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Thermosolutal-Convective Instability through Porous Medium

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Abstract: *An attempt has been made to investigate the thermosolutal convection of a heterogeneous Rivlin-Ericksen viscoelastic fluid layer through porous medium under linear stability theory. A discussion of different modes revealed that the principle of exchange of stabilities is not valid for the problem. Further, it is found that oscillatory modes exist under certain conditions and non-oscillatory modes are unstable.*

Keywords: *Thermosolutal convection; Heterogeneous Rivlin-Ericksen viscoelastic fluid; Porous medium; Linear stability theory*

1. Introduction

The study of onset of convection in a porous medium has attracted considerable interest because of its natural occurrence and of its intrinsic importance in many industrial problems, particularly in petroleum-exploration, chemical and nuclear industries. The derivation of the basic equations of a layer of fluid heated from below in porous medium, using Boussinesq approximation, has been given by Joseph (1976). The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. The theoretical and experimental results of the onset of thermal instability under varying assumptions of hydrodynamics have been discussed in a treatise by Chandrasekhar (1961) in his celebrated monograph. Lapwood (1948) has studied the stability of convective flow in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding (1960).

The investigation of thermosolutal convection is motivated by its interesting complexities as a double diffusion phenomenon as well as by its direct relevance to geophysics and astrophysics. Stomell et al. (1956) did the pioneering work regarding the investigations of thermosolutal convection in non-porous medium. This work was elaborated in different physical situations by Stern (1960), Veronis (1965) and Nield (1967). The problem of thermosolutal convection in a horizontal layer of saturated porous medium has also been studied by several workers [Nield (1968), Sharma and Sharma (1982)].

In all the above studies, the fluid has been considered to be Newtonian. Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies of waves and stability in different viscoelastic fluid dynamical configuration has been carried out by several researchers in the past. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci (1969). The nature of instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner (1972) have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. In another study, Sharma and Sharma (1977) have considered the thermal instability of a rotating Maxwell fluid through porous medium and found that, for stationary convection, the rotation has stabilizing effect whereas the permeability of the medium has both stabilizing as well as destabilizing effect, depending on the magnitude of rotation. In another study, Sharma (1975) has studied the stability of a layer of an electrically conducting Oldroyd fluid (1958) in the presence of a magnetic field and has found that the magnetic field has a stabilizing influence.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of elastico-viscous fluids is Rivlin-Ericksen fluid. Rivlin and Ericksen (1955) have studied the stress deformation relaxations for isotropic materials. Srivastava and Singh (1988) have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of a time-dependent pressure gradient. In another study, Garg et al. (1994) have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar (1996) have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. In another study, Sharma and Kumar (1999) have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid in the presence of suspended particles.

Keeping in mind the importance in various fields particularly in the soil sciences, ground water hydrology, geophysics, astrophysics and bio-mechanics, the thermosolutal convection of a viscoelastic (Rivlin-Ericksen) incompressible and heterogeneous fluid layer saturated with porous medium, where density is $\rho_0 f(z)$, ρ_0 being a positive constant having the dimension of density, and $f(z)$ is a monotonic function of the vertical coordinate z, with $f(0)=1$ has been considered in the present paper.

2. Formulation of the Problem and Basic Equations

Let us consider an infinite horizontal layer of incompressible and heterogeneous Rivlin-Ericksen viscoelastic fluid of thickness 'd ', in porous medium of porosity ε and medium permeability k_1, bounded by the planes $z=0$ and $z=d$. Let z-axis be vertically upwards. The interstitial fluid (which is the fluid in pores) of variable density is viscous, incompressible and heterogeneous. The initial inhomogenenity in the fluid is assumed to be of the form $\rho_0 f(z)$, where ρ_0 is the density at the lower boundary and $f(z)$ be the function of vertical co-ordinate z such that $f(0)=1$. The fluid layer is infinite in horizontal direction and is heated and soluted from below leading to an adverse temperature gradient $\beta = (T_0 - T_1)/d$ and a uniform solute gradient $\beta' = (S_0 - S_1)/d$ where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ and also S_0 and S_1 are the constant solute concentrations of the lower and upper surfaces with $S_0 > S_1$. The effective density is the superposition of the inhomogeneity described by (a) $\rho = \rho_0 f(z)$, and (b) $\rho = \rho_0[1+\alpha(T_0 - T) - \alpha'(S_0 - S)]$ which is caused by temperature gradient and solute gradients. This leads to the effective density

$$\rho = \rho_0[f(z) + \alpha(T_0 - T) - \alpha'(S_0 - S)], \tag{1}$$

where α and α' are the thermal and solute expansion coefficients.

