (1-\alpha_2 T_0)\sum_{n=1}^{n}(-1)^n F_n - \frac{1}{M a^2}(a^2\sum_{n=1}^{n} H_n + \frac{L+1}{2}) \end{vmatrix} = 0 \tag{25}$$

where $E_n = \dfrac{n^2\pi^2(n^2\pi^2 + a^2)}{R_n}$, $F_n = \dfrac{n^2\pi^2}{R_n}$, $H_n = \dfrac{1}{(n^2\pi^2 + a^2)}$

with $R_n = (n^2\pi^2 + a^2)[(n^2\pi^2 + a^2)^2 + n^2\pi^2 Q]$.

From the eigenvalue equation (25), M can be determined as a function of a, Q, $\alpha_2 T_0$ and L as the ratio of two determinants

$$M(a,\,\alpha_2 T_0,\,\mathrm{Q},\,L)\;=\;\frac{1}{a^2(1-\alpha_2 T_0)}\,\frac{\begin{vmatrix}\sum_{n=1}^{n}E_n & 0\\ \sum_{n=1}^{n}(-1)^n F_n & a^2\sum_{n=1}^{n}H_n+\frac{L+1}{2}\end{vmatrix}}{\begin{vmatrix}\sum_{n=1}^{n}E_n & \sum_{n=1}^{n}(-1)^n E_n\\ \sum_{n=1}^{n}(-1)^n F_n & \sum_{n=1}^{n}F_n\end{vmatrix}} \tag{26}$$

4. Numerical Results and Discussion

The numerical calculations may be carried out as follows. For fixed values of $\alpha_2 T_0$, Q and L, expression (26) gives the Marangoni number M as a function of the wave number a. The minimum value of M is the critical Marangoni number M_c and the value of a at which M attains the minimum is the critical wave number a_c. For given values of $\alpha_2 T_0$ and Q it may be mentioned here that the system was found to be less stable for an insulating upper free surface (L = 0), which is as expected, since an conducting upper free surface ($L\to\infty$) yields a completely stable system so far as the surface tension driven convection is concerned. Values of critical Marangoni number M_c and the critical wave number a_c computed from expression (26) for assigned values of $\alpha_2 T_0$ and Q, are listed in Table 1 and Table 2 respectively for L = 0 and $L\to\infty$.

In either case it is evident that for a given value of $\alpha_2 T_0$ both M_c and a_c increase monotonically with Q, and that for a fixed value of Q, an increase in the value of $\alpha_2 T_0$ leads to an increased value of M_c while the value of critical wave number a_c remains unchanged. When $\alpha_2 T_0$ = 0, the critical Marangoni number and the corresponding wave number obtained here are identical to the results obtained by Nield [14] (for the case when the Rayleigh number R = 0).

Table1. *Numerical values of M_c and a_c, for various values of Q when L=0*

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	M_c	a_c	M_c	a_c	M_c	a_c
0	79.607	1.933	113.784	1.933	159.213	1.933
10	104.223	2.181	148.890	2.181	208.445	2.181
50	189.873	2.630	271.246	2.630	379.745	2.630
100	284.222	2.959	406.032	2.959	568.445	2.959
500	919.777	4.080	1313.97	4.080	1839.55	4.080
1000	1632.47	4.745	2332.1	4.745	3264.94	4.745
2500	3624.93	5.838	5178.47	5.838	7249.85	5.838
10000	12830.2	8.092	18328.8	8.092	25660.3	8.092

For a fixed value of $\alpha_2 T_0$, the asymptotic behavior of M_c and a_c when $Q\to\infty$ depend critically on the thermal conditions at the upper surface. We found from the eigenvalue equation (28) that when L = 0

$$\left(1-\alpha_2 T_0\right)\frac{M_c}{Q}\sim 1.28 \text{ and } a_c\to 0.81Q^{\frac{1}{4}},$$

Table2. *Numerical values of Mc and ac , for various values of Q when* $L\to\infty$

Q	$\alpha_2 T_0 = 0$		$\alpha_2 T_0 = 0.3$		$\alpha_2 T_0 = 0.5$	
	M_c / L	a_c	M_c / L	a_c	M_c / L	a_c
0	32.073	3.014	45.819	3.014	64.146	3.014
10	38.068	3.880	54.382	3.880	76.135	3.880
50	55.744	4.361	79.635	4.361	111.489	4.361
100	71.989	5.198	102.842	5.198	143.979	5.198
500	149.519	9.123	213.599	9.123	299.039	9.123
1000	210.699	12.499	300.999	12.499	421.398	12.499

2500	333.026	19.655	475.752	19.655	666.053	19.655
10000	666.064	39.306	951.520	39.306	1332.130	39.306

and when $L \to \infty$

$$(1-\alpha_2 T_0)\frac{M_c}{LQ^{\frac{1}{2}}} \sim 6.66 \ \text{ and } a_c \to 0.393Q^{\frac{1}{2}}.$$

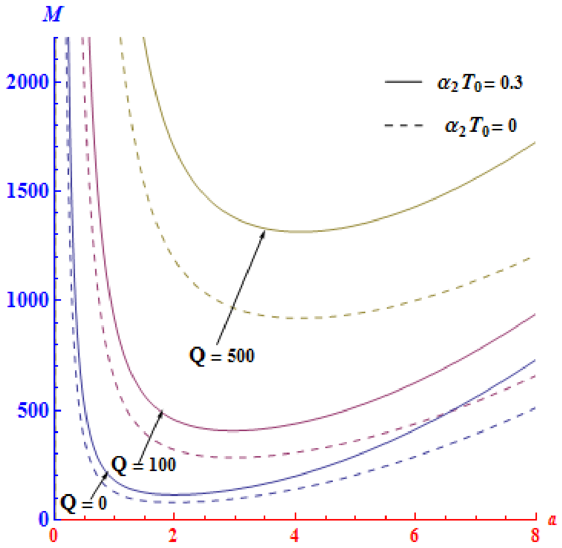

Figure1. *Neutral stability curves for various values of Q when* $\alpha_2 T_0$ *= 0 and* $\alpha_2 T_0$ *= 0.3 with L = 0.*

In Figure 1, the neutral stability curves are plotted for insulating upper free surface (*L* = 0), and $\alpha_2 T_0 = 0$, $\alpha_2 T_0 = 0.3$ using relation (28) for various values of Q. The region below each curve represents the stable state. From Fig. 1, we observed that the neutral stability curves move upwards for increasing values of *Q*, clearly showing the stabilizing effect of *Q*, irrespective of whether the liquid layer is relatively hotter ($\alpha_2 T_0 = 0.3$) or cooler ($\alpha_2 T_0 = 0$).

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