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Plane Symmetric Cosmological Model with Interacting Dark Matter and Holographic Dark Energy Using Power-Law Volumetric Expansion

V. B. Raut[1], K. S. Adhav[2], D. K. Joshi[3]

Department of Mathematics,
Sant Gadge Baba Amravati University,
Amravati, INDIA.
[2]*ati_ksadhav@yahoo.co.in*

Abstract: *The plane symmetric cosmological model filled with interacting Dark matter and Holographic dark energy has been studied. The solutions of the field equations have been obtained under the assumption of constant deceleration parameter using power law volumetric expansion, in particular. The physical and geometrical aspects of the model are also discussed.*

Keywords: *Plane symmetric space-time, Interacting dark fluids, deceleration parameter, State finder parameters, Coincidence problem.*

1. Introduction

Riess *et al*. (1998) and Perlmutter *et al.* (1999)] have shown in their cosmological observations of Type Ia supernovae (SNeIa) that the universe is currently accelerating. The observations of cosmic microwave background (CMB) [Bennett *et al.* (2003), Spergel *et al.*(2003)] and large scale structure (LSS) [Tegmark *et al* (2004), Tegmark *et al.* (2004)], strongly suggest that the universe is spatially flat and dominated by an exotic component with large negative pressure called as dark energy (DE) [Weinberg (1989) , Carroll (2001) , Peebles and Ratra (2003), Padmanabhan (2003)].

A special class of cosmological models in which holographic Dark Energy is allowed to interact with Dark Matter have been studied by Carvalho and Saa, 2004; Gong, 2004; Pavon and Zimdahl, 2005; Gong and Zhang, 2005; Wang *et al*. ,2006; Nojiri and Odintsov, 2006; Guberina, *et al*. 2006 ; Guo, *et al*. 2007 ; Li, *et al*. 2006; Setare, 2006; Sadjadi, 2007; Banerjee and Pavon, 2007; Zimdahl, 2008).

Sarkar (2014a, 2014b, 2014c) studied non-interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type-I and V universe and interacting holographic dark energy in Bianchi type-II respectively.

Here, we have considered the plane symmetric cosmological model filled with interacting Dark matter and Holographic dark energy. The solutions of the field equations have been obtained under the assumption of constant deceleration parameter using power law volumetric expansion**.** The physical and geometrical aspects of the model are also discussed.

2. Metric and Field Equations

We have considered the line element for plane symmetric in the form as

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 , \tag{2.1}$$

Where A and B are the scale factors and functions of the cosmic time t only.

This form (2.1) has been used by Zhang & Noh 2009, Setare & Momeni 2010, Shen & Zhao 2012.

The Einstein's field equations are ($8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = -{}^{m}T_{ij} + {}^{\Lambda}T_{ij} \quad , \tag{2.2}$$

Where ${}^{m}T_{ij} = \rho_m \, u_i \, u_j$ and ${}^{\Lambda}T_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} \, p_\Lambda$ (2.3)

Are matter tensors for dark matter (pressure less i.e. $w_m = 0$) and holographic dark energy. Here ρ_m is the energy density of dark matter and ρ_Λ and p_Λ are the energy density and pressure of holographic dark energy.

The Einstein's field equations (2.2) for metric (2.1) with the help of equations (2.3) can be written as

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho_m + \rho_\Lambda \ , \tag{2.4}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -p_\Lambda \, , \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p_\Lambda \, , \tag{2.6}$$

Where overhead dot ($\cdot$) represents derivative with respect to time *t*.

We further assume that both components do not conserve separately but interact with each other in such a manner that the balance equations reduce to the form

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q \tag{2.7}$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1 + w_\Lambda)\rho_\Lambda = -Q \ , \tag{2.8}$$

Where $w_\Lambda = p_\Lambda / \rho_\Lambda$ is the equation of state parameter for holographic dark energy and $Q > 0$ measures the strength of the interaction.

Models with interaction of dark matter and dark energy were introduced by Wetterich (1988, 1995); Billyarb & Coley (2000) and Horvat (2004).

A vanishing Q implies that matter and dark energy remain separately conserved. Guo *et al.*, 2007 and Amendola *et al.*, 2007 have shown that the interaction between dark energy and dark matter could be expressed as

$$Q = 3b^2 H \rho_m = b^2 \frac{\dot{V}}{V} \rho_m \quad , \tag{2.9}$$

Where b^2 is coupling constant.