The relevant Brinkman-Oberbeck-Boussinesq equations describing our problem are:

$$\rho_0 \frac{D\vec{q}}{Dt} = -grad\ p + \rho\vec{g} + \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left[\nabla^2\vec{q} - \frac{1}{k_1}\vec{q}\right], \tag{2}$$

$$div\ \vec{q} = 0, \tag{3}$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0, \tag{4}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = K\nabla^2 T, \qquad (5)$$

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = K'\nabla^2 S, \qquad (6)$$

where $\vec{q}, \mu, \mu', \rho$ and p are the velocity, coefficient of viscosity, viscoelastcity, density and pressure of the fluid, T the temperature, S the solute concentration, $\vec{g}(0, 0, -g)$ is the acceleration due to gravity, K and K' are the thermal and solute diffusivities and k_1 is the intrinsic permeability of the medium ($k_1 \to \infty$ corresponds to non-porous medium).

Here in writing equations (2)-(6), porosity ε ($0 < \varepsilon < 1$ and $\varepsilon \to 1$corresponds to non-porous medium) corrections have not been included for avoiding the involvement of too many constants. In fact it does not affect the essence of discussions of the results. Strictly speaking, a constant factor $E[= \varepsilon(1-\varepsilon)\rho_S C_S / \rho_0 C]$ multiplies in the first term of equation (4) and a term $\frac{1}{\varepsilon}$ multiplies in the velocity term except in the Darcy's resistance term $\left(-\frac{\mu}{k_1}\vec{q}\right)$. Here ρ_S and C_S are respectively the density and heat capacity of the solid material which forms the porous matrix and C is the heat capacity of the liquid. The thermal diffusivity K is defined as $K = \frac{\lambda^*}{\rho_0 C}$ where $\lambda^* = \varepsilon\lambda + (1-\varepsilon)\lambda_S$ is the effective thermal conductivity and λ and λ_S are the thermal conductivities of the fluid and solid respectively. The solute diffusivity K' is defined analogously. Also a factor E' analogous to E is multiplied in the first term of equation (5).

The initial state whose stability is to be examined is characterized by

$$\vec{q} = 0, T = T_0 - \beta z, S = S_0 - \beta' z, \rho = \rho_0[f(z) + \alpha\beta z - \alpha'\beta' z], p = p_0 - \int_0^z g\rho\, dz, \qquad (7)$$

Where p_0 is the pressure at $\rho = \rho_0$.

Let the system be slightly disturbed and as a result of this small perturbation, the various physical quantities undergo a change

$$\begin{aligned} &\vec{q} \to \vec{0} + \delta\vec{q}, T \to T + \theta, S \to S + \gamma, p \to p + \delta p \\ &and\ \rho \to \rho_0[f(z) + \alpha(T_0 - T - \theta) - \alpha'(S_0 - S - \gamma)] + \delta\rho \end{aligned} . \qquad (8)$$

Substituting (8) in equations (1)-(6) and linearizing them by neglecting second and higher terms and retaining only relevant terms appropriate to physical conditions, we obtain the linearized perturbations equations in component form as

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}u - \nabla^2 u\right], \qquad (9)$$

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}v - \nabla^2 v\right], \qquad (10)$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - g(\delta\rho - \alpha\rho_0\theta + \alpha'\rho_0\gamma) - \left(\mu + \mu'\frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}w - \nabla^2 w\right], \qquad (11)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \ , \tag{12}$$

$$\frac{\partial}{\partial x}\delta\rho+\rho_0 w\frac{df}{dz}=0 \ , \tag{13}$$

$$\frac{\partial\theta}{\partial t}-\beta w=K\nabla^2\theta \ , \tag{14}$$

$$\frac{\partial\gamma}{\partial t}-\beta' w=K'\,\nabla^2\gamma \ , \tag{15}$$

Where $\delta\vec{q}=(u,\, v,\, w)$. (16)

3. Analysis in Terms of Normal Modes

The analysis of an arbitrary disturbance is carried out in terms of normal modes following Chandrasekhar (1961). The stability of each of the modes is discussed separately. We seek solutions of the equations (9)-(15) whose dependence on space-time coordinates are of the form

$$[u,\, v,\, w,\, \theta,\, \gamma,\, \delta p,\, \delta\rho]=[U(z), V(z), W(z), \Theta(z), \Gamma(z), L(z), Y(z)]\exp\left[ik_x x+ik_y y+nt\right], \tag{17}$$

Where k_x and k_y are the horizontal wave numbers and n is the frequency of the harmonic disturbances. Also

$$k=\sqrt{\left(k_x^2+k_y^2\right)} \ , \tag{18}$$

gives the wave number of the perturbation propagation.