Cai & Wang (2005) used same relation for interacting phantom dark energy and dark matter in order to avoid the coincidence problem.

From equations (2.7) and (2.9), we get the energy density of dark matter as

$$\rho_m = \rho_0 V^{(b^2 - 1)} \quad , \tag{2.10}$$

Where $\rho_0 > 0$ is a real constant of integration.

Using equations (2.9) and (2.10), we get the interacting term as

$$Q = 3 \, \rho_0 b^2 H V^{(b^2 - 1)} \ . \tag{2.11}$$

3. COSMOLOGICAL SOLUTIONS FOR CONSTANT DECELERATION PARAMETER

In order to obtain the solutions of the field equations (2.4)-(2.6), we impose a law of variation for the Hubble parameter which yields the constant value of deceleration parameter. This law was first introduced by Berman, 1983.

According to this law the variation of the mean Hubble parameter for plane symmetric space-time is given by

$$H = k(A^2B)^{-m/3}, \tag{3.1}$$

Where $k > 0$ and $m \geq 0$ are constants.

The volume V in terms of scale factors *is* given by

$$V = a^3 = A^2B. \tag{3.2}$$

The directional Hubble parameters in the directions of x, y and z axes respectively are defined as

$$H_x = H_y = \frac{\dot{A}}{A}, \quad H_z = \frac{\dot{B}}{B}. \tag{3.3}$$

The mean Hubble parameter H is given by

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{3.4}$$

The deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{3.5}$$

Equating equation (3.1) with (3.4) and integrating we get

$$V = A^2B = c_1e^{3kt} \quad \text{, For } m = 0, \tag{3.6}$$

And

$$V = A^2B = \ mkt + c_2\ ^{\frac{3}{m}} \text{, For } m \neq 0 , \tag{3.7}$$

Where c_1 and c_2 are positive constant of integration.

Using (3.1) with (3.6) for $m = 0$ and with (3.7) for $m \neq 0$, the mean Hubble parameters are

$$H = k \text{, For } m = 0, \tag{3.8}$$

And

$$H = k\ \ mkt + c_2\ ^{-1}, \text{ For } m \neq 0. \tag{3.9}$$

Using (3.2), (3.6) and (3.7) in (3.5), we get constant values for the deceleration parameter for mean scale factor as

$$q = -1 \text{, For } m = 0 , \tag{3.10}$$

And

$$q = m - 1 \text{, For } m \neq 0. \tag{3.11}$$

The sign of q indicates whether the model accelerates or not. The positive sign if q $m > 1$ corresponds to decelerating models where as the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicates acceleration and $q = 0$ for $m = 1$ corresponds to expansion with constant velocity.

Model for $m \neq 0$ **[Power *Law Volumetric Expansion Model]*:**

Subtracting equation (2.6) from equation (2.5) and using equation (3.2), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0\ . \qquad (3.12)$$

On integration of equation (3.12) and considering equation (3.7), we obtain

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=c_3\ \ mkt+c_2\ ^{\frac{-3}{m}}, \qquad (3.13)$$

On integration of (3.13) and using (3.2), we get exact values of the scale factors as:

$$A=\ c_4\ ^{\frac{1}{3}}\,(mkt+c_2)^{1/m}\exp\left(\frac{c_3}{3k(m-3)}(mkt+c_3)^{\frac{m-3}{3}}\right), \qquad (3.14)$$

$$B=\ c_4\ ^{-\frac{2}{3}}\,(mkt+c_2)^{1/m}\exp\left(-\frac{2c_3}{3k(m-3)}(mkt+c_3)^{\frac{m-3}{3}}\right), \qquad (3.15)$$

Where c_3 and c_4 is constant of integration.