Using expression (17), equations (9)-(15), on simplification, give

$$-k^2L=\rho_0\left[\left(n+\frac{\nu+\nu' n}{k_1}\right)-(\nu+\nu' n)\left(D^2-k^2\right)\right]DW \ , \tag{19}$$

$$\rho_0\left(n+\frac{\nu+\nu' n}{k_1}\right)W=-DL+g\rho_0\left[\frac{1}{n}\left(\frac{df}{dz}\right)W+\alpha\Theta-\alpha'\,\Gamma\right]+\rho_0(\nu+\nu' n)\left(D^2-k^2\right)W \ , \tag{20}$$

$$n\Theta-\beta W=K\left(D^2-k^2\right)\Theta \ , \tag{21}$$

$$n\Gamma-\beta' W=K'\left(D^2-k^2\right)\Gamma \ , \tag{22}$$

Where $\nu\left(=\dfrac{\mu}{\rho_0}\right)$ and $\nu'\left(=\dfrac{\mu'}{\rho_0}\right)$ are respectively the kinematic viscosity and kinematic viscoelasticity.

Elimination of L from equations (19) and (20) gives

$$n\left[\left(n+\frac{\nu+\nu' n}{k_1}\right)-(\nu+\nu' n)\left(D^2-k^2\right)\right]\left(D^2-k^2\right)W+gk^2\left(\frac{df}{dz}\right)W+g\alpha nk^2\Theta-g\alpha' nk^2\Gamma=0. \tag{23}$$

Equations (21)-(23) in non-dimensional form can be written as

$$\left[D^2-a^2-\sigma p_1\right]\Theta=-\left(\frac{\beta d^2}{K}\right)W \ , \tag{24}$$

$$\left[\tau\left(D^2-a^2\right)-\sigma p_1\right]\Gamma=-\left(\frac{\beta' d^2}{K'}\right)W\,, \tag{25}$$

$$\sigma\nu\left(D^2-a^2\right)\left[\left(1+A\sigma\right)\left(D^2-a^2\right)-\left(\sigma+B+B'A\sigma\right)\right]W-\frac{a^2gd^4}{\nu}\left(\frac{df}{dz}\right)W-g\alpha\sigma\, a^2d^2\Theta$$
$$+g\alpha'\sigma\, a^2d^2\Gamma=0 \tag{26}$$

Here we have put

$\hat{D}=dD,\ \hat{a}=kd,\ \hat{\sigma}=\dfrac{nd^2}{\nu}$ and thereafter dropping the caps for convenience. Also we have put $p_1=\dfrac{\nu}{K},\ \tau=\dfrac{K'}{K},\ A=\dfrac{\nu'}{d^2},\ B=\dfrac{d^2}{k_1},\ R=\dfrac{g\alpha\beta d^4}{K\nu},\ R'=\dfrac{g\alpha'\beta' d^4}{K'\nu},\ R_2=\dfrac{gd^4\left(\dfrac{df}{dz}\right)}{K\nu}$.

(27)

Equation (26) with the help of equations (24) and (25) is written as

$$\sigma\ p_1\left(D^2-a^2\right)\left[D^2-a^2-\sigma\ p_1\right]\left[\tau\left(D^2-a^2\right)-\sigma\ p_1\right]\left[\left(1+A\sigma\right)\left(D^2-a^2\right)-\left(\sigma+B+B'A\sigma\right)\right]W$$
$$-a^2R_2\left[D^2-a^2-\sigma\ p_1\right]\left[\tau\left(D^2-a^2\right)-\sigma\ p_1\right]W+a^2\sigma\ p_1R\left[\tau\left(D^2-a^2\right)-\sigma\ p_1\right]W-a^2\sigma\ p_1R'$$
$$\left[D^2-a^2-\sigma\ p_1\right]W=0. \tag{28}$$

The equations (24)-(26) and (28) are to be solved using boundary conditions. Here we consider the case where both the boundaries are free, following Chandrasekhar (1961), the appropriate boundary conditions for this case are

$$W=D^2W=0,\ \Theta=0,\ \Gamma=0 \text{ at } z=0 \text{ and } z=1. \tag{29}$$

4. Results and Discussion of Marginal States

1. Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Hence the substitution of $\sigma=0$ in equations (21)-(23) gives

$$\left.\begin{aligned}&\left(D^2-a^2\right)\Theta=-\left(\frac{\beta d^2}{K}\right)W\\&\tau\left(D^2-a^2\right)\Gamma=-\left(\frac{\beta' d^2}{K'}\right)W\\&\left(\frac{df}{dz}\right)W=0.\end{aligned}\right\} \tag{30}$$

Integrating equation (30) and using the boundary conditions (29), we see that $W=0,\ \Theta=0,\ \Gamma=0$ etc. is the only possible solutions which led to contradiction to the hypothesis that initial state solutions are perturbed.