Using equations (3.14) and (3.15) in equation (3.2), the volume V of the universe is given by

$$V=\ \ mkt+c_2\ ^{\frac{3}{m}}\ . \qquad (3.16)$$

Using equations (3.14) and equation (3.15) in equation (3.4) and equation (3.5), we get the mean Hubble parameter and deceleration parameter as

$$H=k\ \ mkt+c_2\ ^{-1}.$$
(3.17)

$$q=m-1. \qquad (3.18)$$

The mean anisotropy parameter of expansion is defined as $\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2$ and given by

$$\Delta=2\left(\frac{c_3 m}{9k}\right)^2\ \ mkt+c_2\ ^{\frac{2(m-3)}{3}}\ . \qquad (3.19)$$

Using equation (3.16) in equations (2.10) and (2.11), we get

$$\rho_m=\rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}}\ . \qquad (3.20)$$

$$Q=3kb^2\rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}-1}\ . \qquad (3.21)$$

Using equations (3.14) and (3.15) and (3.20) in the equation (2.4), we obtain the energy density of holographic dark energy as

$$\rho_\Lambda=3k^2(mkt+c_2)^{-2}-\frac{1}{27}c_3^2m^2(mkt+c_2)^{\frac{2(m-6)}{3}}-\rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}}\ . \qquad (3.22)$$

Using equations (3.14) and (3.15) in the equation (2.5), we obtain the pressure of holographic dark energy as

$$p_\Lambda = \frac{(2m-3)k^2}{(mkt+c_2)^2} - \frac{mc_3}{27}\left[2k(m-3)^2(mkt+c_2)^{\frac{(m-9)}{3}} + mc_3(mkt+c_2)^{\frac{2(m-6)}{3}}\right]. \tag{3.23}$$

The EoS parameter of holographic dark energy is given by

$$w_\Lambda = \frac{27(2m-3)k^2(mkt+c_2)^{-2} - mc_3\left[2k(m-3)^2(mkt+c_2)^{\frac{(m-9)}{3}} + mc_3(mkt+c_2)^{\frac{2(m-6)}{3}}\right]}{81k^2(mkt+c_2)^{-2} - c_3^2m^2(mkt+c_2)^{\frac{2(m-6)}{3}} - 27\rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}}} \tag{3.24}$$

The coincidence parameter $\bar{r} = \rho_m / \rho_\Lambda$ i.e. the ratio of dark matter energy density to the dark energy density is given by

$$\bar{r} = \frac{\rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}}}{3k^2(mkt+c_2)^{-2} - \frac{1}{27}c_3^2m^2(mkt+c_2)^{\frac{2(m-6)}{3}} - \rho_0(mkt+c_2)^{\frac{3(b^2-1)}{m}}}. \tag{3.25}$$

4. State Finder Diagnostic

Sahni *et al.*, 2003 has defined the sate finder parameter pair $\{r, s\}$ as

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}.$$

The state finder pair for the spatially flat Λ CDM scenario corresponds to a fixed point in the diagram $\{s,r\}\big|_{\Lambda CDM} = \{0,1\}$.

The state finder parameters r and s for power-law volumetric expansion model (i.e model for $m \neq 0$) are given by

$$r = (1-m)(1-2m) \quad \text{And} \quad s = \frac{2m}{3}.$$

5. Conclusion

5.1. The Anisotropy Parameter of Expansion (Δ)

In figure-1, we plot anisotropy parameter of expansion Δ against cosmic time t for power-law volumetric expansion model. It is observed that anisotropy decreases and becomes zero after some time.

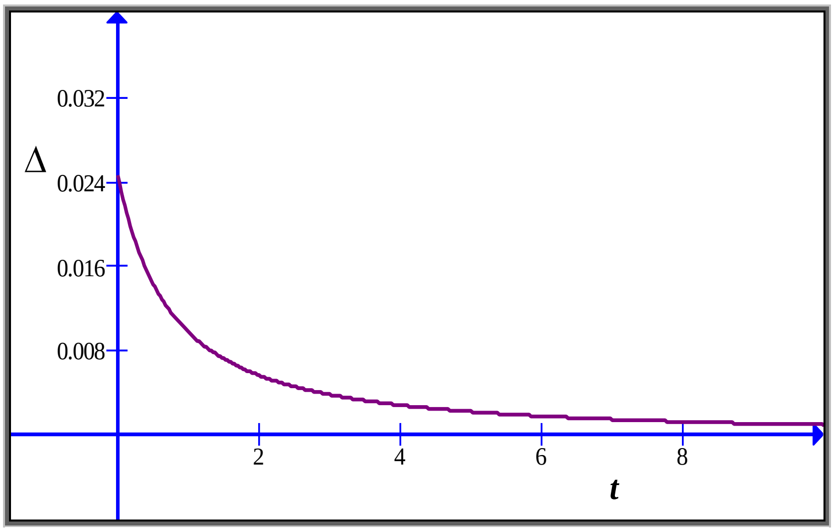

Figure1. *Evolution of anisotropy parameter of expansion* Δ *vs.* t.

Hence, the model reaches to isotropy after some finite time which matches with the recent observations that the universe is isotropic at large scale.

5.2. The Equation of State Parameter (w_Λ)

The figure-2 shows the variation of EoS parameter (w_Λ) with cosmic time *t* for power-law volumetric expansion model. In power-law volumetric expansion model, w_Λ starts phantom region ($w_\Lambda < -1$) and attains the value $w_\Lambda = -1$ after some finite *t*.

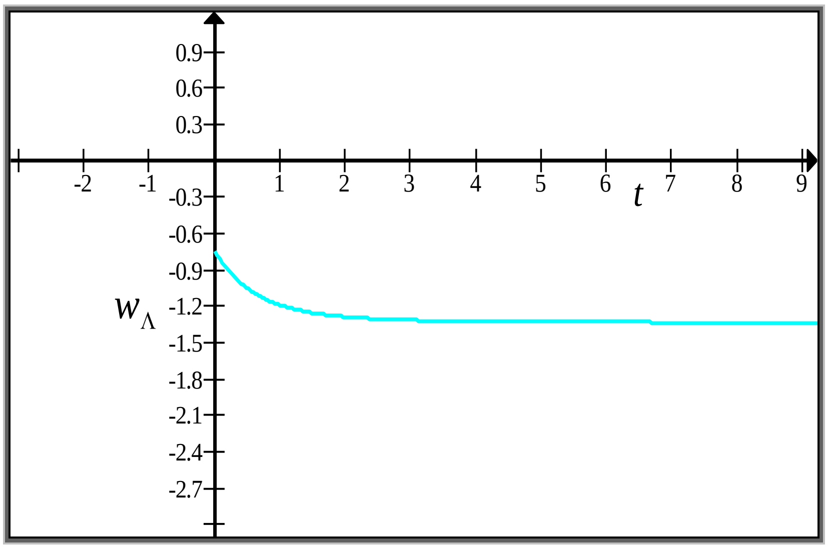

Figure2. *Evolution of EoS parameter (w_Λ).*

Therefore, the model approaches to $\wedge$CDM model after some finite *t*.

5.3. Coincidence Parameter

The variation of coincidence parameter $\bar{r}$ with respect to cosmic time *t* is as shown in figure-3. It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution in exponential volumetric expansion model provided that $b^2 = 1$.

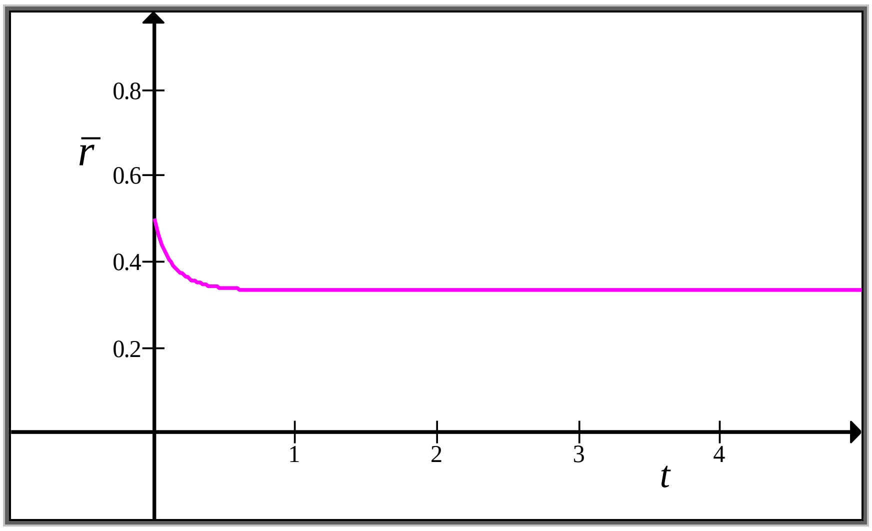

Figure3. *Coincidence parameter $\bar{r}$ verses time t.*

Thus, a suitable kind of interaction between holographic dark energy and dark matter can make the ratio of their densities possible to attain a stationary value during the course of evolution and consequently can help alleviating the coincidence problem which appears in the ΛCDM model.

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