Therefore, the instability can not set in as stationary convection or in the other words the principle of exchange of stabilities is not valid for our problem.

2. Oscillatory Convection

Now for the proper solution of equation (28) for W belonging to the lowest mode, we follow Chandrasekhar (1961), and find that $W=W_0\sin\pi z$, where W_0 is constant. Then, from equation (28), we get

$$a^2R - \frac{a^2R'\left[\pi^2 + a^2 + \sigma\ p_1\right]}{\left[\tau\left(\pi^2 + a^2\right) + \sigma\ p_1\right]} + \frac{a^2R_2\left[\pi^2 + a^2 + \sigma\ p_1\right]}{\sigma\ p_1} \tag{31}$$
$$= \left[\pi^2 + a^2\right]\left[\pi^2 + a^2 + \sigma\ p_1\right]\left[(1 + A\sigma)\left(\pi^2 + a^2\right) + \left(\sigma + B + B'A\sigma\right)\right]$$

As discussed earlier, the principle of exchange of stabilities being not valid for the present problem, the marginal state is governed by $\sigma = i\sigma_2'$ where σ_2' is real. Now letting

$$R_3 = \frac{R_2}{\pi^4},\ R_1 = \frac{R}{\pi^4},\ R_4 = \frac{R'}{\pi^4},\ x = \frac{a^2}{\pi^2},\ \sigma_2 = \frac{\sigma_2'}{\pi^2}\ and\ B_1 = \frac{B}{\pi^2}. \tag{32}$$

Substituting (32) in equation (31), we get

$$xR_1 - \frac{xR_4\left[1 + x + i\sigma_2\ p_1\right]}{\left[\tau(1 + x) + i\sigma_2 p_1\right]} + \frac{xR_3\left[1 + x + i\sigma_2\ p_1\right]}{i\sigma_2\ p_1} \tag{33}$$
$$= [1 + x]\left[1 + x + i\sigma_2\ p_1\right]\left[\left(1 + iA\pi^2\sigma_2\right)(1 + x) + \left(i\sigma_2 + B_1 + B_2 i\pi^2\sigma_2\right)\right]$$

Separating equation (33) in real and imaginary parts, we obtain

$$R_1 = \frac{1}{x}\left[\begin{array}{c}(1 + x)^2\left\{1 + x + B_1 - \sigma_2^2 p_1 A\pi^2\right\} - (1 + x)\sigma_2^2 p_1\left\{1 + B_2\pi^2\right\} - xR_3 \\ + \dfrac{xR_4\left\{\tau(1 + x)^2 + \sigma_2^2 p_1^2\right\}}{\left\{\tau^2(1 + x)^2 + \sigma_2^2 p_1^2\right\}}\end{array}\right], \tag{34}$$

and

$$A_0\sigma_2^4 + A_1\sigma_2^2 + A_2 = 0, \tag{35}$$

Where

$$\left.\begin{array}{l} A_0 = p_1^3(1 + x)\left\{\pi^2 A(1 + x)^2 + (1 + x)\left(1 + B_2\pi^2 + p_1\right) + p_1 B_1\right\} \\ A_1 = \tau^2 p_1 (1 + x)^3\left\{\pi^2 A(1 + x)^2 + (1 + x) + B_2\pi^2(1 + x) + p_1(1 + x) + B_1 p_1(1 + x)\right\} \\ \qquad + xp_1^2(1 + x)\left\{R_4(\tau - 1) + R_3\right\} \\ A_2 = xR_3(1 + x)^3\tau^2 \end{array}\right\}. \tag{36}$$

From (35) and (36), the frequency of oscillations σ_2 in marginal state is given by

$$\sigma_2^2 = \frac{-A_1 + \sqrt{\left(A_1^2 - 4A_0A_2\right)}}{2A_0}, \tag{37}$$

and from (27) and (34), Rayleigh number R is given by

$$R = \pi^4 R_1 = \pi^4\left[\begin{array}{l}\dfrac{1}{x}\left\{(1 + x)^2\left(1 + x + B_1 - \sigma_2^2 p_1 A\pi^2\right) - (1 + x)\sigma_2^2 p_1\left(1 + B_1\pi^2\right)\right\} \\ - R_3 + R_4\dfrac{\left\{\tau(1 + x)^2 + \sigma_2^2 p_1^2\right\}}{\left\{\tau^2(1 + x)^2 + \sigma_2^2 p_1^2\right\}}\end{array}\right]. \tag{38}$$

We now discuss the existence of overstable marginal states for various cases:

Case (A): When $R_3 > 0$, i.e. $\frac{df}{dz} > 0$

Since A_0 is always positive and $R_3 > 0$ implies $A_2 > 0$, therefore, if $A_1 > 0$ i.e. $\tau - 1 > 0$ i.e. $K' > K$ then there will be no real σ_2 resulting non-occurrence of overstable marginal state. But, if R_4 satisfies the inequality

$$R_4 > \frac{1}{xp_1^2(1-\tau)}\left[\tau^2 p_1 (1+x)^2 \left\{(1+x)^2 \pi^2 A + (1+x)\left(1 + B_2\pi^2 + p_1\right) + p_1 B_1\right\} + xR_3 p_1^2\right], \qquad (39)$$

besides $K' < K$ and $A_1^2 - 4A_0A_2 > 0$, then the marginal state may exist even when $R_3 > 0$.

Case (B): When $R_3 < 0$, i.e. $\dfrac{df}{dz} < 0$

When $R_3 < 0$, the marginal state always exist whatever be the values of other parameters provided $A_1^2 - 4A_0A_2 > 0$ and then σ_2 is given by equation (37).

3. Nature of Non-Oscillatory Modes

For $R_3 > 0$ i.e. $R_2 > 0$ and $K' > K$, the only modes that may exist are non-oscillatory modes for which $\sigma_2 = 0$ and $\sigma = \sigma_1$ (σ_1 is real). Hence substitution of $\sigma = \sigma_1$ and $W = W_0 \sin \pi z$ in equation (28) gives

$$D_0\sigma_1^4 + D_1\sigma_1^3 + D_2\sigma_1^2 + D_3\sigma_1 + D_4 = 0, \qquad (40)$$

Where

$$\left.\begin{aligned}
D_0 &= p_1^3\left(\pi^2 + a^2\right)\left[A\left(\pi^2 + a^2\right) + 1 + B'A\right] \\
D_1 &= p_1^2\left(\pi^2 + a^2\right)^2\left[A\left(\pi^2 + a^2\right) + 1 + B'A\right](1+\tau) + p_1^3\left(\pi^2 + a^2\right)\left[\left(\pi^2 + a^2\right) + B\right] \\
D_2 &= p_1\left(\pi^2 + a^2\right)^3\left[\tau\left\{A\left(\pi^2 + a^2\right) + 1 + B'A\right\} + p_1(1+\tau)\right] + p_1^2\left(\pi^2 + a^2\right)^2 B(1+\tau) \\
&\quad - a^2 p_1^2\left(R_2 + R - R'\right) \\
D_3 &= p_1\tau\left(\pi^2 + a^2\right)^3\left\{\left(\pi^2 + a^2\right) + B\right\} - a^2 p_1\left(\pi^2 + a^2\right)\left\{R_2\tau + R\tau - R'\right\} - a^2 R_2\tau\, p_1 \\
D_4 &= -a^2\left(\pi^2 + a^2\right)R_2\tau^2
\end{aligned}\right\} \qquad (41)$$

Equation (40) is the fourth degree characteristic equation in σ_1 with real coefficients and has four roots, which may be real. The constant term in the characteristic equation being negative, at least two of the roots are real, one positive and one negative. Thus, we have non-oscillatory modes, one of which essentially grows in time making the system unstable.

5. Conclusions

The thermosolutal convection in a layer of heterogeneous Rivlin-Ericksen viscoelastic fluid heated and soluted from below through porous medium is considered in the present paper. The investigation of thermosolutal convection is motivated by its complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. Thermosolutal convection problems arise in oceanography, limnology and engineering. Ponds built to trap solar heat and some Antarctic lakes provide examples of particular interest. The main conclusions from the analysis of this paper are as follows:

- The principle of exchange of stabilities is not valid for this problem.
- Frequency of oscillation and the Rayleigh number in the marginal state are given by equations (35) and (34), respectively.
- For density distribution with positive gradient and $K' > K$, the over stable marginal state do not exist and we have only non-oscillatory modes which make the system unstable.

- While for positive density gradient and $K' < K$, the over stability may occur for the solute Rayleigh numbers satisfying (39).
- For density distribution with negative gradient, the marginal state and overstable solution exist, irrespective of the values of other parameters.

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